Computer Algebra Independent Integration Tests

Summer 2023 edition

5-Inverse-trig-functions/5.2-Inverse-cosine/145-5.2.2-d-x-^m-a+b-arccos-c-x-^n

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September 5, 2023 Compiled on September 5, 2023 at 11:55am

Contents

CHAPTER 1

INTRODUCTION

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [227]. This is test number [145].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (227)	0.00 (0)
Mathematica	98.24 (223)	1.76 (4)
Maple	94.71 (215)	5.29 (12)
Giac	71.81 (163)	28.19 (64)
Sympy	44.49 (101)	55.51 (126)
Fricas	37.44 (85)	62.56 (142)
Maxima	33.04 (75)	66.96 (152)
Mupad	32.16 (73)	67.84 (154)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

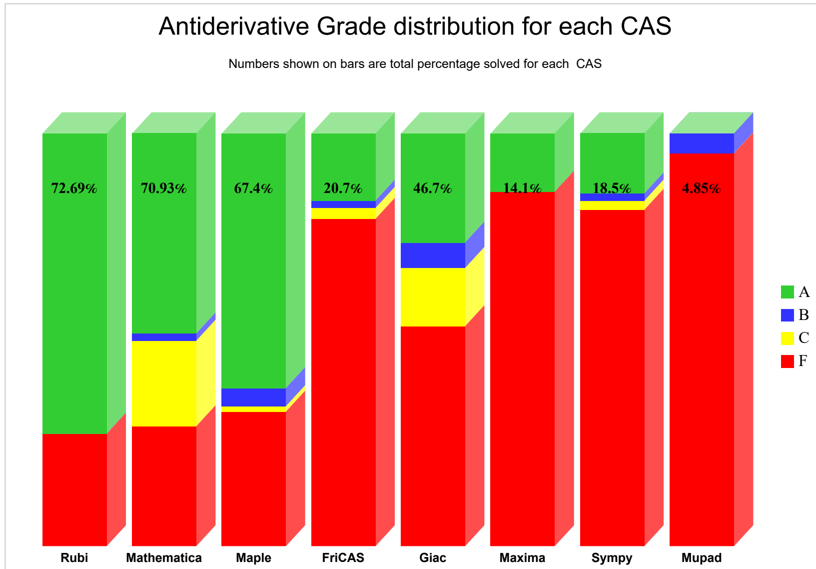
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

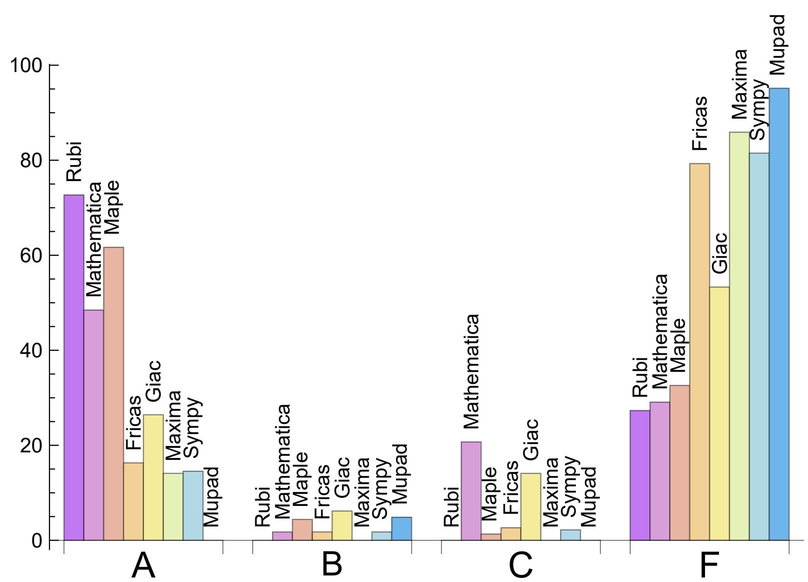
System	% A grade	% B grade	% C grade	% F grade
Rubi	72.687	0.000	0.000	27.313
Maple	61.674	4.405	1.322	32.599
Mathematica	48.458	1.762	20.705	29.075
Giac	26.432	6.167	14.097	53.304
Fricas	16.300	1.762	2.643	79.295
Sympy	14.537	1.762	2.203	81.498
Maxima	14.097	0.000	0.000	85.903
Mupad	0.000	4.846	0.000	95.154

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	4	100.00	0.00	0.00
Maple	12	100.00	0.00	0.00
Giac	64	79.69	0.00	20.31
Fricas	142	44.37	0.00	55.63
Sympy	126	91.27	1.59	7.14
Maxima	152	60.53	0.00	39.47
Mupad	154	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.10
Fricas	0.24
Mupad	0.27
Giac	0.47
Maxima	0.90
Maple	1.04
Mathematica	3.44
Sympy	7.26

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	21.11	1.07	16.00	1.00
Fricas	53.99	1.30	48.00	1.14
Sympy	54.93	1.09	17.00	1.00
Rubi	86.04	1.00	75.00	1.00
Maple	96.86	1.05	66.00	0.95
Mathematica	100.98	1.13	68.00	1.11
Maxima	109.77	5.56	69.00	1.00
Giac	173.02	1.79	56.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

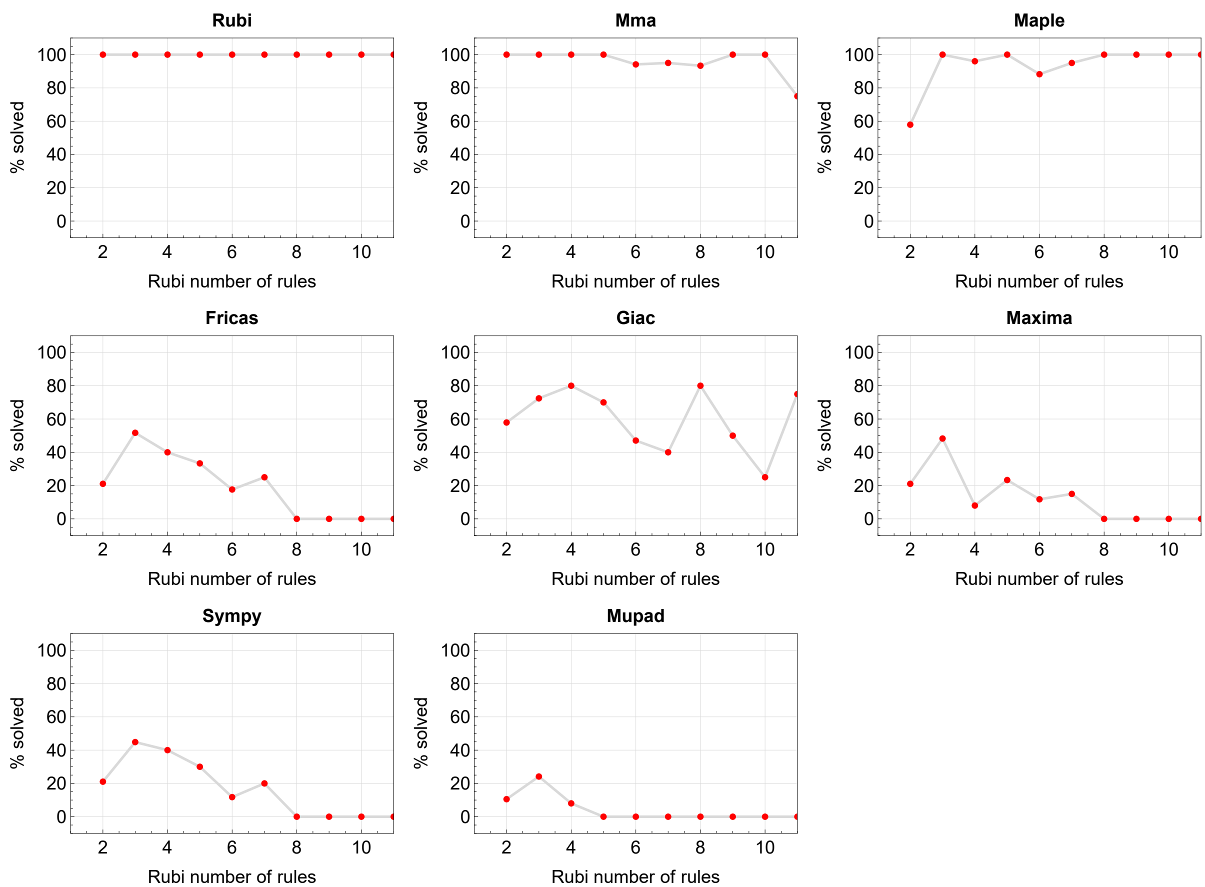


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

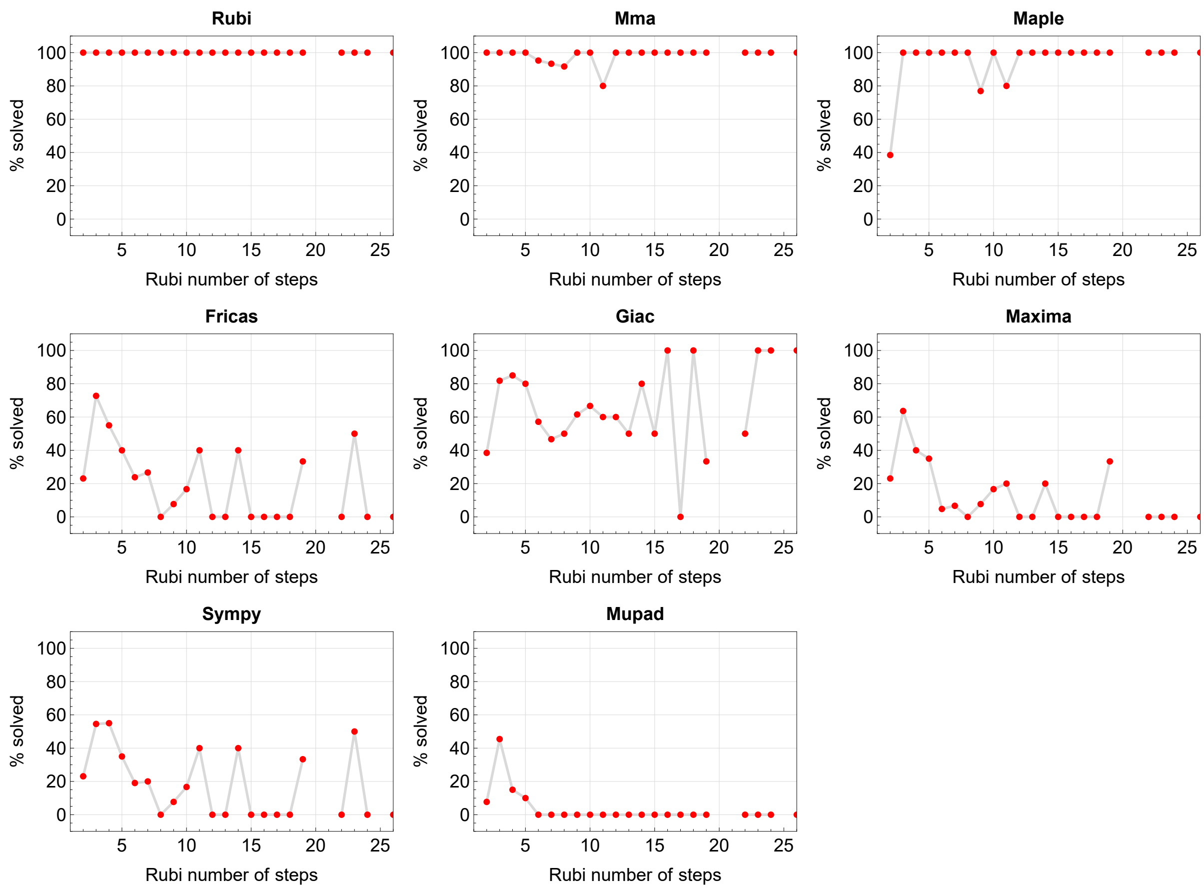


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

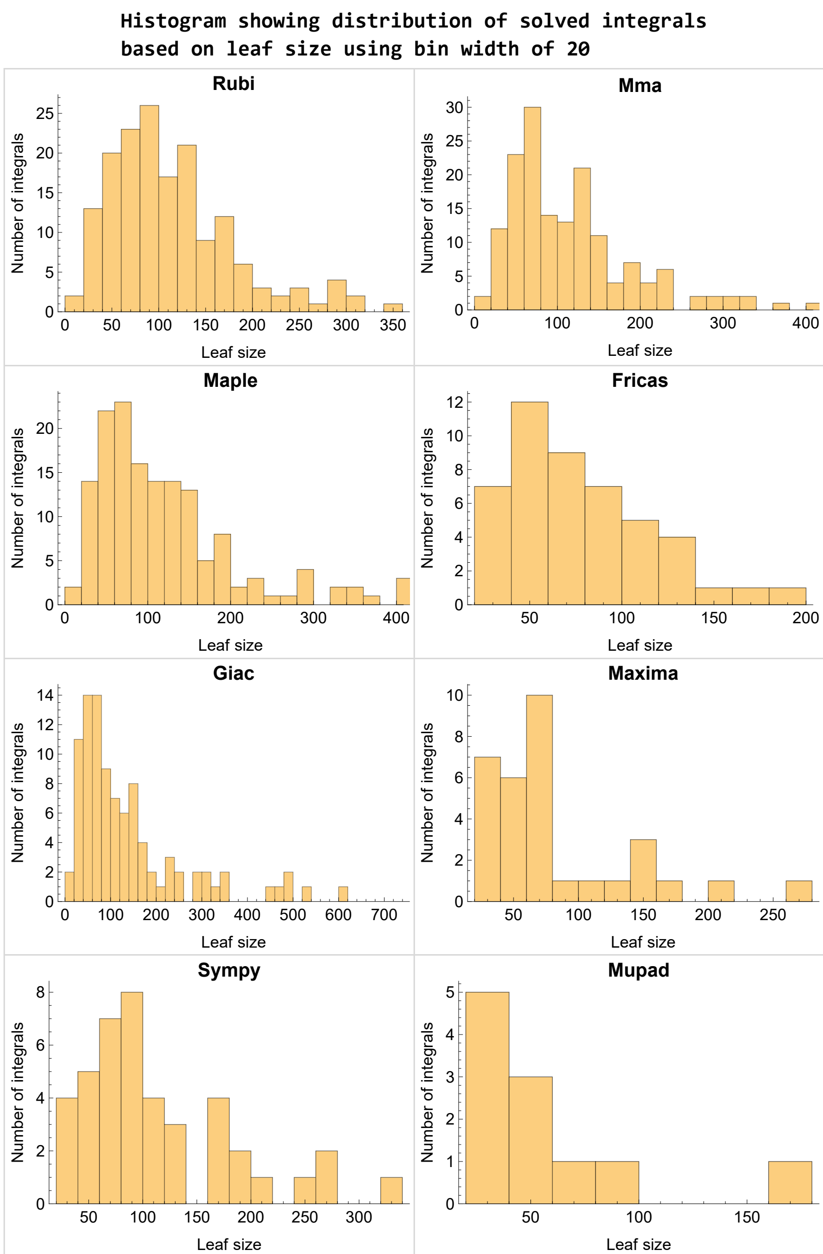


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

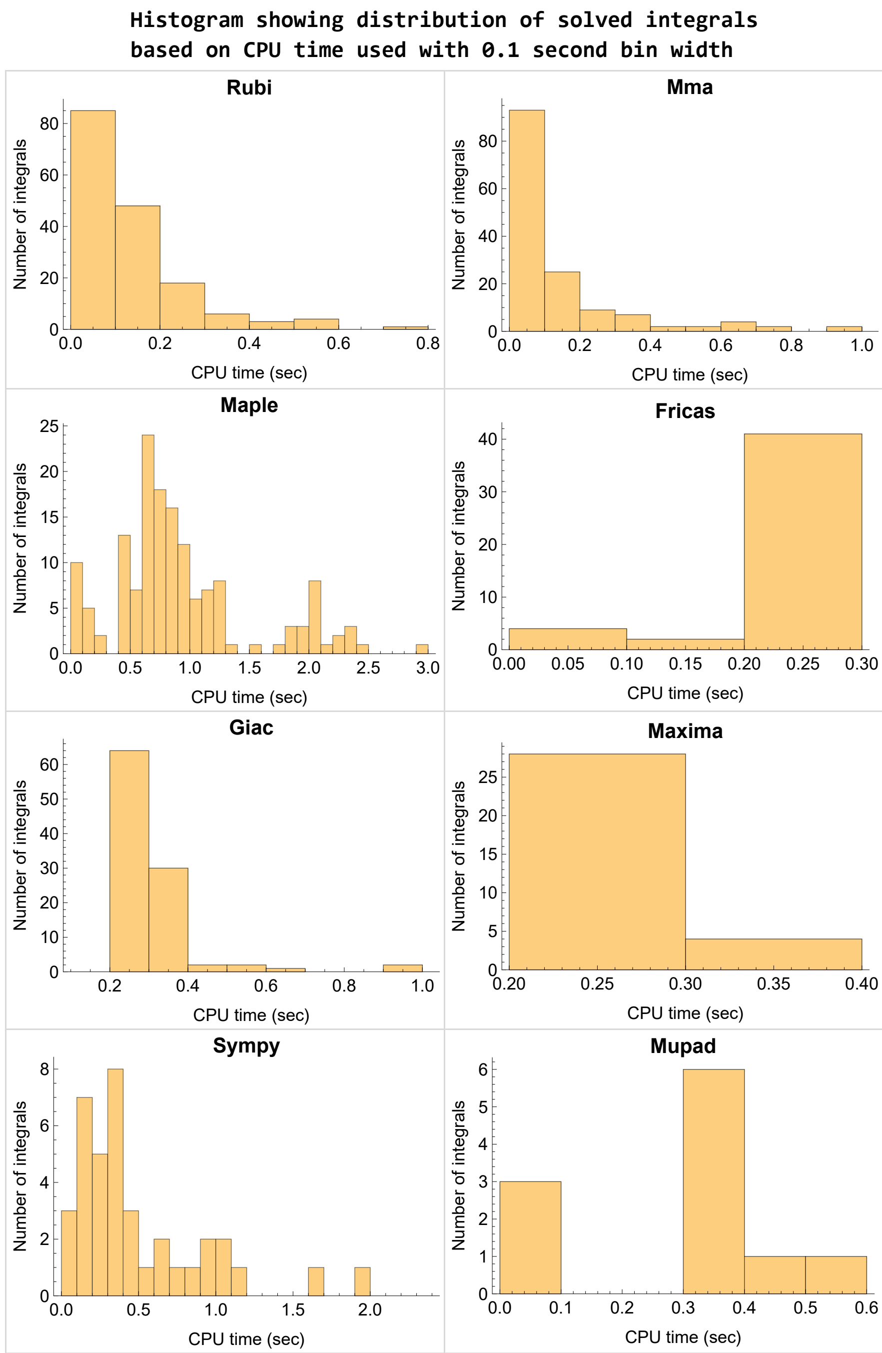


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

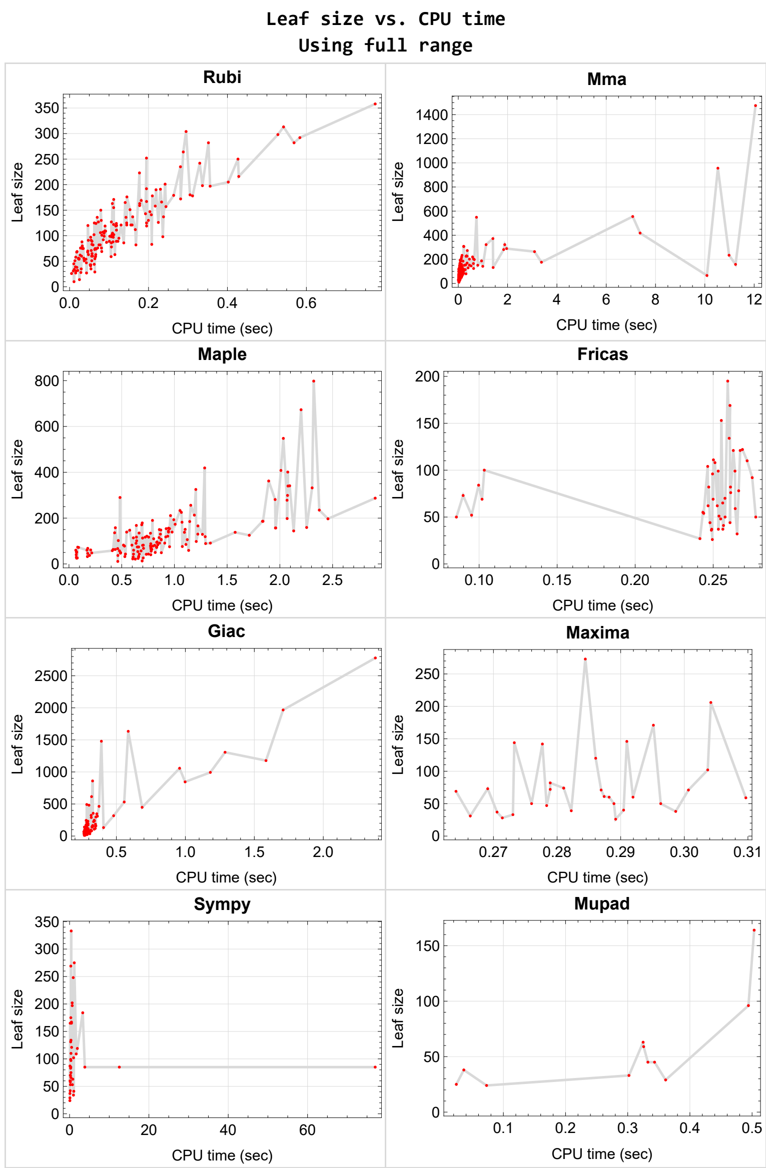


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{49, 50, 58, 59, 65, 66, 72, 73, 79, 85, 91, 97, 98, 106, 112, 118, 119, 120, 123, 124, 125, 126, 127, 128, 129, 134, 135, 136, 137, 138, 139, 161, 162, 166, 167, 171, 172, 176, 177, 181, 182, 186, 187, 191, 192, 196, 197, 201, 202, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

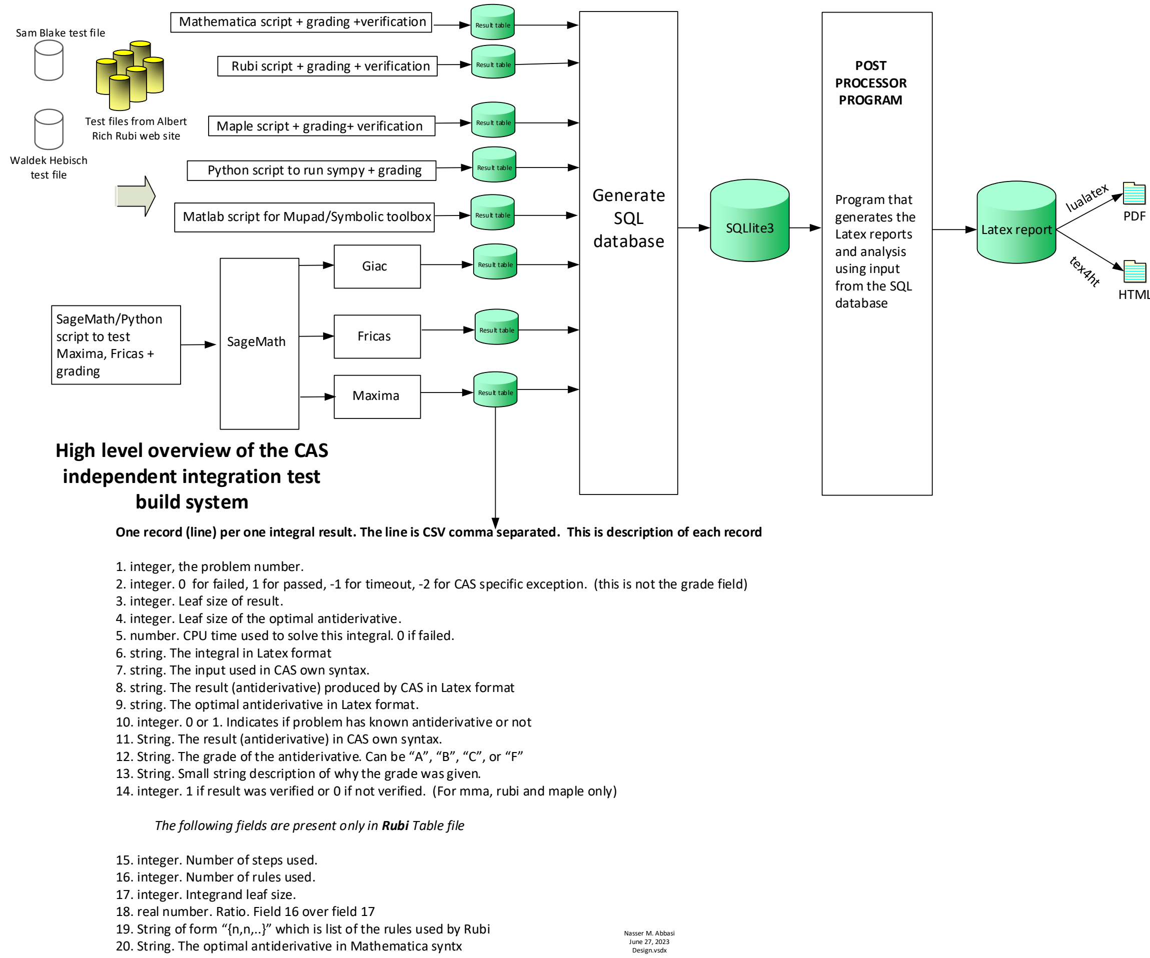
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 121, 122, 130, 131, 132, 133, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 77, 83, 89, 95, 104, 110, 116, 122, 130, 131, 132, 133, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 158, 159, 160, 163, 164, 165, 168, 169, 170, 174, 179, 184, 189, 210, 211, 212, 213, 214 }

B grade { 39, 41, 157, 209 }

C grade { 74, 75, 76, 78, 80, 81, 82, 84, 86, 87, 88, 90, 92, 93, 94, 96, 99, 100, 101, 102, 103, 105, 107, 108, 109, 111, 113, 114, 115, 117, 121, 173, 175, 178, 180, 183, 185, 188, 190, 193, 198, 203, 204, 205, 206, 207, 208 }

F normal fail { 194, 195, 199, 200 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 188, 189, 190, 193, 194, 195, 203, 204, 205, 206, 207, 208 }

B grade { 156, 178, 179, 180, 183, 184, 185, 198, 199, 200 }

C grade { 130, 132, 133 }

F normal fail { 28, 39, 121, 122, 131, 157, 209, 210, 211, 212, 213, 214 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 8, 10, 11, 12, 13, 14, 15, 16, 19, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 140, 141, 142, 143, 146, 148, 149, 150, 153, 154, 155 }

B grade { 7, 9, 145, 147 }

C grade { 203, 204, 205, 206, 207, 208 }

F normal fail { 6, 17, 18, 20, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 121, 122, 130, 131, 132, 133, 144, 151, 152, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 209, 210, 211, 212, 213, 214 }

F(-1) timedout fail { }

F(-2) exception fail { 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 125, 126, 127, 128, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202 }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 14, 16, 19, 21, 22, 24, 26, 33, 35, 37, 140, 141, 142, 143, 145, 146, 147, 148, 150, 153, 155 }

B grade { }

C grade { }

F normal fail { 6, 13, 15, 17, 18, 20, 23, 25, 27, 28, 29, 30, 31, 32, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 121, 122, 144, 149, 151, 152, 154, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214 }

F(-1) timedout fail { }

F(-2) exception fail { 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139 }

Giac

A grade { 1, 2, 3, 4, 5, 7, 9, 11, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 140, 141, 142, 143, 148, 149, 150, 153, 155, 158, 159, 160 }

B grade { 8, 10, 19, 21, 145, 146, 147, 154, 163, 164, 165, 168, 169, 170 }

C grade { 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190 }

F normal fail { 6, 17, 18, 20, 27, 28, 29, 30, 38, 39, 40, 41, 99, 101, 103, 104, 105, 107, 109, 110, 111, 113, 115, 116, 117, 121, 122, 130, 131, 132, 133, 144, 151, 152, 156, 157, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 212, 213, 214 }

F(-1) timedout fail { }

F(-2) exception fail { 31, 100, 102, 108, 114, 161, 196, 201, 209, 210, 211, 215, 216 }

Mupad

A grade { }

B grade { 4, 5, 7, 16, 26, 37, 142, 143, 145, 150, 155 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 6, 8, 9, 10, 11, 12, 13, 14, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 86, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 121, 122, 130, 131, 132, 133, 140, 141, 144, 146, 147, 148, 149, 151, 152, 153, 154, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 12, 13, 14, 15, 16, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 140, 141, 142, 143, 145, 146, 147, 148, 153, 203, 204, 205 }

B grade { 149, 150, 154, 155 }

C grade { 7, 8, 9, 10, 11 }

F normal fail { 6, 17, 18, 19, 20, 21, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 53, 54, 55, 56, 57, 60, 61, 62, 63, 64, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 80, 81, 82, 83, 84, 87, 88, 89, 90, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 121, 122, 130, 131, 132, 133, 144, 151, 152, 156, 157, 158, 159, 160, 163, 164, 165, 168, 169, 170, 173, 174, 175, 178, 179, 180, 183, 184, 185, 188, 189, 190, 193, 194, 195, 198, 199, 200, 210, 211 }

F(-1) timedout fail { 86, 209 }

F(-2) exception fail { 206, 207, 208, 212, 213, 214, 217, 218, 219 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	51	72	71	50	75	67	0
N.S.	1	1.00	0.68	0.96	0.95	0.67	1.00	0.89	0.00
time (sec)	N/A	0.032	0.026	0.091	0.301	0.277	0.343	0.269	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	54	60	61	48	66	57	0
N.S.	1	1.00	0.78	0.87	0.88	0.70	0.96	0.83	0.00
time (sec)	N/A	0.019	0.025	0.065	0.287	0.255	0.328	0.274	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	42	52	50	41	53	47	0
N.S.	1	1.00	0.78	0.96	0.93	0.76	0.98	0.87	0.00
time (sec)	N/A	0.024	0.020	0.072	0.296	0.257	0.205	0.278	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	42	40	40	37	42	37	38
N.S.	1	1.00	0.93	0.89	0.89	0.82	0.93	0.82	0.84
time (sec)	N/A	0.011	0.013	0.069	0.290	0.256	0.169	0.274	0.036

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	26	25	26	26	24	26	24
N.S.	1	1.00	1.00	0.96	1.00	1.00	0.92	1.00	0.92
time (sec)	N/A	0.005	0.006	0.073	0.289	0.249	0.076	0.269	0.073

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	51	68	0	0	0	0	0
N.S.	1	1.00	1.00	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.037	0.014	0.707	0.000	0.000	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	34	26	38	82	34	48	25
N.S.	1	1.00	1.26	0.96	1.41	3.04	1.26	1.78	0.93
time (sec)	N/A	0.015	0.010	0.079	0.299	0.261	0.983	0.279	0.024

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	29	28	27	53	68	0
N.S.	1	1.00	0.91	0.85	0.82	0.79	1.56	2.00	0.00
time (sec)	N/A	0.012	0.014	0.070	0.271	0.242	0.719	0.286	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	67	50	60	110	109	77	0
N.S.	1	1.00	1.20	0.89	1.07	1.96	1.95	1.38	0.00
time (sec)	N/A	0.023	0.017	0.073	0.288	0.272	1.675	0.267	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	41	52	50	37	102	130	0
N.S.	1	1.00	0.71	0.90	0.86	0.64	1.76	2.24	0.00
time (sec)	N/A	0.016	0.020	0.073	0.276	0.253	1.006	0.278	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	C	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	72	73	82	122	184	101	0
N.S.	1	1.00	0.90	0.91	1.02	1.52	2.30	1.26	0.00
time (sec)	N/A	0.032	0.048	0.083	0.279	0.269	3.320	0.270	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	82	76	102	76	121	100	0
N.S.	1	1.00	0.68	0.63	0.85	0.63	1.01	0.83	0.00
time (sec)	N/A	0.119	0.043	1.075	0.304	0.261	0.516	0.289	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	74	91	0	70	97	87	0
N.S.	1	1.00	0.76	0.93	0.00	0.71	0.99	0.89	0.00
time (sec)	N/A	0.106	0.027	0.908	0.000	0.258	0.335	0.297	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	63	59	72	59	83	68	0
N.S.	1	1.00	0.77	0.72	0.88	0.72	1.01	0.83	0.00
time (sec)	N/A	0.078	0.033	1.135	0.279	0.264	0.265	0.293	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	57	63	0	51	58	55	0
N.S.	1	1.00	0.95	1.05	0.00	0.85	0.97	0.92	0.00
time (sec)	N/A	0.058	0.019	0.419	0.000	0.254	0.197	0.294	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	37	33	36	37	33	45
N.S.	1	1.00	1.00	1.06	0.94	1.03	1.06	0.94	1.29
time (sec)	N/A	0.030	0.015	0.464	0.273	0.249	0.090	0.274	0.343

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	73	101	0	0	0	0	0
N.S.	1	1.00	1.00	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.064	0.016	0.642	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	98	136	0	0	0	0	0
N.S.	1	1.00	1.32	1.84	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.067	0.111	0.433	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	43	47	39	44	0	82	0
N.S.	1	1.00	1.00	1.09	0.91	1.02	0.00	1.91	0.00
time (sec)	N/A	0.052	0.021	0.488	0.282	0.261	0.000	0.320	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	152	166	0	0	0	0	0
N.S.	1	1.00	1.23	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	0.452	1.223	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	69	82	74	62	0	185	0
N.S.	1	1.00	0.79	0.94	0.85	0.71	0.00	2.13	0.00
time (sec)	N/A	0.094	0.031	0.517	0.281	0.253	0.000	0.348	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	122	159	171	104	202	175	0
N.S.	1	1.00	0.61	0.79	0.85	0.52	1.00	0.87	0.00
time (sec)	N/A	0.242	0.044	2.254	0.295	0.246	0.657	0.285	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	115	151	0	96	167	141	0
N.S.	1	1.00	0.69	0.90	0.00	0.57	1.00	0.84	0.00
time (sec)	N/A	0.195	0.052	1.102	0.000	0.250	0.477	0.268	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	95	106	120	78	134	117	0
N.S.	1	1.00	0.70	0.78	0.88	0.57	0.99	0.86	0.00
time (sec)	N/A	0.143	0.038	1.122	0.286	0.266	0.352	0.295	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	85	90	0	69	99	83	0
N.S.	1	1.00	0.86	0.91	0.00	0.70	1.00	0.84	0.00
time (sec)	N/A	0.102	0.030	0.636	0.000	0.250	0.274	0.275	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	57	59	44	60	56	59
N.S.	1	1.00	1.00	0.95	0.98	0.73	1.00	0.93	0.98
time (sec)	N/A	0.056	0.016	0.479	0.310	0.248	0.117	0.269	0.326

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	101	135	0	0	0	0	0
N.S.	1	1.00	1.00	1.34	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.082	0.018	0.665	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	139	0	0	0	0	0	0
N.S.	1	1.00	1.14	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.111	0.084	0.000	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	92	117	0	0	0	0	0
N.S.	1	1.00	0.90	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.112	0.155	0.756	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	165	256	0	0	0	0	0
N.S.	1	1.00	0.86	1.33	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.612	1.155	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	151	190	0	0	0	0	0
N.S.	1	1.00	0.89	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.182	0.245	0.792	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	167	332	0	153	275	245	0
N.S.	1	1.00	0.59	1.18	0.00	0.54	0.98	0.87	0.00
time (sec)	N/A	0.569	0.064	2.306	0.000	0.255	1.184	0.281	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	150	197	206	134	248	212	0
N.S.	1	1.00	0.60	0.79	0.82	0.54	0.99	0.85	0.00
time (sec)	N/A	0.427	0.051	2.456	0.304	0.260	0.923	0.285	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	135	213	0	121	197	173	0
N.S.	1	1.00	0.68	1.08	0.00	0.61	0.99	0.87	0.00
time (sec)	N/A	0.337	0.046	1.189	0.000	0.267	0.667	0.280	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	114	130	146	99	165	140	0
N.S.	1	1.00	0.69	0.78	0.88	0.60	0.99	0.84	0.00
time (sec)	N/A	0.233	0.062	1.222	0.291	0.264	0.473	0.285	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	96	113	0	82	110	101	0
N.S.	1	1.00	0.86	1.01	0.00	0.73	0.98	0.90	0.00
time (sec)	N/A	0.167	0.030	0.698	0.000	0.247	0.354	0.293	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	67	74	55	70	65	63
N.S.	1	1.00	1.00	0.97	1.07	0.80	1.01	0.94	0.91
time (sec)	N/A	0.081	0.021	0.462	0.281	0.243	0.184	0.274	0.325

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	119	168	0	0	0	0	0
N.S.	1	1.00	1.00	1.41	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.017	0.639	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	176	176	549	0	0	0	0	0	0
N.S.	1	1.00	3.12	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	0.738	0.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	115	150	0	0	0	0	0
N.S.	1	1.00	0.95	1.24	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.145	0.283	0.857	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	1475	419	0	0	0	0	0
N.S.	1	1.00	4.85	1.38	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.295	12.057	1.287	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	40	40	0	0	0	47	0
N.S.	1	1.00	0.73	0.73	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.064	0.068	0.776	0.000	0.000	0.000	0.281	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	33	33	0	0	0	37	0
N.S.	1	1.00	0.77	0.77	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.054	0.073	0.661	0.000	0.000	0.000	0.267	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	31	31	0	0	0	35	0
N.S.	1	1.00	0.76	0.76	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.053	0.053	0.662	0.000	0.000	0.000	0.265	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	24	24	0	0	0	25	0
N.S.	1	1.00	0.83	0.83	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.064	0.056	0.656	0.000	0.000	0.000	0.285	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	27	27	20	22	0	0	0	23	0
N.S.	1	1.00	0.74	0.81	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.043	0.038	0.605	0.000	0.000	0.000	0.289	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	14	13	0	0	0	12	0
N.S.	1	1.00	1.00	0.93	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.025	0.022	0.694	0.000	0.000	0.000	0.274	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	10	11	0	0	0	10	0
N.S.	1	1.00	1.00	1.10	0.00	0.00	0.00	1.00	0.00
time (sec)	N/A	0.011	0.027	0.463	0.000	0.000	0.000	0.267	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	8	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	0.80	1.20	1.20
time (sec)	N/A	0.010	0.182	3.198	0.329	0.233	0.310	0.289	0.256

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	12	12	10	12	12
N.S.	1	1.00	1.20	1.00	1.20	1.20	1.00	1.20	1.20
time (sec)	N/A	0.011	0.792	1.747	0.335	0.228	0.331	0.322	0.254

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	86	105	0	0	0	72	0
N.S.	1	1.00	1.05	1.28	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.059	0.138	0.875	0.000	0.000	0.000	0.286	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	63	78	0	0	0	62	0
N.S.	1	1.00	0.90	1.11	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.046	0.140	0.796	0.000	0.000	0.000	0.276	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	61	81	0	0	0	60	0
N.S.	1	1.00	0.90	1.19	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.046	0.148	0.698	0.000	0.000	0.000	0.282	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	50	54	0	0	0	50	0
N.S.	1	1.00	0.89	0.96	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.036	0.127	0.653	0.000	0.000	0.000	0.275	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	57	0	0	0	48	0
N.S.	1	1.00	0.93	1.06	0.00	0.00	0.00	0.89	0.00
time (sec)	N/A	0.037	0.109	0.684	0.000	0.000	0.000	0.280	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	37	30	0	0	0	36	0
N.S.	1	1.00	0.97	0.79	0.00	0.00	0.00	0.95	0.00
time (sec)	N/A	0.019	0.075	0.609	0.000	0.000	0.000	0.283	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	35	32	0	0	0	33	0
N.S.	1	1.00	1.00	0.91	0.00	0.00	0.00	0.94	0.00
time (sec)	N/A	0.054	0.038	0.530	0.000	0.000	0.000	0.273	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	127	12	10	12	12
N.S.	1	1.00	1.20	1.00	12.70	1.20	1.00	1.20	1.20
time (sec)	N/A	0.009	1.140	4.814	0.536	0.239	0.395	0.307	0.244

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	136	12	12	12	12
N.S.	1	1.00	1.20	1.00	13.60	1.20	1.20	1.20	1.20
time (sec)	N/A	0.009	14.827	1.755	0.647	0.238	0.441	0.362	0.254

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	103	121	0	0	0	86	0
N.S.	1	1.00	1.05	1.23	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.236	0.100	0.739	0.000	0.000	0.000	0.297	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	70	82	0	0	0	75	0
N.S.	1	1.00	0.84	0.99	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.208	0.112	0.597	0.000	0.000	0.000	0.290	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	65	82	0	0	0	72	0
N.S.	1	1.00	0.79	1.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.168	0.103	0.705	0.000	0.000	0.000	0.302	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	63	43	0	0	0	57	0
N.S.	1	1.00	1.00	0.68	0.00	0.00	0.00	0.90	0.00
time (sec)	N/A	0.115	0.037	0.612	0.000	0.000	0.000	0.299	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	47	43	0	0	0	43	0
N.S.	1	1.00	0.92	0.84	0.00	0.00	0.00	0.84	0.00
time (sec)	N/A	0.056	0.034	0.536	0.000	0.000	0.000	0.274	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	124	12	10	12	12
N.S.	1	1.00	1.20	1.00	12.40	1.20	1.00	1.20	1.20
time (sec)	N/A	0.010	0.611	3.166	1.384	0.236	0.506	0.314	0.253

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	143	12	12	12	12
N.S.	1	1.00	1.20	1.00	14.30	1.20	1.20	1.20	1.20
time (sec)	N/A	0.009	7.671	1.667	1.703	0.235	0.592	0.388	0.245

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	159	171	0	0	0	138	0
N.S.	1	1.00	1.01	1.08	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.217	0.130	0.690	0.000	0.000	0.000	0.283	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	107	114	0	0	0	125	0
N.S.	1	1.00	0.75	0.80	0.00	0.00	0.00	0.87	0.00
time (sec)	N/A	0.192	0.219	0.611	0.000	0.000	0.000	0.281	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	112	117	0	0	0	121	0
N.S.	1	1.00	0.79	0.83	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.207	0.130	0.640	0.000	0.000	0.000	0.280	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	86	60	0	0	0	83	0
N.S.	1	1.00	0.89	0.62	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.111	0.086	0.613	0.000	0.000	0.000	0.280	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	71	63	0	0	0	66	0
N.S.	1	1.00	0.91	0.81	0.00	0.00	0.00	0.85	0.00
time (sec)	N/A	0.106	0.041	0.438	0.000	0.000	0.000	0.264	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	200	12	10	12	12
N.S.	1	1.00	1.20	1.00	20.00	1.20	1.00	1.20	1.20
time (sec)	N/A	0.009	3.525	3.005	4.424	0.230	0.578	0.317	0.246

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	229	12	12	12	12
N.S.	1	1.00	1.20	1.00	22.90	1.20	1.20	1.20	1.20
time (sec)	N/A	0.009	17.336	1.615	5.042	0.240	0.802	0.436	0.255

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	194	143	0	0	0	247	0
N.S.	1	1.00	1.60	1.18	0.00	0.00	0.00	2.04	0.00
time (sec)	N/A	0.155	0.084	0.943	0.000	0.000	0.000	0.339	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	131	91	0	0	0	153	0
N.S.	1	1.00	1.38	0.96	0.00	0.00	0.00	1.61	0.00
time (sec)	N/A	0.125	0.057	0.859	0.000	0.000	0.000	0.328	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	128	96	0	0	0	165	0
N.S.	1	1.00	1.49	1.12	0.00	0.00	0.00	1.92	0.00
time (sec)	N/A	0.137	0.070	0.835	0.000	0.000	0.000	0.328	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	49	43	0	0	0	71	0
N.S.	1	1.00	0.83	0.73	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.105	0.027	0.667	0.000	0.000	0.000	0.312	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	69	49	0	0	0	83	0
N.S.	1	1.00	1.57	1.11	0.00	0.00	0.00	1.89	0.00
time (sec)	N/A	0.062	0.023	0.876	0.000	0.000	0.000	0.298	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	12	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	1.00	1.00
time (sec)	N/A	0.009	0.204	1.050	0.000	0.000	0.401	0.454	0.255

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	282	282	185	193	0	0	0	355	0
N.S.	1	1.00	0.66	0.68	0.00	0.00	0.00	1.26	0.00
time (sec)	N/A	0.352	0.094	0.994	0.000	0.000	0.000	0.332	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	128	118	0	0	0	225	0
N.S.	1	1.00	0.82	0.75	0.00	0.00	0.00	1.43	0.00
time (sec)	N/A	0.244	0.054	0.918	0.000	0.000	0.000	0.334	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	125	130	0	0	0	237	0
N.S.	1	1.00	0.85	0.88	0.00	0.00	0.00	1.61	0.00
time (sec)	N/A	0.203	0.076	0.867	0.000	0.000	0.000	0.341	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	64	64	0	0	0	107	0
N.S.	1	1.00	0.72	0.72	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.122	0.048	0.786	0.000	0.000	0.000	0.340	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	66	72	0	0	0	119	0
N.S.	1	1.00	0.88	0.96	0.00	0.00	0.00	1.59	0.00
time (sec)	N/A	0.065	0.027	0.776	0.000	0.000	0.000	0.347	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	12	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	1.00	1.00
time (sec)	N/A	0.011	0.204	1.045	0.000	0.000	1.526	0.533	0.245

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F(-1)	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	298	298	194	233	0	0	0	463	0
N.S.	1	1.00	0.65	0.78	0.00	0.00	0.00	1.55	0.00
time (sec)	N/A	0.528	0.081	1.053	0.000	0.000	0.000	0.374	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	131	154	0	0	0	297	0
N.S.	1	1.00	0.64	0.75	0.00	0.00	0.00	1.45	0.00
time (sec)	N/A	0.402	0.055	0.954	0.000	0.000	0.000	0.351	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	128	156	0	0	0	309	0
N.S.	1	1.00	0.72	0.88	0.00	0.00	0.00	1.74	0.00
time (sec)	N/A	0.312	0.065	0.944	0.000	0.000	0.000	0.362	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	73	79	0	0	0	143	0
N.S.	1	1.00	0.61	0.66	0.00	0.00	0.00	1.20	0.00
time (sec)	N/A	0.195	0.069	0.773	0.000	0.000	0.000	0.326	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	69	88	0	0	0	155	0
N.S.	1	1.00	0.78	1.00	0.00	0.00	0.00	1.76	0.00
time (sec)	N/A	0.110	0.023	0.776	0.000	0.000	0.000	0.348	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	10	12	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	0.83	1.00	1.00
time (sec)	N/A	0.010	0.200	1.146	0.000	0.000	19.287	0.553	0.244

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	192	72	0	0	0	139	0
N.S.	1	1.00	1.81	0.68	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.078	0.078	0.831	0.000	0.000	0.000	0.323	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	130	43	0	0	0	81	0
N.S.	1	1.00	2.00	0.66	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.057	0.054	0.753	0.000	0.000	0.000	0.319	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	126	50	0	0	0	93	0
N.S.	1	1.00	1.77	0.70	0.00	0.00	0.00	1.31	0.00
time (sec)	N/A	0.055	0.065	0.818	0.000	0.000	0.000	0.314	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	0	0	0	35	0
N.S.	1	1.00	1.00	0.75	0.00	0.00	0.00	1.25	0.00
time (sec)	N/A	0.029	0.020	0.629	0.000	0.000	0.000	0.307	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	68	26	0	0	0	47	0
N.S.	1	1.00	2.19	0.84	0.00	0.00	0.00	1.52	0.00
time (sec)	N/A	0.017	0.021	0.704	0.000	0.000	0.000	0.290	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	12	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.009	0.200	0.993	0.000	0.000	0.400	0.384	0.246

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	14	12	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.17	1.00	1.00
time (sec)	N/A	0.009	1.961	1.080	0.000	0.000	0.525	0.402	0.246

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	306	182	0	0	0	0	0
N.S.	1	1.00	1.79	1.06	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.112	0.222	1.072	0.000	0.000	0.000	0.000	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	226	121	0	0	0	0	0
N.S.	1	1.00	1.78	0.95	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.073	0.358	0.937	0.000	0.000	0.000	0.000	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	233	139	0	0	0	0	0
N.S.	1	1.00	1.71	1.02	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.070	0.152	0.944	0.000	0.000	0.000	0.000	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	154	81	0	0	0	0	0
N.S.	1	1.00	1.69	0.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	0.300	0.813	0.000	0.000	0.000	0.000	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	159	94	0	0	0	0	0
N.S.	1	1.00	1.64	0.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.053	0.088	0.832	0.000	0.000	0.000	0.000	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	44	42	0	0	0	0	0
N.S.	1	1.00	0.80	0.76	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.022	0.041	0.733	0.000	0.000	0.000	0.000	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	86	66	0	0	0	0	0
N.S.	1	1.00	1.46	1.12	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.066	0.033	0.777	0.000	0.000	0.000	0.000	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	12	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.009	0.221	1.043	0.000	0.000	1.016	0.404	0.242

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	322	173	0	0	0	0	0
N.S.	1	1.00	1.37	0.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.281	1.128	1.007	0.000	0.000	0.000	0.000	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	203	107	0	0	0	0	0
N.S.	1	1.00	1.61	0.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.225	0.618	0.875	0.000	0.000	0.000	0.000	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	220	115	0	0	0	0	0
N.S.	1	1.00	1.76	0.92	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.579	0.855	0.000	0.000	0.000	0.000	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	61	56	0	0	0	0	0
N.S.	1	1.00	0.69	0.63	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.119	0.071	0.780	0.000	0.000	0.000	0.000	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	122	83	0	0	0	0	0
N.S.	1	1.00	1.61	1.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.071	0.153	0.842	0.000	0.000	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	12	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.233	1.042	0.000	0.000	7.777	0.406	0.246

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	264	264	418	225	0	0	0	0	0
N.S.	1	1.00	1.58	0.85	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.288	7.374	1.066	0.000	0.000	0.000	0.000	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	264	139	0	0	0	0	0
N.S.	1	1.00	1.39	0.73	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.219	3.092	0.978	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	191	191	281	154	0	0	0	0	0
N.S.	1	1.00	1.47	0.81	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.230	1.840	0.922	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	75	73	0	0	0	0	0
N.S.	1	1.00	0.63	0.61	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.117	0.079	0.832	0.000	0.000	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F(-2)	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	151	110	0	0	0	0	0
N.S.	1	1.00	1.44	1.05	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.109	0.790	0.803	0.000	0.000	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	10	0	0	12	12	12
N.S.	1	1.00	1.17	0.83	0.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.009	0.235	1.056	0.000	0.000	68.438	0.424	0.247

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	115	14	12	14	14
N.S.	1	1.00	1.17	1.00	9.58	1.17	1.00	1.17	1.17
time (sec)	N/A	0.081	0.578	2.383	0.702	0.257	5.391	0.652	0.256

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	115	14	12	14	14
N.S.	1	1.00	1.17	1.00	9.58	1.17	1.00	1.17	1.17
time (sec)	N/A	0.079	0.564	1.862	0.681	0.257	2.904	0.642	0.242

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	150	150	132	0	0	0	0	0	0
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.079	1.413	0.000	0.000	0.000	0.000	0.000	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	68	68	54	0	0	0	0	0	0
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.018	0.036	0.000	0.000	0.000	0.000	0.000	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	14	10	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.17	0.83	1.17	1.17
time (sec)	N/A	0.011	0.369	2.144	0.335	0.236	0.391	0.405	0.251

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	156	14	12	14	14
N.S.	1	1.00	1.17	1.00	13.00	1.17	1.00	1.17	1.17
time (sec)	N/A	0.010	0.387	2.289	0.951	0.253	0.727	0.436	0.252

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	0	14	14	14
N.S.	1	1.00	1.14	0.86	0.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.012	0.644	0.540	0.000	0.000	64.633	1.486	0.258

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	0	14	14	14
N.S.	1	1.00	1.14	0.86	0.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.011	0.726	0.525	0.000	0.000	1.300	0.999	0.245

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	0	14	14	14
N.S.	1	1.00	1.14	0.86	0.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.784	0.559	0.000	0.000	0.580	0.873	0.253

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	0	14	14	14
N.S.	1	1.00	1.14	0.86	0.00	0.00	1.00	1.00	1.00
time (sec)	N/A	0.010	0.713	0.559	0.000	0.000	4.140	0.775	0.267

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	0	14	12	14	14
N.S.	1	1.00	1.17	1.00	0.00	1.17	1.00	1.17	1.17
time (sec)	N/A	0.011	0.586	2.872	0.000	0.262	4.075	0.824	0.251

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	130	287	0	0	0	0	0
N.S.	1	1.00	0.79	1.74	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.141	0.083	2.904	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	163	163	152	0	0	0	0	0	0
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.108	0.140	0.000	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	74	138	0	0	0	0	0
N.S.	1	1.00	0.89	1.66	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.056	0.042	1.095	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	70	148	0	0	0	0	0
N.S.	1	1.00	0.93	1.97	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.035	0.030	0.871	0.000	0.000	0.000	0.000	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	0	12	8	12	12
N.S.	1	1.00	1.20	1.00	0.00	1.20	0.80	1.20	1.20
time (sec)	N/A	0.011	0.255	0.993	0.000	0.276	0.403	0.331	0.262

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	0	12	10	12	12
N.S.	1	1.00	1.20	1.00	0.00	1.20	1.00	1.20	1.20
time (sec)	N/A	0.011	0.648	0.770	0.000	0.252	0.561	0.320	0.258

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	16	14	14	14
N.S.	1	1.00	1.14	0.86	0.00	1.14	1.00	1.00	1.00
time (sec)	N/A	0.013	1.920	0.486	0.000	0.258	162.715	0.595	0.250

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	14	14	14	14
N.S.	1	1.00	1.14	0.86	0.00	1.00	1.00	1.00	1.00
time (sec)	N/A	0.011	3.152	0.563	0.000	0.268	3.963	0.595	0.251

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	20	14	14	14
N.S.	1	1.00	1.14	0.86	0.00	1.43	1.00	1.00	1.00
time (sec)	N/A	0.012	1.288	0.889	0.000	0.255	1.617	0.471	0.267

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	0	20	14	14	14
N.S.	1	1.00	1.14	0.86	0.00	1.43	1.00	1.00	1.00
time (sec)	N/A	0.014	1.211	0.935	0.000	0.262	13.714	0.509	0.259

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	68	68	71	62	85	67	0
N.S.	1	1.00	0.89	0.89	0.93	0.82	1.12	0.88	0.00
time (sec)	N/A	0.028	0.045	0.178	0.287	0.247	0.303	0.279	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	55	60	60	54	70	56	0
N.S.	1	1.00	0.92	1.00	1.00	0.90	1.17	0.93	0.00
time (sec)	N/A	0.030	0.039	0.171	0.292	0.244	0.213	0.276	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	56	48	50	50	60	46	45
N.S.	1	1.00	1.10	0.94	0.98	0.98	1.18	0.90	0.88
time (sec)	N/A	0.014	0.030	0.216	0.289	0.258	0.198	0.279	0.332

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	31	32	31	32	29	31	29
N.S.	1	1.00	1.00	1.03	1.00	1.03	0.94	1.00	0.94
time (sec)	N/A	0.010	0.008	0.175	0.266	0.265	0.086	0.271	0.361

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	58	75	0	0	0	0	0
N.S.	1	1.00	0.92	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.048	0.021	0.954	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	43	37	47	92	41	347	33
N.S.	1	1.00	1.34	1.16	1.47	2.88	1.28	10.84	1.03
time (sec)	N/A	0.021	0.023	0.195	0.278	0.275	1.069	0.358	0.302

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	44	46	37	37	63	492	0
N.S.	1	1.00	1.13	1.18	0.95	0.95	1.62	12.62	0.00
time (sec)	N/A	0.015	0.025	0.178	0.271	0.249	0.815	0.285	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	79	61	69	121	119	1634	0
N.S.	1	1.00	1.27	0.98	1.11	1.95	1.92	26.35	0.00
time (sec)	N/A	0.027	0.027	0.206	0.264	0.263	1.961	0.586	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	121	125	142	111	175	143	0
N.S.	1	1.00	1.19	1.23	1.39	1.09	1.72	1.40	0.00
time (sec)	N/A	0.107	0.095	1.710	0.278	0.250	0.315	0.278	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	104	117	0	99	131	119	0
N.S.	1	1.00	1.37	1.54	0.00	1.30	1.72	1.57	0.00
time (sec)	N/A	0.081	0.140	0.723	0.000	0.253	0.275	0.285	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	76	74	73	65	87	75	96
N.S.	1	1.00	1.62	1.57	1.55	1.38	1.85	1.60	2.04
time (sec)	N/A	0.043	0.082	0.661	0.269	0.256	0.105	0.278	0.494

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	128	185	0	0	0	0	0
N.S.	1	1.00	1.39	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.085	0.089	1.145	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	134	179	0	0	0	0	0
N.S.	1	1.00	1.51	2.01	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.101	0.164	0.695	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	218	235	273	195	333	289	0
N.S.	1	1.00	1.22	1.32	1.53	1.10	1.87	1.62	0.00
time (sec)	N/A	0.210	0.132	2.374	0.284	0.259	0.411	0.318	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	185	211	0	169	269	231	0
N.S.	1	1.00	1.48	1.69	0.00	1.35	2.15	1.85	0.00
time (sec)	N/A	0.145	0.191	0.963	0.000	0.261	0.309	0.294	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	128	134	144	108	165	150	164
N.S.	1	1.00	1.56	1.63	1.76	1.32	2.01	1.83	2.00
time (sec)	N/A	0.074	0.126	0.773	0.273	0.251	0.157	0.324	0.503

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	204	325	0	0	0	0	0
N.S.	1	1.00	1.61	2.56	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.106	0.120	1.203	0.000	0.000	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	151	151	308	0	0	0	0	0	0
N.S.	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.153	0.220	0.000	0.000	0.000	0.000	0.000	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	91	102	0	0	0	172	0
N.S.	1	1.00	0.75	0.84	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.130	0.129	0.468	0.000	0.000	0.000	0.295	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	56	58	0	0	0	86	0
N.S.	1	1.00	0.89	0.92	0.00	0.00	0.00	1.37	0.00
time (sec)	N/A	0.066	0.057	0.411	0.000	0.000	0.000	0.291	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	46	49	0	0	0	50	0
N.S.	1	1.00	0.85	0.91	0.00	0.00	0.00	0.93	0.00
time (sec)	N/A	0.042	0.052	0.491	0.000	0.000	0.000	0.285	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	15	12	0	16
N.S.	1	1.00	1.14	1.00	1.14	1.07	0.86	0.00	1.14
time (sec)	N/A	0.018	0.252	1.470	0.336	0.240	0.750	0.000	0.287

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	19	14	16	16
N.S.	1	1.00	1.14	1.00	1.14	1.36	1.00	1.14	1.14
time (sec)	N/A	0.018	2.815	0.603	0.338	0.236	0.742	0.578	0.291

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	124	147	0	0	0	615	0
N.S.	1	1.00	0.80	0.95	0.00	0.00	0.00	3.97	0.00
time (sec)	N/A	0.112	0.622	0.576	0.000	0.000	0.000	0.320	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	80	78	0	0	0	323	0
N.S.	1	1.00	0.88	0.86	0.00	0.00	0.00	3.55	0.00
time (sec)	N/A	0.055	0.313	0.527	0.000	0.000	0.000	0.317	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	72	74	0	0	0	193	0
N.S.	1	1.00	0.84	0.86	0.00	0.00	0.00	2.24	0.00
time (sec)	N/A	0.106	0.176	0.648	0.000	0.000	0.000	0.295	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	166	30	14	16	16
N.S.	1	1.00	1.14	1.00	11.86	2.14	1.00	1.14	1.14
time (sec)	N/A	0.017	7.272	1.177	0.596	0.242	1.209	0.551	0.283

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	181	36	15	16	16
N.S.	1	1.00	1.14	1.00	12.93	2.57	1.07	1.14	1.14
time (sec)	N/A	0.018	56.932	0.631	0.707	0.234	1.116	0.816	0.278

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	169	290	0	0	0	1479	0
N.S.	1	1.00	0.86	1.47	0.00	0.00	0.00	7.51	0.00
time (sec)	N/A	0.356	0.358	0.484	0.000	0.000	0.000	0.391	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	107	158	0	0	0	860	0
N.S.	1	1.00	0.82	1.22	0.00	0.00	0.00	6.62	0.00
time (sec)	N/A	0.200	0.227	0.440	0.000	0.000	0.000	0.327	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	89	139	0	0	0	481	0
N.S.	1	1.00	0.80	1.25	0.00	0.00	0.00	4.33	0.00
time (sec)	N/A	0.119	0.196	0.547	0.000	0.000	0.000	0.301	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	251	45	14	16	16
N.S.	1	1.00	1.14	1.00	17.93	3.21	1.00	1.14	1.14
time (sec)	N/A	0.018	2.629	1.658	2.399	0.248	1.809	0.849	0.283

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	284	53	15	16	16
N.S.	1	1.00	1.14	1.00	20.29	3.79	1.07	1.14	1.14
time (sec)	N/A	0.017	24.820	1.928	2.895	0.239	1.720	1.471	0.304

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	229	362	0	0	0	1057	0
N.S.	1	1.00	0.95	1.50	0.00	0.00	0.00	4.37	0.00
time (sec)	N/A	0.330	0.331	1.894	0.000	0.000	0.000	0.958	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	117	186	0	0	0	448	0
N.S.	1	1.00	0.85	1.36	0.00	0.00	0.00	3.27	0.00
time (sec)	N/A	0.238	0.221	1.840	0.000	0.000	0.000	0.687	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	122	186	0	0	0	531	0
N.S.	1	1.00	1.01	1.54	0.00	0.00	0.00	4.39	0.00
time (sec)	N/A	0.158	0.090	1.835	0.000	0.000	0.000	0.557	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	14	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.88	1.00	1.00
time (sec)	N/A	0.025	1.508	1.181	0.545	0.000	0.452	0.919	0.254

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	1.00
time (sec)	N/A	0.024	7.217	1.214	0.551	0.000	0.407	0.970	0.273

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	555	548	0	0	0	1967	0
N.S.	1	1.00	1.77	1.75	0.00	0.00	0.00	6.28	0.00
time (sec)	N/A	0.542	7.075	2.033	0.000	0.000	0.000	1.709	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	145	281	0	0	0	845	0
N.S.	1	1.00	0.84	1.63	0.00	0.00	0.00	4.91	0.00
time (sec)	N/A	0.281	0.519	1.955	0.000	0.000	0.000	0.998	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	289	278	0	0	0	993	0
N.S.	1	1.00	1.82	1.75	0.00	0.00	0.00	6.25	0.00
time (sec)	N/A	0.177	1.961	2.068	0.000	0.000	0.000	1.181	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	14	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.88	1.00	1.00
time (sec)	N/A	0.029	0.684	1.189	0.637	0.000	15.353	1.009	0.270

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	1.00
time (sec)	N/A	0.028	7.140	1.195	0.647	0.000	3.131	1.065	0.258

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	358	358	956	798	0	0	0	2778	0
N.S.	1	1.00	2.67	2.23	0.00	0.00	0.00	7.76	0.00
time (sec)	N/A	0.774	10.530	2.321	0.000	0.000	0.000	2.378	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	216	216	187	408	0	0	0	1307	0
N.S.	1	1.00	0.87	1.89	0.00	0.00	0.00	6.05	0.00
time (sec)	N/A	0.429	0.943	2.010	0.000	0.000	0.000	1.288	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	372	401	0	0	0	1177	0
N.S.	1	1.00	2.08	2.24	0.00	0.00	0.00	6.58	0.00
time (sec)	N/A	0.264	1.404	2.075	0.000	0.000	0.000	1.585	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	14	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.88	1.00	1.00
time (sec)	N/A	0.029	0.788	1.233	0.756	0.000	38.817	1.109	0.261

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	1.00
time (sec)	N/A	0.028	6.913	1.159	0.763	0.000	24.824	1.146	0.275

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	225	198	0	0	0	317	0
N.S.	1	1.00	1.01	0.89	0.00	0.00	0.00	1.42	0.00
time (sec)	N/A	0.177	0.316	2.069	0.000	0.000	0.000	0.480	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	85	91	0	0	0	132	0
N.S.	1	1.00	0.86	0.92	0.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.088	0.217	1.341	0.000	0.000	0.000	0.407	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	118	89	0	0	0	159	0
N.S.	1	1.00	1.16	0.87	0.00	0.00	0.00	1.56	0.00
time (sec)	N/A	0.062	0.079	1.296	0.000	0.000	0.000	0.351	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	1.00	1.00
time (sec)	N/A	0.025	1.307	1.180	0.593	0.000	0.448	0.583	0.253

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	17	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	1.06	1.00	1.00
time (sec)	N/A	0.024	7.387	1.171	0.618	0.000	0.570	0.647	0.277

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	252	252	273	299	0	0	0	0	0
N.S.	1	1.00	1.08	1.19	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.195	0.365	2.073	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	0	157	0	0	0	0	0
N.S.	1	1.00	0.00	1.21	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.080	0.000	1.961	0.000	0.000	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	A	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	0	157	0	0	0	0	0
N.S.	1	1.00	0.00	1.15	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.160	0.000	1.958	0.000	0.000	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	0	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	0.00	1.00
time (sec)	N/A	0.030	1.182	0.924	0.595	0.000	1.814	0.000	0.275

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	17	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	1.06	1.00	1.00
time (sec)	N/A	0.029	7.337	1.364	0.587	0.000	2.503	1.155	0.264

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	322	673	0	0	0	0	0
N.S.	1	1.00	1.10	2.30	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.583	1.885	2.201	0.000	0.000	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	0	340	0	0	0	0	0
N.S.	1	1.00	0.00	1.89	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.305	0.000	2.077	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F	B	F	F(-2)	F	F	F(-1)
verified	N/A	Yes	N/A	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	0	341	0	0	0	0	0
N.S.	1	1.00	0.00	2.09	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.178	0.000	2.093	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	15	0	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	0.94	0.00	1.00
time (sec)	N/A	0.030	1.217	0.961	0.642	0.000	8.329	0.000	0.293

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	0	17	16	16
N.S.	1	1.00	1.12	0.88	1.00	0.00	1.06	1.00	1.00
time (sec)	N/A	0.030	7.736	1.179	0.655	0.000	15.375	1.559	0.263

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	158	144	0	100	85	0	0
N.S.	1	1.00	1.32	1.20	0.00	0.83	0.71	0.00	0.00
time (sec)	N/A	0.046	11.246	2.131	0.000	0.103	77.054	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	66	138	0	84	85	0	0
N.S.	1	1.00	0.53	1.11	0.00	0.68	0.69	0.00	0.00
time (sec)	N/A	0.067	10.086	1.576	0.000	0.100	12.547	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	113	119	0	69	85	0	0
N.S.	1	1.00	1.28	1.35	0.00	0.78	0.97	0.00	0.00
time (sec)	N/A	0.031	0.160	1.292	0.000	0.102	3.842	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	45	98	0	52	0	0	0
N.S.	1	1.00	0.51	1.10	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.057	0.038	1.207	0.000	0.095	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	93	85	0	50	0	0	0
N.S.	1	1.00	1.69	1.55	0.00	0.91	0.00	0.00	0.00
time (sec)	N/A	0.024	0.134	1.109	0.000	0.085	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	A	F	C	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	68	129	0	73	0	0	0
N.S.	1	1.00	0.54	1.03	0.00	0.58	0.00	0.00	0.00
time (sec)	N/A	0.063	0.081	1.266	0.000	0.090	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	F	F	F	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	234	0	0	0	0	0	0
N.S.	1	1.00	2.15	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.089	10.982	0.000	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	176	0	0	0	0	0	0
N.S.	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.089	3.373	0.000	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	202	0	0	0	0	0	0
N.S.	1	1.00	1.85	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.090	0.453	0.000	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	107	107	142	0	0	0	0	0	0
N.S.	1	1.00	1.33	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.084	0.993	0.000	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	105	105	129	0	0	0	0	0	0
N.S.	1	1.00	1.23	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.088	0.398	0.000	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	109	109	198	0	0	0	0	0	0
N.S.	1	1.00	1.82	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.091	0.650	0.000	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	441	53	17	0	18
N.S.	1	1.00	1.11	0.89	24.50	2.94	0.94	0.00	1.00
time (sec)	N/A	0.102	45.481	0.819	3.500	0.251	80.913	0.000	0.317

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	418	44	17	0	18
N.S.	1	1.00	1.11	0.89	23.22	2.44	0.94	0.00	1.00
time (sec)	N/A	0.103	141.232	1.385	3.425	0.250	8.492	0.000	0.339

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	458	50	0	18	18
N.S.	1	1.00	1.11	0.89	25.44	2.78	0.00	1.00	1.00
time (sec)	N/A	0.092	73.008	0.312	3.395	0.254	0.000	0.682	0.320

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	489	50	0	18	18
N.S.	1	1.00	1.11	0.89	27.17	2.78	0.00	1.00	1.00
time (sec)	N/A	0.095	51.877	2.240	3.414	0.255	0.000	0.800	0.324

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	491	50	0	18	18
N.S.	1	1.00	1.11	0.89	27.28	2.78	0.00	1.00	1.00
time (sec)	N/A	0.105	37.277	1.589	3.472	0.246	0.000	0.757	0.309

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	20	15	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.11	0.83	1.00	1.00
time (sec)	N/A	0.020	2.899	0.441	0.403	0.231	4.130	0.283	0.255

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	18	15	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.00	0.83	1.00	1.00
time (sec)	N/A	0.019	2.519	0.959	0.413	0.243	0.511	0.287	0.283

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	23	17	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.28	0.94	1.00	1.00
time (sec)	N/A	0.019	1.473	0.967	0.445	0.234	1.451	0.282	0.267

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	31	17	18	18
N.S.	1	1.00	1.11	0.89	1.00	1.72	0.94	1.00	1.00
time (sec)	N/A	0.022	1.400	0.938	0.497	0.234	3.529	0.284	0.281

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	181	34	17	18	18
N.S.	1	1.00	1.11	0.89	10.06	1.89	0.94	1.00	1.00
time (sec)	N/A	0.019	16.110	0.526	1.853	0.236	10.481	0.337	0.286

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	181	32	17	18	18
N.S.	1	1.00	1.11	0.89	10.06	1.78	0.94	1.00	1.00
time (sec)	N/A	0.016	16.350	0.967	1.875	0.237	2.066	0.321	0.285

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	196	39	19	18	18
N.S.	1	1.00	1.11	0.89	10.89	2.17	1.06	1.00	1.00
time (sec)	N/A	0.017	40.224	0.984	1.683	0.243	4.302	0.303	0.299

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	218	51	19	18	18
N.S.	1	1.00	1.11	0.89	12.11	2.83	1.06	1.00	1.00
time (sec)	N/A	0.019	22.856	0.980	1.934	0.235	11.732	0.299	0.322

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [30] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	8	0.375
2	A	4	3	1.00	8	0.375
3	A	4	3	1.00	8	0.375
4	A	3	3	1.00	6	0.500
5	A	2	2	1.00	4	0.500
6	A	5	5	1.00	8	0.625
7	A	4	4	1.00	8	0.500
8	A	2	2	1.00	8	0.250
9	A	5	5	1.00	8	0.625
10	A	3	3	1.00	8	0.375
11	A	6	5	1.00	8	0.625
12	A	7	5	1.00	10	0.500
13	A	6	4	1.00	10	0.400
14	A	5	5	1.00	10	0.500
15	A	4	4	1.00	8	0.500
16	A	3	3	1.00	6	0.500
17	A	6	6	1.00	10	0.600
18	A	7	5	1.00	10	0.500
19	A	3	3	1.00	10	0.300
20	A	9	7	1.00	10	0.700
21	A	5	5	1.00	10	0.500
22	A	14	7	1.00	10	0.700
23	A	11	5	1.00	10	0.500
24	A	9	7	1.00	10	0.700

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
25	A	6	5	1.00	8	0.625
26	A	4	3	1.00	6	0.500
27	A	7	7	1.00	10	0.700
28	A	9	6	1.00	10	0.600
29	A	7	7	1.00	10	0.700
30	A	14	10	1.00	10	1.000
31	A	10	9	1.00	10	0.900
32	A	23	4	1.00	10	0.400
33	A	19	6	1.00	10	0.600
34	A	14	4	1.00	10	0.400
35	A	11	6	1.00	10	0.600
36	A	7	4	1.00	8	0.500
37	A	5	3	1.00	6	0.500
38	A	8	7	1.00	10	0.700
39	A	11	7	1.00	10	0.700
40	A	8	8	1.00	10	0.800
41	A	19	10	1.00	10	1.000
42	A	7	3	1.00	10	0.300
43	A	6	3	1.00	10	0.300
44	A	6	3	1.00	10	0.300
45	A	5	3	1.00	10	0.300
46	A	5	3	1.00	10	0.300
47	A	4	4	1.00	8	0.500
48	A	2	2	1.00	6	0.333
49	N/A	0	0	1.00	10	0.000
50	N/A	0	0	1.00	10	0.000
51	A	6	2	1.00	10	0.200
52	A	5	2	1.00	10	0.200
53	A	5	2	1.00	10	0.200
54	A	4	2	1.00	10	0.200
55	A	4	2	1.00	10	0.200
56	A	2	2	1.00	8	0.250
57	A	3	3	1.00	6	0.500
58	N/A	0	0	1.00	10	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	N/A	0	0	1.00	10	0.000
60	A	14	5	1.00	10	0.500
61	A	12	6	1.00	10	0.600
62	A	10	6	1.00	10	0.600
63	A	7	7	1.00	8	0.875
64	A	4	4	1.00	6	0.667
65	N/A	0	0	1.00	10	0.000
66	N/A	0	0	1.00	10	0.000
67	A	12	4	1.00	10	0.400
68	A	9	4	1.00	10	0.400
69	A	10	6	1.00	10	0.600
70	A	5	5	1.00	8	0.625
71	A	5	4	1.00	6	0.667
72	N/A	0	0	1.00	10	0.000
73	N/A	0	0	1.00	10	0.000
74	A	10	5	1.00	12	0.417
75	A	8	5	1.00	12	0.417
76	A	8	5	1.00	12	0.417
77	A	6	5	1.00	10	0.500
78	A	4	4	1.00	8	0.500
79	N/A	0	0	1.00	12	0.000
80	A	23	8	1.00	12	0.667
81	A	16	8	1.00	12	0.667
82	A	13	8	1.00	12	0.667
83	A	8	8	1.00	10	0.800
84	A	5	5	1.00	8	0.625
85	N/A	0	0	1.00	12	0.000
86	A	26	8	1.00	12	0.667
87	A	18	7	1.00	12	0.583
88	A	15	8	1.00	12	0.667
89	A	9	7	1.00	10	0.700
90	A	6	5	1.00	8	0.625
91	N/A	0	0	1.00	12	0.000
92	A	9	4	1.00	12	0.333

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
93	A	7	4	1.00	12	0.333
94	A	7	4	1.00	12	0.333
95	A	5	5	1.00	10	0.500
96	A	3	3	1.00	8	0.375
97	N/A	0	0	1.00	12	0.000
98	N/A	0	0	1.00	12	0.000
99	A	10	3	1.00	12	0.250
100	A	8	3	1.00	12	0.250
101	A	8	3	1.00	12	0.250
102	A	6	3	1.00	12	0.250
103	A	6	3	1.00	12	0.250
104	A	3	3	1.00	10	0.300
105	A	4	4	1.00	8	0.500
106	N/A	0	0	1.00	12	0.000
107	A	19	6	1.00	12	0.500
108	A	15	7	1.00	12	0.583
109	A	13	7	1.00	12	0.583
110	A	8	8	1.00	10	0.800
111	A	5	5	1.00	8	0.625
112	N/A	0	0	1.00	12	0.000
113	A	17	5	1.00	12	0.417
114	A	12	5	1.00	12	0.417
115	A	13	7	1.00	12	0.583
116	A	6	6	1.00	10	0.600
117	A	6	5	1.00	8	0.625
118	N/A	0	0	1.00	12	0.000
119	N/A	0	0	1.00	12	0.000
120	N/A	0	0	1.00	12	0.000
121	A	2	2	1.00	12	0.167
122	A	2	2	1.00	10	0.200
123	N/A	0	0	1.00	12	0.000
124	N/A	0	0	1.00	12	0.000
125	N/A	0	0	1.00	14	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
126	N/A	0	0	1.00	14	0.000
127	N/A	0	0	1.00	14	0.000
128	N/A	0	0	1.00	14	0.000
129	N/A	0	0	1.00	12	0.000
130	A	9	4	1.00	10	0.400
131	A	9	4	1.00	10	0.400
132	A	6	5	1.00	8	0.625
133	A	4	3	1.00	6	0.500
134	N/A	0	0	1.00	10	0.000
135	N/A	0	0	1.00	10	0.000
136	N/A	0	0	1.00	14	0.000
137	N/A	0	0	1.00	14	0.000
138	N/A	0	0	1.00	14	0.000
139	N/A	0	0	1.00	14	0.000
140	A	4	3	1.00	12	0.250
141	A	4	3	1.00	12	0.250
142	A	3	3	1.00	10	0.300
143	A	3	2	1.00	8	0.250
144	A	5	5	1.00	12	0.417
145	A	4	4	1.00	12	0.333
146	A	2	2	1.00	12	0.167
147	A	5	5	1.00	12	0.417
148	A	5	5	1.00	14	0.357
149	A	4	4	1.00	12	0.333
150	A	3	3	1.00	10	0.300
151	A	6	6	1.00	14	0.429
152	A	7	5	1.00	14	0.357
153	A	10	7	1.00	14	0.500
154	A	6	5	1.00	12	0.417
155	A	5	3	1.00	10	0.300
156	A	7	7	1.00	14	0.500
157	A	9	6	1.00	14	0.429
158	A	9	5	1.00	14	0.357
159	A	6	6	1.00	12	0.500

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
160	A	4	4	1.00	10	0.400
161	N/A	0	0	1.00	14	0.000
162	N/A	0	0	1.00	14	0.000
163	A	8	4	1.00	14	0.286
164	A	4	4	1.00	12	0.333
165	A	5	5	1.00	10	0.500
166	N/A	0	0	1.00	14	0.000
167	N/A	0	0	1.00	14	0.000
168	A	16	8	1.00	14	0.571
169	A	9	9	1.00	12	0.750
170	A	6	6	1.00	10	0.600
171	N/A	0	0	1.00	14	0.000
172	N/A	0	0	1.00	14	0.000
173	A	14	8	1.00	16	0.500
174	A	9	8	1.00	14	0.571
175	A	7	7	1.00	12	0.583
176	N/A	0	0	1.00	16	0.000
177	N/A	0	0	1.00	16	0.000
178	A	22	11	1.00	16	0.688
179	A	11	11	1.00	14	0.786
180	A	8	8	1.00	12	0.667
181	N/A	0	0	1.00	16	0.000
182	N/A	0	0	1.00	16	0.000
183	A	24	11	1.00	16	0.688
184	A	12	10	1.00	14	0.714
185	A	9	8	1.00	12	0.667
186	N/A	0	0	1.00	16	0.000
187	N/A	0	0	1.00	16	0.000
188	A	13	7	1.00	16	0.438
189	A	8	8	1.00	14	0.571
190	A	6	6	1.00	12	0.500
191	N/A	0	0	1.00	16	0.000
192	N/A	0	0	1.00	16	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
193	A	12	6	1.00	16	0.375
194	A	6	6	1.00	14	0.429
195	A	7	7	1.00	12	0.583
196	N/A	0	0	1.00	16	0.000
197	N/A	0	0	1.00	16	0.000
198	A	22	10	1.00	16	0.625
199	A	11	11	1.00	14	0.786
200	A	8	8	1.00	12	0.667
201	N/A	0	0	1.00	16	0.000
202	N/A	0	0	1.00	16	0.000
203	A	5	4	1.00	16	0.250
204	A	7	7	1.00	16	0.438
205	A	4	4	1.00	16	0.250
206	A	6	6	1.00	16	0.375
207	A	3	3	1.00	16	0.188
208	A	7	7	1.00	16	0.438
209	A	2	2	1.00	18	0.111
210	A	2	2	1.00	18	0.111
211	A	2	2	1.00	18	0.111
212	A	2	2	1.00	18	0.111
213	A	2	2	1.00	18	0.111
214	A	2	2	1.00	18	0.111
215	N/A	0	0	1.00	18	0.000
216	N/A	0	0	1.00	18	0.000
217	N/A	0	0	1.00	18	0.000
218	N/A	0	0	1.00	18	0.000
219	N/A	0	0	1.00	18	0.000
220	N/A	0	0	1.00	18	0.000
221	N/A	0	0	1.00	18	0.000
222	N/A	0	0	1.00	18	0.000
223	N/A	0	0	1.00	18	0.000
224	N/A	0	0	1.00	18	0.000
225	N/A	0	0	1.00	18	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
226	N/A	0	0	1.00	18	0.000
227	N/A	0	0	1.00	18	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^4 \arccos(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 75

$$\int x^4 \arccos(ax)\,dx = -\frac{\sqrt{1-a^2x^2}}{5a^5} + \frac{2(1-a^2x^2)^{3/2}}{15a^5} - \frac{(1-a^2x^2)^{5/2}}{25a^5} + \frac{1}{5}x^5 \arccos(ax)$$

```
[Out] 2/15*(-a^2*x^2+1)^(3/2)/a^5-1/25*(-a^2*x^2+1)^(5/2)/a^5+1/5*x^5*arccos(a*x)
-1/5*(-a^2*x^2+1)^(1/2)/a^5
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4724, 272, 45}

$$\int x^4 \arccos(ax)\,dx = -\frac{(1-a^2x^2)^{5/2}}{25a^5} + \frac{2(1-a^2x^2)^{3/2}}{15a^5} - \frac{\sqrt{1-a^2x^2}}{5a^5} + \frac{1}{5}x^5 \arccos(ax)$$

```
[In] Int[x^4*ArcCos[a*x],x]
```

```
[Out] -1/5*Sqrt[1 - a^2*x^2]/a^5 + (2*(1 - a^2*x^2)^(3/2))/(15*a^5) - (1 - a^2*x^
2)^(5/2)/(25*a^5) + (x^5*ArcCos[a*x])/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\arccos(ax) + \frac{1}{5}a\int \frac{x^5}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{1}{5}x^5\arccos(ax) + \frac{1}{10}a\text{Subst}\left(\int \frac{x^2}{\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{1}{5}x^5\arccos(ax) + \frac{1}{10}a\text{Subst}\left(\int \left(\frac{1}{a^4\sqrt{1-a^2x}} - \frac{2\sqrt{1-a^2x}}{a^4} + \frac{(1-a^2x)^{3/2}}{a^4}\right)\,dx, x, x^2\right) \\
&= -\frac{\sqrt{1-a^2x^2}}{5a^5} + \frac{2(1-a^2x^2)^{3/2}}{15a^5} - \frac{(1-a^2x^2)^{5/2}}{25a^5} + \frac{1}{5}x^5\arccos(ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int x^4\arccos(ax)\,dx = -\frac{\sqrt{1-a^2x^2}(8+4a^2x^2+3a^4x^4)}{75a^5} + \frac{1}{5}x^5\arccos(ax)$$

```
[In] Integrate[x^4*ArcCos[a*x],x]
[Out] -1/75*(Sqrt[1 - a^2*x^2]*(8 + 4*a^2*x^2 + 3*a^4*x^4))/a^5 + (x^5*ArcCos[a*x
])/5
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{a^5x^5\arccos(ax)}{5}-\frac{a^4x^4\sqrt{-a^2x^2+1}}{25}-\frac{4a^2x^2\sqrt{-a^2x^2+1}}{75}-\frac{8\sqrt{-a^2x^2+1}}{75}}{a^5}$	72
default	$\frac{\frac{a^5x^5\arccos(ax)}{5}-\frac{a^4x^4\sqrt{-a^2x^2+1}}{25}-\frac{4a^2x^2\sqrt{-a^2x^2+1}}{75}-\frac{8\sqrt{-a^2x^2+1}}{75}}{a^5}$	72
parts	$\frac{x^5\arccos(ax)}{5}+\frac{a\left(-\frac{x^4\sqrt{-a^2x^2+1}}{5a^2}+\frac{-\frac{4x^2\sqrt{-a^2x^2+1}}{15a^2}-\frac{8\sqrt{-a^2x^2+1}}{15a^4}}{a^2}\right)}{5}$	78

[In] int(x^4*arccos(a*x),x,method=_RETURNVERBOSE)

[Out] 1/a^5*(1/5*a^5*x^5*arccos(a*x)-1/25*a^4*x^4*(-a^2*x^2+1)^(1/2)-4/75*a^2*x^2*(-a^2*x^2+1)^(1/2)-8/75*(-a^2*x^2+1)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int x^4\arccos(ax)\,dx=\frac{15\,a^5x^5\arccos{(ax)}-(3\,a^4x^4+4\,a^2x^2+8)\sqrt{-a^2x^2+1}}{75\,a^5}$$

[In] integrate(x^4*arccos(a*x),x, algorithm="fricas")

[Out] 1/75*(15*a^5*x^5*arccos(a*x) - (3*a^4*x^4 + 4*a^2*x^2 + 8)*sqrt(-a^2*x^2 + 1))/a^5

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^4\arccos(ax)\,dx=\begin{cases}\frac{x^5\operatorname{acos}{(ax)}}{5}-\frac{x^4\sqrt{-a^2x^2+1}}{25a}-\frac{4x^2\sqrt{-a^2x^2+1}}{75a^3}-\frac{8\sqrt{-a^2x^2+1}}{75a^5} & \text{for } a\neq 0\\ \frac{\pi x^5}{10} & \text{otherwise}\end{cases}$$

[In] integrate(x**4*acos(a*x),x)

[Out] Piecewise((x**5*acos(a*x)/5 - x**4*sqrt(-a**2*x**2 + 1)/(25*a) - 4*x**2*sqrt(-a**2*x**2 + 1)/(75*a**3) - 8*sqrt(-a**2*x**2 + 1)/(75*a**5), Ne(a, 0)), (pi*x**5/10, True))

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int x^4 \arccos(ax)\,dx = \frac{1}{5}\,x^5 \arccos\left(ax\right) - \frac{1}{75}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2} + \frac{4\sqrt{-a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{-a^2x^2+1}}{a^6}\right)a$$

```
[In] integrate(x^4*arccos(a*x),x, algorithm="maxima")
[Out] 1/5*x^5*arccos(a*x) - 1/75*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^2
+ 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.89

$$\int x^4 \arccos(ax)\,dx = \frac{1}{5}\,x^5 \arccos\left(ax\right) - \frac{\sqrt{-a^2x^2+1}x^4}{25\,a} - \frac{4\sqrt{-a^2x^2+1}x^2}{75\,a^3} - \frac{8\sqrt{-a^2x^2+1}}{75\,a^5}$$

```
[In] integrate(x^4*arccos(a*x),x, algorithm="giac")
[Out] 1/5*x^5*arccos(a*x) - 1/25*sqrt(-a^2*x^2 + 1)*x^4/a - 4/75*sqrt(-a^2*x^2 +
1)*x^2/a^3 - 8/75*sqrt(-a^2*x^2 + 1)/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)\,dx = \int x^4\,\mathrm{acos}(a\,x)\,dx$$

```
[In] int(x^4*acos(a*x),x)
[Out] int(x^4*acos(a*x), x)
```

3.2 $\int x^3 \arccos(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 69

$$\int x^3 \arccos(ax)\,dx = -\frac{3x\sqrt{1-a^2x^2}}{32a^3} - \frac{x^3\sqrt{1-a^2x^2}}{16a} + \frac{1}{4}x^4 \arccos(ax) + \frac{3\arcsin(ax)}{32a^4}$$

```
[Out] 1/4*x^4*arccos(a*x)+3/32*arcsin(a*x)/a^4-3/32*x*(-a^2*x^2+1)^(1/2)/a^3-1/16
*x^3*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4724, 327, 222}

$$\int x^3 \arccos(ax)\,dx = \frac{3\arcsin(ax)}{32a^4} - \frac{x^3\sqrt{1-a^2x^2}}{16a} - \frac{3x\sqrt{1-a^2x^2}}{32a^3} + \frac{1}{4}x^4 \arccos(ax)$$

```
[In] Int[x^3*ArcCos[a*x],x]
```

```
[Out] (-3*x*Sqrt[1 - a^2*x^2])/(32*a^3) - (x^3*Sqrt[1 - a^2*x^2])/(16*a) + (x^4*A
rcCos[a*x])/4 + (3*ArcSin[a*x])/(32*a^4)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4 \arccos(ax) + \frac{1}{4}a \int \frac{x^4}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{x^3\sqrt{1-a^2x^2}}{16a} + \frac{1}{4}x^4 \arccos(ax) + \frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx}{16a} \\
&= -\frac{3x\sqrt{1-a^2x^2}}{32a^3} - \frac{x^3\sqrt{1-a^2x^2}}{16a} + \frac{1}{4}x^4 \arccos(ax) + \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{32a^3} \\
&= -\frac{3x\sqrt{1-a^2x^2}}{32a^3} - \frac{x^3\sqrt{1-a^2x^2}}{16a} + \frac{1}{4}x^4 \arccos(ax) + \frac{3\arcsin(ax)}{32a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int x^3 \arccos(ax)\,dx = \frac{-ax\sqrt{1-a^2x^2}(3+2a^2x^2) + 8a^4x^4 \arccos(ax) + 3\arcsin(ax)}{32a^4}$$

```
[In] Integrate[x^3*ArcCos[a*x],x]
[Out] (-(a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)) + 8*a^4*x^4*ArcCos[a*x] + 3*ArcSi
n[a*x])/(32*a^4)
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

method	result	size
derivativedivides	$\frac{\frac{a^4x^4\arccos(ax)}{4}-\frac{a^3x^3\sqrt{-a^2x^2+1}}{16}-\frac{3ax\sqrt{-a^2x^2+1}}{32}+\frac{3\arcsin(ax)}{32}}{a^4}$	60
default	$\frac{\frac{a^4x^4\arccos(ax)}{4}-\frac{a^3x^3\sqrt{-a^2x^2+1}}{16}-\frac{3ax\sqrt{-a^2x^2+1}}{32}+\frac{3\arcsin(ax)}{32}}{a^4}$	60
parts	$\frac{x^4\arccos(ax)}{4}+\frac{a\left(-\frac{x^3\sqrt{-a^2x^2+1}}{4a^2}+\frac{-\frac{3x\sqrt{-a^2x^2+1}}{8a^2}+\frac{3\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{8a^2\sqrt{a^2}}}{a^2}\right)}{4}$	89

[In] int(x^3*arccos(a*x),x,method=_RETURNVERBOSE)

[Out] 1/a^4*(1/4*a^4*x^4*arccos(a*x)-1/16*a^3*x^3*(-a^2*x^2+1)^(1/2)-3/32*a*x*(-a^2*x^2+1)^(1/2)+3/32*arcsin(a*x))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.70

$$\int x^3 \arccos(ax)\,dx = \frac{(8\,a^4x^4-3)\arccos{(ax)}-(2\,a^3x^3+3\,ax)\sqrt{-a^2x^2+1}}{32\,a^4}$$

[In] integrate(x^3*arccos(a*x),x, algorithm="fricas")

[Out] 1/32*((8*a^4*x^4 - 3)*arccos(a*x) - (2*a^3*x^3 + 3*a*x)*sqrt(-a^2*x^2 + 1))/a^4

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int x^3 \arccos(ax)\,dx = \begin{cases} \frac{x^4\operatorname{acos}{(ax)}}{4}-\frac{x^3\sqrt{-a^2x^2+1}}{16a}-\frac{3x\sqrt{-a^2x^2+1}}{32a^3}-\frac{3\operatorname{acos}{(ax)}}{32a^4} & \text{for}\ a\neq 0\\ \frac{\pi x^4}{8} & \text{otherwise}\end{cases}$$

[In] integrate(x**3*acos(a*x),x)

[Out] Piecewise((x**4*acos(a*x)/4 - x**3*sqrt(-a**2*x**2 + 1)/(16*a) - 3*x*sqrt(-a**2*x**2 + 1)/(32*a**3) - 3*acos(a*x)/(32*a**4), Ne(a, 0)), (pi*x**4/8, True))

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int x^3 \arccos(ax)\,dx = \frac{1}{4}\,x^4 \arccos\left(ax\right) - \frac{1}{32}\left(\frac{2\sqrt{-a^2x^2+1}x^3}{a^2} + \frac{3\sqrt{-a^2x^2+1}x}{a^4} - \frac{3\,\arcsin\left(ax\right)}{a^5}\right)a$$

```
[In] integrate(x^3*arccos(a*x),x, algorithm="maxima")
[Out] 1/4*x^4*arccos(a*x) - 1/32*(2*sqrt(-a^2*x^2 + 1)*x^3/a^2 + 3*sqrt(-a^2*x^2
+ 1)*x/a^4 - 3*arcsin(a*x)/a^5)*a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.83

$$\int x^3 \arccos(ax)\,dx = \frac{1}{4}\,x^4 \arccos\left(ax\right) - \frac{\sqrt{-a^2x^2+1}x^3}{16\,a} - \frac{3\sqrt{-a^2x^2+1}x}{32\,a^3} - \frac{3\,\arccos\left(ax\right)}{32\,a^4}$$

```
[In] integrate(x^3*arccos(a*x),x, algorithm="giac")
[Out] 1/4*x^4*arccos(a*x) - 1/16*sqrt(-a^2*x^2 + 1)*x^3/a - 3/32*sqrt(-a^2*x^2 +
1)*x/a^3 - 3/32*arccos(a*x)/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)\,dx = \int x^3\,\mathrm{acos}(a\,x)\;dx$$

```
[In] int(x^3*acos(a*x),x)
[Out] int(x^3*acos(a*x), x)
```

3.3 $\int x^2 \arccos(ax)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 54

$$\int x^2 \arccos(ax)\,dx = -\frac{\sqrt{1-a^2x^2}}{3a^3} + \frac{(1-a^2x^2)^{3/2}}{9a^3} + \frac{1}{3}x^3 \arccos(ax)$$

```
[Out] 1/9*(-a^2*x^2+1)^(3/2)/a^3+1/3*x^3*arccos(a*x)-1/3*(-a^2*x^2+1)^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4724, 272, 45}

$$\int x^2 \arccos(ax)\,dx = \frac{(1-a^2x^2)^{3/2}}{9a^3} - \frac{\sqrt{1-a^2x^2}}{3a^3} + \frac{1}{3}x^3 \arccos(ax)$$

```
[In] Int[x^2*ArcCos[a*x],x]
```

```
[Out] -1/3*Sqrt[1 - a^2*x^2]/a^3 + (1 - a^2*x^2)^(3/2)/(9*a^3) + (x^3*ArcCos[a*x]
)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arccos(ax)+\frac{1}{3}a\int\frac{x^3}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{1}{3}x^3\arccos(ax)+\frac{1}{6}a\text{Subst}\left(\int\frac{x}{\sqrt{1-a^2x}}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3\arccos(ax)+\frac{1}{6}a\text{Subst}\left(\int\left(\frac{1}{a^2\sqrt{1-a^2x}}-\frac{\sqrt{1-a^2x}}{a^2}\right)\,dx,x,x^2\right)\\
&= -\frac{\sqrt{1-a^2x^2}}{3a^3}+\frac{\left(1-a^2x^2\right)^{3/2}}{9a^3}+\frac{1}{3}x^3\arccos(ax)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int x^2\arccos(ax)\,dx=-\frac{\sqrt{1-a^2x^2}(2+a^2x^2)}{9a^3}+\frac{1}{3}x^3\arccos(ax)$$

```
[In] Integrate[x^2*ArcCos[a*x],x]
[Out] -1/9*(Sqrt[1 - a^2*x^2]*(2 + a^2*x^2))/a^3 + (x^3*ArcCos[a*x])/3
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{a^3x^3 \arccos(ax)}{3}-\frac{a^2x^2\sqrt{-a^2x^2+1}}{9}-\frac{2\sqrt{-a^2x^2+1}}{9}}{a^3}$	52
default	$\frac{\frac{a^3x^3 \arccos(ax)}{3}-\frac{a^2x^2\sqrt{-a^2x^2+1}}{9}-\frac{2\sqrt{-a^2x^2+1}}{9}}{a^3}$	52
parts	$\frac{x^3 \arccos(ax)}{3}+\frac{a\left(-\frac{x^2\sqrt{-a^2x^2+1}}{3a^2}-\frac{2\sqrt{-a^2x^2+1}}{3a^4}\right)}{3}$	52

[In] int(x^2*arccos(a*x),x,method=_RETURNVERBOSE)

[Out] 1/a^3*(1/3*a^3*x^3*arccos(a*x)-1/9*a^2*x^2*(-a^2*x^2+1)^(1/2)-2/9*(-a^2*x^2+1)^(1/2))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int x^2 \arccos(ax)\,dx = \frac{3\,a^3x^3 \arccos{(ax)} - (a^2x^2+2)\sqrt{-a^2x^2+1}}{9\,a^3}$$

[In] integrate(x^2*arccos(a*x),x, algorithm="fricas")

[Out] 1/9*(3*a^3*x^3*arccos(a*x) - (a^2*x^2 + 2)*sqrt(-a^2*x^2 + 1))/a^3

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int x^2 \arccos(ax)\,dx = \begin{cases} \frac{x^3 \operatorname{acos}{(ax)}}{3} - \frac{x^2\sqrt{-a^2x^2+1}}{9a} - \frac{2\sqrt{-a^2x^2+1}}{9a^3} & \text{for } a \neq 0 \\ \frac{\pi x^3}{6} & \text{otherwise} \end{cases}$$

[In] integrate(x**2*acos(a*x),x)

[Out] Piecewise((x**3*acos(a*x)/3 - x**2*sqrt(-a**2*x**2 + 1)/(9*a) - 2*sqrt(-a**2*x**2 + 1)/(9*a**3), Ne(a, 0)), (pi*x**3/6, True))

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int x^2 \arccos(ax)\,dx = \frac{1}{3}\,x^3\arccos\left(ax\right) - \frac{1}{9}\,a\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\,\sqrt{-a^2x^2+1}}{a^4}\right)$$

```
[In] integrate(x^2*arccos(a*x),x, algorithm="maxima")
[Out] 1/3*x^3*arccos(a*x) - 1/9*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2 +
 1)/a^4)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int x^2 \arccos(ax)\,dx = \frac{1}{3}\,x^3\arccos\left(ax\right) - \frac{\sqrt{-a^2x^2+1}x^2}{9\,a} - \frac{2\,\sqrt{-a^2x^2+1}}{9\,a^3}$$

```
[In] integrate(x^2*arccos(a*x),x, algorithm="giac")
[Out] 1/3*x^3*arccos(a*x) - 1/9*sqrt(-a^2*x^2 + 1)*x^2/a - 2/9*sqrt(-a^2*x^2 + 1)
/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)\,dx = \begin{cases} \frac{x^3\operatorname{acos}(a\,x)}{3} - \frac{\sqrt{\frac{1}{a^2}-x^2}\,\left(\frac{2}{a^2}+x^2\right)}{9} & \text{if } 0<a \\ \int x^2\operatorname{acos}(a\,x)\,dx & \text{if } \neg 0<a \end{cases}$$

```
[In] int(x^2*acos(a*x),x)
[Out] piecewise(0 < a, - ((1/a^2 - x^2)^(1/2)*(2/a^2 + x^2))/9 + (x^3*acos(a*x))/
3, ~0 < a, int(x^2*acos(a*x), x))
```

3.4 $\int x \arccos(ax) \, dx$

Optimal result

Integrand size = 6, antiderivative size = 45

$$\int x \arccos(ax) \, dx = -\frac{x\sqrt{1-a^2x^2}}{4a} + \frac{1}{2}x^2 \arccos(ax) + \frac{\arcsin(ax)}{4a^2}$$

```
[Out] 1/2*x^2*arccos(a*x)+1/4*arcsin(a*x)/a^2-1/4*x*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4724, 327, 222}

$$\int x \arccos(ax) \, dx = \frac{\arcsin(ax)}{4a^2} - \frac{x\sqrt{1-a^2x^2}}{4a} + \frac{1}{2}x^2 \arccos(ax)$$

```
[In] Int[x*ArcCos[a*x],x]
```

```
[Out] -1/4*(x*Sqrt[1 - a^2*x^2])/a + (x^2*ArcCos[a*x])/2 + ArcSin[a*x]/(4*a^2)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2\arccos(ax) + \frac{1}{2}a\int\frac{x^2}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{x\sqrt{1-a^2x^2}}{4a} + \frac{1}{2}x^2\arccos(ax) + \frac{\int\frac{1}{\sqrt{1-a^2x^2}}\,dx}{4a}\\ &= -\frac{x\sqrt{1-a^2x^2}}{4a} + \frac{1}{2}x^2\arccos(ax) + \frac{\arcsin(ax)}{4a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x\arccos(ax)\,dx = \frac{-ax\sqrt{1-a^2x^2} + 2a^2x^2\arccos(ax) + \arcsin(ax)}{4a^2}$$

```
[In] Integrate[x*ArcCos[a*x],x]
[Out] (-(a*x*Sqrt[1 - a^2*x^2]) + 2*a^2*x^2*ArcCos[a*x] + ArcSin[a*x])/(4*a^2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{a^2x^2\arccos(ax)}{2}-\frac{ax\sqrt{-a^2x^2+1}}{4}+\frac{\arcsin(ax)}{4}}{a^2}$	40
default	$\frac{\frac{a^2x^2\arccos(ax)}{2}-\frac{ax\sqrt{-a^2x^2+1}}{4}+\frac{\arcsin(ax)}{4}}{a^2}$	40
parts	$\frac{x^2\arccos(ax)}{2}+\frac{a\left(-\frac{x\sqrt{-a^2x^2+1}}{2a^2}+\frac{\arctan\left(\frac{\sqrt{a^2}\,x}{\sqrt{-a^2x^2+1}}\right)}{2a^2\sqrt{a^2}}\right)}{2}$	63

```
[In] int(x*arccos(a*x),x,method=_RETURNVERBOSE)
```

[Out] 1/a^2*(1/2*a^2*x^2*arccos(a*x)-1/4*a*x*(-a^2*x^2+1)^(1/2)+1/4*arcsin(a*x))

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x \arccos(ax)\,dx = -\frac{\sqrt{-a^2x^2+1}ax - (2\,a^2x^2-1)\arccos{(ax)}}{4\,a^2}$$

[In] integrate(x*arccos(a*x),x, algorithm="fricas")

[Out] -1/4*(sqrt(-a^2*x^2 + 1)*a*x - (2*a^2*x^2 - 1)*arccos(a*x))/a^2

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.93

$$\int x \arccos(ax)\,dx = \begin{cases} \frac{x^2\operatorname{acos}{(ax)}}{2} - \frac{x\sqrt{-a^2x^2+1}}{4a} - \frac{\operatorname{acos}{(ax)}}{4a^2} & \text{for}\ a \neq 0 \\ \frac{\pi x^2}{4} & \text{otherwise} \end{cases}$$

[In] integrate(x*acos(a*x),x)

[Out] Piecewise((x**2*acos(a*x)/2 - x*sqrt(-a**2*x**2 + 1)/(4*a) - acos(a*x)/(4*a
2), Ne(a, 0)), (pi*x2/4, True))

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.89

$$\int x \arccos(ax)\,dx = \frac{1}{2}\,x^2\arccos{(ax)} - \frac{1}{4}\,a\left(\frac{\sqrt{-a^2x^2+1}x}{a^2} - \frac{\arcsin{(ax)}}{a^3}\right)$$

[In] integrate(x*arccos(a*x),x, algorithm="maxima")

[Out] 1/2*x^2*arccos(a*x) - 1/4*a*(sqrt(-a^2*x^2 + 1)*x/a^2 - arcsin(a*x)/a^3)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int x \arccos(ax)\, dx = \frac{1}{2} x^2 \arccos(ax) - \frac{\sqrt{-a^2x^2 + 1}x}{4a} - \frac{\arccos(ax)}{4a^2}$$

```
[In] integrate(x*arccos(a*x),x, algorithm="giac")
[Out] 1/2*x^2*arccos(a*x) - 1/4*sqrt(-a^2*x^2 + 1)*x/a - 1/4*arccos(a*x)/a^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.84

$$\int x \arccos(ax)\, dx = \frac{\operatorname{acos}(a\,x)\,\left(2\,a^2\,x^2 - 1\right)}{4\,a^2} - \frac{x\,\sqrt{1 - a^2\,x^2}}{4\,a}$$

```
[In] int(x*acos(a*x),x)
[Out] (acos(a*x)*(2*a^2*x^2 - 1))/(4*a^2) - (x*(1 - a^2*x^2)^(1/2))/(4*a)
```

3.5 $\int \arccos(ax)\,dx$

Optimal result

Integrand size = 4, antiderivative size = 26

$$\int \arccos(ax)\,dx = -\frac{\sqrt{1-a^2x^2}}{a} + x\arccos(ax)$$

```
[Out] x*arccos(a*x)-(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4716, 267}

$$\int \arccos(ax)\,dx = x\arccos(ax) - \frac{\sqrt{1-a^2x^2}}{a}$$

```
[In] Int[ArcCos[a*x],x]
```

```
[Out] -(Sqrt[1 - a^2*x^2]/a) + x*ArcCos[a*x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\arccos(ax) + a\int \frac{x}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{\sqrt{1-a^2x^2}}{a} + x\arccos(ax)\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \arccos(ax)\,dx = -\frac{\sqrt{1-a^2x^2}}{a} + x\arccos(ax)$$

```
[In] Integrate[ArcCos[a*x],x]
[Out] -(Sqrt[1 - a^2*x^2]/a) + x*ArcCos[a*x]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
parts	$x\arccos\left(ax\right)-\frac{\sqrt{-a^2x^2+1}}{a}$	25
derivativedivides	$\frac{ax\arccos(ax)-\sqrt{-a^2x^2+1}}{a}$	27
default	$\frac{ax\arccos(ax)-\sqrt{-a^2x^2+1}}{a}$	27

```
[In] int(arccos(a*x),x,method=_RETURNVERBOSE)
[Out] x*arccos(a*x)-(-a^2*x^2+1)^(1/2)/a
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \arccos(ax)\,dx = \frac{ax\arccos\left(ax\right) - \sqrt{-a^2x^2+1}}{a}$$

```
[In] integrate(arccos(a*x),x, algorithm="fricas")
[Out] (a*x*arccos(a*x) - sqrt(-a^2*x^2 + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \arccos(ax)\,dx = \begin{cases} x\operatorname{acos}(ax) - \frac{\sqrt{-a^2x^2+1}}{a} & \text{for } a \neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}$$

```
[In] integrate(acos(a*x),x)
[Out] Piecewise((x*acos(a*x) - sqrt(-a**2*x**2 + 1)/a, Ne(a, 0)), (pi*x/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \arccos(ax)\,dx = \frac{ax\arccos(ax) - \sqrt{-a^2x^2+1}}{a}$$

```
[In] integrate(arccos(a*x),x, algorithm="maxima")
[Out] (a*x*arccos(a*x) - sqrt(-a^2*x^2 + 1))/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \arccos(ax)\,dx = \frac{ax\arccos(ax) - \sqrt{-a^2x^2+1}}{a}$$

```
[In] integrate(arccos(a*x),x, algorithm="giac")
[Out] (a*x*arccos(a*x) - sqrt(-a^2*x^2 + 1))/a
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int \arccos(ax)\,dx = x\,\mathrm{acos}(a\,x) - \frac{\sqrt{1-a^2\,x^2}}{a}$$

```
[In] int(acos(a*x),x)
[Out] x*acos(a*x) - (1 - a^2*x^2)^(1/2)/a
```

3.6 $\int \frac{\arccos(ax)}{x} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 51

$$\int \frac{\arccos(ax)}{x} \, dx = -\frac{1}{2} i \arccos(ax)^2 + \arccos(ax) \log \left(1 + e^{2i \arccos(ax)}\right) - \frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2i \arccos(ax)}\right)$$

```
[Out] -1/2*I*arccos(a*x)^2+arccos(a*x)*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-1/2*I*p
olylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4722, 3800, 2221, 2317, 2438}

$$\int \frac{\arccos(ax)}{x} \, dx = -\frac{1}{2} i \operatorname{PolyLog}\left(2, -e^{2i \arccos(ax)}\right) - \frac{1}{2} i \arccos(ax)^2 + \arccos(ax) \log \left(1 + e^{2i \arccos(ax)}\right)$$

```
[In] Int[ArcCos[a*x]/x,x]
```

```
[Out] (-1/2*I)*ArcCos[a*x]^2 + ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])] - (I/2)
*PolyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x\tan(x)\,dx, x, \arccos(ax)\right)\\
&= -\frac{1}{2}i\arccos(ax)^2 + 2i\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \arccos(ax)\right)\\
&= -\frac{1}{2}i\arccos(ax)^2 + \arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)\\
&\quad - \text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \arccos(ax)\right)\\
&= -\frac{1}{2}i\arccos(ax)^2 + \arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right)\\
&\quad + \frac{1}{2}i\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\arccos(ax)}\right)\\
&= -\frac{1}{2}i\arccos(ax)^2 + \arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)}{x}\,dx = -\frac{1}{2}i\arccos(ax)^2 + \arccos(ax)\log\left(1 + e^{2i\arccos(ax)}\right) - \frac{1}{2}i\,\text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right)$$

```
[In] Integrate[ArcCos[a*x]/x,x]
[Out] (-1/2*I)*ArcCos[a*x]^2 + ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])] - (I/2)
*PolyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{i\arccos(ax)^2}{2} + \arccos(ax)\ln\left(1 + \left(i\sqrt{-a^2x^2+1} + ax\right)^2\right) - \frac{i\,\text{polylog}\left(2, -\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right)}{2}$
default	$-\frac{i\arccos(ax)^2}{2} + \arccos(ax)\ln\left(1 + \left(i\sqrt{-a^2x^2+1} + ax\right)^2\right) - \frac{i\,\text{polylog}\left(2, -\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right)}{2}$

```
[In] int(arccos(a*x)/x,x,method=_RETURNVERBOSE)
[Out] -1/2*I*arccos(a*x)^2+arccos(a*x)*ln(1+(I*(-a^2*x^2+1)^(1/2)+a*x)^2)-1/2*I*p
olylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)
```

Fricas [F]

$$\int \frac{\arccos(ax)}{x}\,dx = \int \frac{\arccos(ax)}{x}\,dx$$

```
[In] integrate(arccos(a*x)/x,x, algorithm="fricas")
[Out] integral(arccos(a*x)/x, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)}{x}\,dx = \int \frac{\operatorname{acos}(ax)}{x}\,dx$$

```
[In] integrate(acos(a*x)/x,x)
[Out] Integral(acos(a*x)/x, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)}{x}\,dx = \int \frac{\arccos(ax)}{x}\,dx$$

```
[In] integrate(arccos(a*x)/x,x, algorithm="maxima")
[Out] integrate(arccos(a*x)/x, x)
```

Giac [F]

$$\int \frac{\arccos(ax)}{x}\,dx = \int \frac{\arccos(ax)}{x}\,dx$$

```
[In] integrate(arccos(a*x)/x,x, algorithm="giac")
[Out] integrate(arccos(a*x)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x}\,dx = \int \frac{\operatorname{acos}(a\,x)}{x}\,dx$$

```
[In] int(acos(a*x)/x,x)
[Out] int(acos(a*x)/x, x)
```

3.7 $\int \frac{\arccos(ax)}{x^2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{\arccos(ax)}{x^2}\,dx = -\frac{\arccos(ax)}{x} + a\,\mathrm{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -arccos(a*x)/x+a*arctanh((-a^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4724, 272, 65, 214}

$$\int \frac{\arccos(ax)}{x^2}\,dx = a\,\mathrm{arctanh}\left(\sqrt{1-a^2x^2}\right) - \frac{\arccos(ax)}{x}$$

```
[In] Int[ArcCos[a*x]/x^2,x]
```

```
[Out] -(ArcCos[a*x]/x) + a*ArcTanh[Sqrt[1 - a^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)}{x} - a\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\arccos(ax)}{x} - \frac{1}{2}a\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= -\frac{\arccos(ax)}{x} + \frac{\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right)}{a} \\
&= -\frac{\arccos(ax)}{x} + a\text{arctanh}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\arccos(ax)}{x^2}\,dx = -\frac{\arccos(ax)}{x} - a\log(x) + a\log\left(1+\sqrt{1-a^2x^2}\right)$$

```
[In] Integrate[ArcCos[a*x]/x^2,x]
[Out] -(ArcCos[a*x]/x) - a*Log[x] + a*Log[1 + Sqrt[1 - a^2*x^2]]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
parts	$-\frac{\arccos(ax)}{x}+a\ \mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$	26
derivativedivides	$a\left(-\frac{\arccos(ax)}{ax}+\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)$	29
default	$a\left(-\frac{\arccos(ax)}{ax}+\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)\right)$	29

```
[In] int(arccos(a*x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -arccos(a*x)/x+a*arctanh(1/(-a^2*x^2+1)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(25) = 50.

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 3.04

$$\int \frac{\arccos(ax)}{x^2}\,dx = \frac{ax\log\left(\sqrt{-a^2x^2+1}+1\right)-ax\log\left(\sqrt{-a^2x^2+1}-1\right)+2\,(x-1)\arccos\left(ax\right)-2\,x\arctan\left(\frac{\sqrt{-a^2x^2+1}ax}{a^2x^2-1}\right)}{2\,x}$$

```
[In] integrate(arccos(a*x)/x^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*x*log(sqrt(-a^2*x^2 + 1) + 1) - a*x*log(sqrt(-a^2*x^2 + 1) - 1) + 2*
(x - 1)*arccos(a*x) - 2*x*arctan(sqrt(-a^2*x^2 + 1)*a*x/(a^2*x^2 - 1)))/x
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.26

$$\int \frac{\arccos(ax)}{x^2}\,dx = -a\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{ax}\right) & \text{for}\ \frac{1}{|a^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{ax}\right) & \text{otherwise}\end{cases}\right)-\frac{\operatorname{acos}\left(ax\right)}{x}$$

```
[In] integrate(acos(a*x)/x**2,x)
```

```
[Out] -a*Piecewise((-acosh(1/(a*x)), 1/Abs(a**2*x**2) > 1), (I*asin(1/(a*x)), Tru
e)) - acos(a*x)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\arccos(ax)}{x^2}\,dx = a\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\arccos(ax)}{x}$$

```
[In] integrate(arccos(a*x)/x^2,x, algorithm="maxima")
[Out] a*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(a*x)/x
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.78

$$\int \frac{\arccos(ax)}{x^2}\,dx = \frac{1}{2}a\left(\log\left(\sqrt{-a^2x^2+1}+1\right)-\log\left(-\sqrt{-a^2x^2+1}+1\right)\right)-\frac{\arccos(ax)}{x}$$

```
[In] integrate(arccos(a*x)/x^2,x, algorithm="giac")
[Out] 1/2*a*(log(sqrt(-a^2*x^2 + 1) + 1) - log(-sqrt(-a^2*x^2 + 1) + 1)) - arccos
(a*x)/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{\arccos(ax)}{x^2}\,dx = a\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-a^2\,x^2}}\right)-\frac{\mathrm{acos}(a\,x)}{x}$$

```
[In] int(acos(a*x)/x^2,x)
[Out] a*atanh(1/(1 - a^2*x^2)^(1/2)) - acos(a*x)/x
```

3.8 $\int \frac{\arccos(ax)}{x^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 34

$$\int \frac{\arccos(ax)}{x^3} dx = \frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\arccos(ax)}{2x^2}$$

```
[Out] -1/2*arccos(a*x)/x^2+1/2*a*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4724, 270}

$$\int \frac{\arccos(ax)}{x^3} dx = \frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\arccos(ax)}{2x^2}$$

```
[In] Int[ArcCos[a*x]/x^3,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2])/(2*x) - ArcCos[a*x]/(2*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
```

```
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\arccos(ax)}{2x^2} - \frac{1}{2}a\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{a\sqrt{1-a^2x^2}}{2x} - \frac{\arccos(ax)}{2x^2}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int \frac{\arccos(ax)}{x^3}\,dx = \frac{ax\sqrt{1-a^2x^2} - \arccos(ax)}{2x^2}$$

```
[In] Integrate[ArcCos[a*x]/x^3,x]
[Out] (a*x*Sqrt[1 - a^2*x^2] - ArcCos[a*x])/(2*x^2)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

method	result	size
parts	$-\frac{\arccos(ax)}{2x^2}+\frac{a\sqrt{-a^2x^2+1}}{2x}$	29
derivativedivides	$a^2\left(-\frac{\arccos(ax)}{2a^2x^2}+\frac{\sqrt{-a^2x^2+1}}{2ax}\right)$	38
default	$a^2\left(-\frac{\arccos(ax)}{2a^2x^2}+\frac{\sqrt{-a^2x^2+1}}{2ax}\right)$	38

```
[In] int(arccos(a*x)/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*arccos(a*x)/x^2+1/2*a*(-a^2*x^2+1)^(1/2)/x
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.79

$$\int \frac{\arccos(ax)}{x^3}\,dx = \frac{\sqrt{-a^2x^2+1}ax - \arccos{(ax)}}{2\,x^2}$$

```
[In] integrate(arccos(a*x)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-a^2*x^2 + 1)*a*x - arccos(a*x))/x^2
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.72 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.56

$$\int \frac{\arccos(ax)}{x^3}\,dx = -\frac{a\left(\begin{cases} -\frac{i\sqrt{a^2x^2-1}}{x} & \text{for}\ |a^2x^2| > 1 \\ -\frac{\sqrt{-a^2x^2+1}}{x} & \text{otherwise} \end{cases}\right)}{2} - \frac{\operatorname{acos}{(ax)}}{2x^2}$$

```
[In] integrate(acos(a*x)/x**3,x)
```

```
[Out] -a*Piecewise((-I*sqrt(a**2*x**2 - 1)/x, Abs(a**2*x**2) > 1), (-sqrt(-a**2*x
**2 + 1)/x, True))/2 - acos(a*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int \frac{\arccos(ax)}{x^3}\,dx = \frac{\sqrt{-a^2x^2+1}a}{2\,x} - \frac{\arccos{(ax)}}{2\,x^2}$$

```
[In] integrate(arccos(a*x)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(-a^2*x^2 + 1)*a/x - 1/2*arccos(a*x)/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.00

$$\int \frac{\arccos(ax)}{x^3} \, dx = -\frac{1}{4} \left(\frac{a^4 x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|} \right) a - \frac{\arccos(ax)}{2\,x^2}$$

```
[In] integrate(arccos(a*x)/x^3,x, algorithm="giac")
[Out] -1/4*(a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)*
abs(a) + a)/(x*abs(a)))*a - 1/2*arccos(a*x)/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x^3} \, dx = \int \frac{\operatorname{acos}(a\,x)}{x^3} \, dx$$

```
[In] int(acos(a*x)/x^3,x)
[Out] int(acos(a*x)/x^3, x)
```

3.9 $\int \frac{\arccos(ax)}{x^4}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 56

$$\int \frac{\arccos(ax)}{x^4}\,dx = \frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\arccos(ax)}{3x^3} + \frac{1}{6}a^3\text{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/3*arccos(a*x)/x^3+1/6*a^3*arctanh((-a^2*x^2+1)^(1/2))+1/6*a*(-a^2*x^2+1)
^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4724, 272, 44, 65, 214}

$$\int \frac{\arccos(ax)}{x^4}\,dx = \frac{a\sqrt{1-a^2x^2}}{6x^2} + \frac{1}{6}a^3\text{arctanh}\left(\sqrt{1-a^2x^2}\right) - \frac{\arccos(ax)}{3x^3}$$

```
[In] Int[ArcCos[a*x]/x^4,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2])/(6*x^2) - ArcCos[a*x]/(3*x^3) + (a^3*ArcTanh[Sqrt[1 -
 a^2*x^2]])/6
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)}{3x^3} - \frac{1}{3}a\int \frac{1}{x^3\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\arccos(ax)}{3x^3} - \frac{1}{6}a\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\arccos(ax)}{3x^3} - \frac{1}{12}a^3\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\arccos(ax)}{3x^3} + \frac{1}{6}a\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\arccos(ax)}{3x^3} + \frac{1}{6}a^3\text{arctanh}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)}{x^4}\,dx = \frac{a\sqrt{1-a^2x^2}}{6x^2} - \frac{\arccos(ax)}{3x^3} - \frac{1}{6}a^3\log(x) + \frac{1}{6}a^3\log\left(1+\sqrt{1-a^2x^2}\right)$$

```
[In] Integrate[ArcCos[a*x]/x^4,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2])/(6*x^2) - ArcCos[a*x]/(3*x^3) - (a^3*Log[x])/6 + (a^3
*Log[1 + Sqrt[1 - a^2*x^2]])/6
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

method	result	size
parts	$-\frac{\arccos(ax)}{3x^3} - \frac{a\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2} - \frac{a^2\,\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)}{3}$	50
derivativedivides	$a^3\left(-\frac{\arccos(ax)}{3a^3x^3} + \frac{\sqrt{-a^2x^2+1}}{6a^2x^2} + \frac{\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{6}\right)$	53
default	$a^3\left(-\frac{\arccos(ax)}{3a^3x^3} + \frac{\sqrt{-a^2x^2+1}}{6a^2x^2} + \frac{\mathrm{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{6}\right)$	53

```
[In] int(arccos(a*x)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*arccos(a*x)/x^3-1/3*a*(-1/2/x^2*(-a^2*x^2+1)^(1/2)-1/2*a^2*arctanh(1/(
-a^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(46) = 92$.

Time = 0.27 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.96

$$\int \frac{\arccos(ax)}{x^4}\,dx = \frac{a^3x^3\log\left(\sqrt{-a^2x^2+1}+1\right) - a^3x^3\log\left(\sqrt{-a^2x^2+1}-1\right) - 4\,x^3\arctan\left(\frac{\sqrt{-a^2x^2+1}ax}{a^2x^2-1}\right) + 2\sqrt{-a^2x^2+1}ax +}{12\,x^3}$$

```
[In] integrate(arccos(a*x)/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(a^3*x^3*log(sqrt(-a^2*x^2 + 1) + 1) - a^3*x^3*log(sqrt(-a^2*x^2 + 1)
- 1) - 4*x^3*arctan(sqrt(-a^2*x^2 + 1)*a*x/(a^2*x^2 - 1)) + 2*sqrt(-a^2*x^2
 + 1)*a*x + 4*(x^3 - 1)*arccos(a*x))/x^3
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.68 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.95

$$\int \frac{\arccos(ax)}{x^4}\,dx = -\frac{a\left(\begin{cases} -\frac{a^2\operatorname{acosh}\left(\frac{1}{ax}\right)}{2} + \frac{a}{2x\sqrt{-1+\frac{1}{a^2x^2}}} - \frac{1}{2ax^3\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for } \frac{1}{|a^2x^2|} > 1 \\ \frac{ia^2\operatorname{asin}\left(\frac{1}{ax}\right)}{2} - \frac{ia\sqrt{1-\frac{1}{a^2x^2}}}{2x} & \text{otherwise} \end{cases}\right)}{3} - \frac{\operatorname{acos}(ax)}{3x^3}$$

```
[In] integrate(acos(a*x)/x**4,x)
```

```
[Out] -a*Piecewise((-a**2*acosh(1/(a*x))/2 + a/(2*x*sqrt(-1 + 1/(a**2*x**2))) - 1
/(2*a*x**3*sqrt(-1 + 1/(a**2*x**2))), 1/Abs(a**2*x**2) > 1), (I*a**2*asin(1
/(a*x))/2 - I*a*sqrt(1 - 1/(a**2*x**2))/(2*x), True))/3 - acos(a*x)/(3*x**3
)
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{\arccos(ax)}{x^4}\,dx = \frac{1}{6}\left(a^2\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-a^2x^2+1}}{x^2}\right)a - \frac{\arccos(ax)}{3\,x^3}$$

```
[In] integrate(arccos(a*x)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*(a^2*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2 + 1)/x
^2)*a - 1/3*arccos(a*x)/x^3
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int \frac{\arccos(ax)}{x^4}\,dx = \frac{a^4 \log\left(\sqrt{-a^2x^2+1}+1\right) - a^4 \log\left(-\sqrt{-a^2x^2+1}+1\right) + \frac{2\sqrt{-a^2x^2+1}a^2}{x^2}}{12\,a} - \frac{\arccos\left(ax\right)}{3\,x^3}$$

```
[In] integrate(arccos(a*x)/x^4,x, algorithm="giac")
```

```
[Out] 1/12*(a^4*log(sqrt(-a^2*x^2 + 1) + 1) - a^4*log(-sqrt(-a^2*x^2 + 1) + 1) +
2*sqrt(-a^2*x^2 + 1)*a^2/x^2)/a - 1/3*arccos(a*x)/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x^4}\,dx = \int \frac{\mathrm{acos}(a\,x)}{x^4}\,dx$$

```
[In] int(acos(a*x)/x^4,x)
```

```
[Out] int(acos(a*x)/x^4, x)
```

3.10 $\int \frac{\arccos(ax)}{x^5} dx$

Optimal result

Integrand size = 8, antiderivative size = 58

$$\int \frac{\arccos(ax)}{x^5} dx = \frac{a\sqrt{1-a^2x^2}}{12x^3} + \frac{a^3\sqrt{1-a^2x^2}}{6x} - \frac{\arccos(ax)}{4x^4}$$

```
[Out] -1/4*arccos(a*x)/x^4+1/12*a*(-a^2*x^2+1)^(1/2)/x^3+1/6*a^3*(-a^2*x^2+1)^(1/
2)/x
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4724, 277, 270}

$$\int \frac{\arccos(ax)}{x^5} dx = \frac{a\sqrt{1-a^2x^2}}{12x^3} + \frac{a^3\sqrt{1-a^2x^2}}{6x} - \frac{\arccos(ax)}{4x^4}$$

```
[In] Int[ArcCos[a*x]/x^5,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2])/(12*x^3) + (a^3*Sqrt[1 - a^2*x^2])/(6*x) - ArcCos[a*x
]/(4*x^4)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
```

```
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)}{4x^4} - \frac{1}{4}a\int \frac{1}{x^4\sqrt{1-a^2x^2}}\,dx \\
&= \frac{a\sqrt{1-a^2x^2}}{12x^3} - \frac{\arccos(ax)}{4x^4} - \frac{1}{6}a^3\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= \frac{a\sqrt{1-a^2x^2}}{12x^3} + \frac{a^3\sqrt{1-a^2x^2}}{6x} - \frac{\arccos(ax)}{4x^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.71

$$\int \frac{\arccos(ax)}{x^5}\,dx = \frac{ax\sqrt{1-a^2x^2}(1+2a^2x^2) - 3\arccos(ax)}{12x^4}$$

```
[In] Integrate[ArcCos[a*x]/x^5,x]
[Out] (a*x*Sqrt[1 - a^2*x^2]*(1 + 2*a^2*x^2) - 3*ArcCos[a*x])/(12*x^4)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

method	result	size
parts	$-\frac{\arccos(ax)}{4x^4} - \frac{a\left(-\frac{\sqrt{-a^2x^2+1}}{3x^3} - \frac{2a^2\sqrt{-a^2x^2+1}}{3x}\right)}{4}$	52
derivativedivides	$a^4\left(-\frac{\arccos(ax)}{4a^4x^4} + \frac{\sqrt{-a^2x^2+1}}{12a^3x^3} + \frac{\sqrt{-a^2x^2+1}}{6ax}\right)$	58
default	$a^4\left(-\frac{\arccos(ax)}{4a^4x^4} + \frac{\sqrt{-a^2x^2+1}}{12a^3x^3} + \frac{\sqrt{-a^2x^2+1}}{6ax}\right)$	58

```
[In] int(arccos(a*x)/x^5,x,method=_RETURNVERBOSE)
[Out] -1/4*arccos(a*x)/x^4-1/4*a*(-1/3/x^3*(-a^2*x^2+1)^(1/2)-2/3*a^2/x*(-a^2*x^2
+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.64

$$\int \frac{\arccos(ax)}{x^5}\,dx = \frac{(2\,a^3x^3 + ax)\sqrt{-a^2x^2+1} - 3\,\arccos(ax)}{12\,x^4}$$

```
[In] integrate(arccos(a*x)/x^5,x, algorithm="fricas")
[Out] 1/12*((2*a^3*x^3 + a*x)*sqrt(-a^2*x^2 + 1) - 3*arccos(a*x))/x^4
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.76

$$\int \frac{\arccos(ax)}{x^5}\,dx = -\frac{a\left(\begin{cases} -\frac{2ia^2\sqrt{a^2x^2-1}}{3x} - \frac{i\sqrt{a^2x^2-1}}{3x^3} & \text{for } |a^2x^2| > 1 \\ -\frac{2a^2\sqrt{-a^2x^2+1}}{3x} - \frac{\sqrt{-a^2x^2+1}}{3x^3} & \text{otherwise} \end{cases}\right)}{4} - \frac{\operatorname{acos}(ax)}{4x^4}$$

```
[In] integrate(acos(a*x)/x**5,x)
[Out] -a*Piecewise((-2*I*a**2*sqrt(a**2*x**2 - 1)/(3*x) - I*sqrt(a**2*x**2 - 1)/(
3*x**3), Abs(a**2*x**2) > 1), (-2*a**2*sqrt(-a**2*x**2 + 1)/(3*x) - sqrt(-a
**2*x**2 + 1)/(3*x**3), True))/4 - acos(a*x)/(4*x**4)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.86

$$\int \frac{\arccos(ax)}{x^5}\,dx = \frac{1}{12}\left(\frac{2\sqrt{-a^2x^2+1}a^2}{x} + \frac{\sqrt{-a^2x^2+1}}{x^3}\right)a - \frac{\arccos(ax)}{4\,x^4}$$

```
[In] integrate(arccos(a*x)/x^5,x, algorithm="maxima")
[Out] 1/12*(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a^2*x^2 + 1)/x^3)*a - 1/4*arccos(a
*x)/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. 2(48) = 96.

Time = 0.28 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.24

$$\int \frac{\arccos(ax)}{x^5}\,dx = -\frac{1}{96}\left(\frac{\left(a^4+\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}\right)a^6x^3}{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|}-\frac{\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4}{x}+\frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{x^3}}{a^2|a|}\right)a - \frac{\arccos(ax)}{4\,x^4}$$

```
[In] integrate(arccos(a*x)/x^5,x, algorithm="giac")
```

```
[Out] -1/96*((a^4 + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/x^2)*a^6*x^3/((sqrt(-a^2*
x^2 + 1)*abs(a) + a)^3*abs(a)) - (9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x +
 (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/x^3)/(a^2*abs(a)))*a - 1/4*arccos(a*x)/x
^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x^5}\,dx = \int \frac{\mathrm{acos}(a\,x)}{x^5}\,dx$$

```
[In] int(acos(a*x)/x^5,x)
```

```
[Out] int(acos(a*x)/x^5, x)
```

3.11 $\int \frac{\arccos(ax)}{x^6} dx$

Optimal result

Integrand size = 8, antiderivative size = 80

$$\int \frac{\arccos(ax)}{x^6} dx = \frac{a\sqrt{1-a^2x^2}}{20x^4} + \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\arccos(ax)}{5x^5} + \frac{3}{40}a^5 \operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right)$$

```
[Out] -1/5*arccos(a*x)/x^5+3/40*a^5*arctanh((-a^2*x^2+1)^(1/2))+1/20*a*(-a^2*x^2+
1)^(1/2)/x^4+3/40*a^3*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4724, 272, 44, 65, 214}

$$\int \frac{\arccos(ax)}{x^6} dx = \frac{a\sqrt{1-a^2x^2}}{20x^4} + \frac{3}{40}a^5 \operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right) + \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\arccos(ax)}{5x^5}$$

```
[In] Int[ArcCos[a*x]/x^6,x]
```

```
[Out] (a*Sqrt[1 - a^2*x^2])/(20*x^4) + (3*a^3*Sqrt[1 - a^2*x^2])/(40*x^2) - ArcCo
s[a*x]/(5*x^5) + (3*a^5*ArcTanh[Sqrt[1 - a^2*x^2]])/40
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)}{5x^5} - \frac{1}{5}a\int \frac{1}{x^5\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\arccos(ax)}{5x^5} - \frac{1}{10}a\text{Subst}\left(\int \frac{1}{x^3\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{20x^4} - \frac{\arccos(ax)}{5x^5} - \frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{20x^4} + \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\arccos(ax)}{5x^5} - \frac{1}{80}\left(3a^5\right)\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{20x^4} + \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\arccos(ax)}{5x^5} + \frac{1}{40}\left(3a^3\right)\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) \\
&= \frac{a\sqrt{1-a^2x^2}}{20x^4} + \frac{3a^3\sqrt{1-a^2x^2}}{40x^2} - \frac{\arccos(ax)}{5x^5} + \frac{3}{40}a^5\text{arctanh}\left(\sqrt{1-a^2x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.90

$$\int \frac{\arccos(ax)}{x^6}\,dx = \frac{1}{40}\left(\frac{a\sqrt{1-a^2x^2}(2+3a^2x^2)}{x^4} - \frac{8\arccos(ax)}{x^5} - 3a^5\log(x) + 3a^5\log\left(1+\sqrt{1-a^2x^2}\right)\right)$$

```
[In] Integrate[ArcCos[a*x]/x^6,x]
```

```
[Out] ((a*Sqrt[1 - a^2*x^2]*(2 + 3*a^2*x^2))/x^4 - (8*ArcCos[a*x])/x^5 - 3*a^5*Lo
g[x] + 3*a^5*Log[1 + Sqrt[1 - a^2*x^2]])/40
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$a^5\left(-\frac{\arccos(ax)}{5a^5x^5}+\frac{\sqrt{-a^2x^2+1}}{20a^4x^4}+\frac{3\sqrt{-a^2x^2+1}}{40a^2x^2}+\frac{3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{40}\right)$	73
default	$a^5\left(-\frac{\arccos(ax)}{5a^5x^5}+\frac{\sqrt{-a^2x^2+1}}{20a^4x^4}+\frac{3\sqrt{-a^2x^2+1}}{40a^2x^2}+\frac{3\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{40}\right)$	73
parts	$-\frac{\arccos(ax)}{5x^5}-\frac{a\left(-\frac{\sqrt{-a^2x^2+1}}{4x^4}+\frac{3a^2\left(-\frac{\sqrt{-a^2x^2+1}}{2x^2}-\frac{a^2\,\operatorname{arctanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)}{2}\right)}{4}\right)}{5}$	73

```
[In] int(arccos(a*x)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*(-1/5/a^5/x^5*arccos(a*x)+1/20/a^4/x^4*(-a^2*x^2+1)^(1/2)+3/40/a^2/x^2*
(-a^2*x^2+1)^(1/2)+3/40*arctanh(1/(-a^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.52

$$\int \frac{\arccos(ax)}{x^6}\,dx = \frac{3\,a^5x^5\log\left(\sqrt{-a^2x^2+1}+1\right) - 3\,a^5x^5\log\left(\sqrt{-a^2x^2+1}-1\right) - 16\,x^5\arctan\left(\frac{\sqrt{-a^2x^2+1}ax}{a^2x^2-1}\right) + 16\left(x^5-1\right)\mathrm{arc}}{80\,x^5}$$

```
[In] integrate(arccos(a*x)/x^6,x, algorithm="fricas")
```

```
[Out] 1/80*(3*a^5*x^5*log(sqrt(-a^2*x^2 + 1) + 1) - 3*a^5*x^5*log(sqrt(-a^2*x^2 +
 1) - 1) - 16*x^5*arctan(sqrt(-a^2*x^2 + 1)*a*x/(a^2*x^2 - 1)) + 16*(x^5 -
1)*arccos(a*x) + 2*(3*a^3*x^3 + 2*a*x)*sqrt(-a^2*x^2 + 1))/x^5
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 3.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.30

$$\int \frac{\arccos(ax)}{x^6}\,dx = -\frac{a\left(\begin{cases} -\frac{3a^4\operatorname{acosh}\left(\frac{1}{ax}\right)}{8} + \frac{3a^3}{8x\sqrt{-1+\frac{1}{a^2x^2}}} - \frac{a}{8x^3\sqrt{-1+\frac{1}{a^2x^2}}} - \frac{1}{4ax^5\sqrt{-1+\frac{1}{a^2x^2}}} & \text{for}\ \frac{1}{|a^2x^2|} > 1 \\ \frac{3ia^4\operatorname{asin}\left(\frac{1}{ax}\right)}{8} - \frac{3ia^3}{8x\sqrt{1-\frac{1}{a^2x^2}}} + \frac{ia}{8x^3\sqrt{1-\frac{1}{a^2x^2}}} + \frac{i}{4ax^5\sqrt{1-\frac{1}{a^2x^2}}} & \text{otherwise} \end{cases}\right)}{5} - \frac{\operatorname{acos}(ax)}{5x^5}$$

```
[In] integrate(acos(a*x)/x**6,x)
```

```
[Out] -a*Piecewise((-3*a**4*acosh(1/(a*x))/8 + 3*a**3/(8*x*sqrt(-1 + 1/(a**2*x**2
))) - a/(8*x**3*sqrt(-1 + 1/(a**2*x**2))) - 1/(4*a*x**5*sqrt(-1 + 1/(a**2*x
**2))), 1/Abs(a**2*x**2) > 1), (3*I*a**4*asin(1/(a*x))/8 - 3*I*a**3/(8*x*sq
rt(1 - 1/(a**2*x**2))) + I*a/(8*x**3*sqrt(1 - 1/(a**2*x**2))) + I/(4*a*x**5
*sqrt(1 - 1/(a**2*x**2))), True))/5 - acos(a*x)/(5*x**5)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.02

$$\int \frac{\arccos(ax)}{x^6}\,dx = \frac{1}{40}\left(3\,a^4\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|}+\frac{2}{|x|}\right)+\frac{3\sqrt{-a^2x^2+1}a^2}{x^2}+\frac{2\sqrt{-a^2x^2+1}}{x^4}\right)a - \frac{\arccos(ax)}{5\,x^5}$$

```
[In] integrate(arccos(a*x)/x^6,x, algorithm="maxima")
```

```
[Out] 1/40*(3*a^4*log(2*sqrt(-a^2*x^2 + 1)/abs(x) + 2/abs(x)) + 3*sqrt(-a^2*x^2 +
 1)*a^2/x^2 + 2*sqrt(-a^2*x^2 + 1)/x^4)*a - 1/5*arccos(a*x)/x^5
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{\arccos(ax)}{x^6}\,dx = \frac{3\,a^6\log\left(\sqrt{-a^2x^2+1}+1\right)-3\,a^6\log\left(-\sqrt{-a^2x^2+1}+1\right)-\frac{2\left(3\,(-a^2x^2+1)^{\frac{3}{2}}a^6-5\sqrt{-a^2x^2+1}a^6\right)}{a^4x^4}}{80\,a} - \frac{\arccos(ax)}{5\,x^5}$$

```
[In] integrate(arccos(a*x)/x^6,x, algorithm="giac")
```

```
[Out] 1/80*(3*a^6*log(sqrt(-a^2*x^2 + 1) + 1) - 3*a^6*log(-sqrt(-a^2*x^2 + 1) + 1
) - 2*(3*(-a^2*x^2 + 1)^(3/2)*a^6 - 5*sqrt(-a^2*x^2 + 1)*a^6)/(a^4*x^4))/a
- 1/5*arccos(a*x)/x^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)}{x^6}\,dx = \int \frac{\operatorname{acos}(a\,x)}{x^6}\,dx$$

```
[In] int(acos(a*x)/x^6,x)
```

```
[Out] int(acos(a*x)/x^6, x)
```

3.12 $\int x^4 \arccos(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 120

$$\int x^4 \arccos(ax)^2 \, dx = -\frac{16x}{75a^4} - \frac{8x^3}{225a^2} - \frac{2x^5}{125} - \frac{16\sqrt{1-a^2x^2}\arccos(ax)}{75a^5} - \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{75a^3} - \frac{2x^4\sqrt{1-a^2x^2}\arccos(ax)}{25a} + \frac{1}{5}x^5\arccos(ax)^2$$

```
[Out] -16/75*x/a^4-8/225*x^3/a^2-2/125*x^5+1/5*x^5*arccos(a*x)^2-16/75*arccos(a*x
)*(-a^2*x^2+1)^(1/2)/a^5-8/75*x^2*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a^3-2/25*x
^4*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4724, 4796, 4768, 8, 30}

$$\int x^4 \arccos(ax)^2 \, dx = -\frac{16x}{75a^4} - \frac{2x^4\sqrt{1-a^2x^2}\arccos(ax)}{25a} - \frac{8x^3}{225a^2} - \frac{16\sqrt{1-a^2x^2}\arccos(ax)}{75a^5} - \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{75a^3} + \frac{1}{5}x^5\arccos(ax)^2 - \frac{2x^5}{125}$$

```
[In] Int[x^4*ArcCos[a*x]^2,x]
```

```
[Out] (-16*x)/(75*a^4) - (8*x^3)/(225*a^2) - (2*x^5)/125 - (16*Sqrt[1 - a^2*x^2]*
ArcCos[a*x])/(75*a^5) - (8*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(75*a^3) - (2
*x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(25*a) + (x^5*ArcCos[a*x]^2)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\arccos(ax)^2 + \frac{1}{5}(2a)\int \frac{x^5\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{2x^4\sqrt{1-a^2x^2}\arccos(ax)}{25a} + \frac{1}{5}x^5\arccos(ax)^2 - \frac{2\int x^4\,dx}{25} + \frac{8\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{25a} \\
&= -\frac{2x^5}{125} - \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{75a^3} - \frac{2x^4\sqrt{1-a^2x^2}\arccos(ax)}{25a} \\
&\quad + \frac{1}{5}x^5\arccos(ax)^2 + \frac{16\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{75a^3} - \frac{8\int x^2\,dx}{75a^2}
\end{aligned}$$

$$= -\frac{8x^3}{225a^2} - \frac{2x^5}{125} - \frac{16\sqrt{1-a^2x^2}\arccos(ax)}{75a^5} - \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{75a^3} - \frac{2x^4\sqrt{1-a^2x^2}\arccos(ax)}{25a} + \frac{1}{5}x^5\arccos(ax)^2 - \frac{16\int 1\,dx}{75a^4}$$

$$= -\frac{16x}{75a^4} - \frac{8x^3}{225a^2} - \frac{2x^5}{125} - \frac{16\sqrt{1-a^2x^2}\arccos(ax)}{75a^5} - \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{75a^3} - \frac{2x^4\sqrt{1-a^2x^2}\arccos(ax)}{25a} + \frac{1}{5}x^5\arccos(ax)^2$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

$$\int x^4\arccos(ax)^2\,dx = -\frac{16x}{75a^4} - \frac{8x^3}{225a^2} - \frac{2x^5}{125} - \frac{2\sqrt{1-a^2x^2}(8+4a^2x^2+3a^4x^4)\arccos(ax)}{75a^5} + \frac{1}{5}x^5\arccos(ax)^2$$

```
[In] Integrate[x^4*ArcCos[a*x]^2,x]
```

```
[Out] (-16*x)/(75*a^4) - (8*x^3)/(225*a^2) - (2*x^5)/125 - (2*Sqrt[1 - a^2*x^2]*(
8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcCos[a*x])/(75*a^5) + (x^5*ArcCos[a*x]^2)/5
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.63

method	result	size
derivativedivides	$\frac{\frac{a^5x^5\arccos(ax)^2}{5}-\frac{2\arccos(ax)\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{75}-\frac{2a^5x^5}{125}-\frac{8a^3x^3}{225}-\frac{16ax}{75}}{a^5}$	76
default	$\frac{\frac{a^5x^5\arccos(ax)^2}{5}-\frac{2\arccos(ax)\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{75}-\frac{2a^5x^5}{125}-\frac{8a^3x^3}{225}-\frac{16ax}{75}}{a^5}$	76

```
[In] int(x^4*arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*a^5*x^5*arccos(a*x)^2-2/75*arccos(a*x)*(3*a^4*x^4+4*a^2*x^2+8)*(
-a^2*x^2+1)^(1/2)-2/125*a^5*x^5-8/225*a^3*x^3-16/75*a*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.63

$$\int x^4 \arccos(ax)^2\,dx$$
$$= \frac{225\,a^5x^5 \arccos{(ax)}^2 - 18\,a^5x^5 - 40\,a^3x^3 - 30\,(3\,a^4x^4 + 4\,a^2x^2 + 8)\sqrt{-a^2x^2+1} \arccos{(ax)} - 240\,ax}{1125\,a^5}$$

```
[In] integrate(x^4*arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/1125*(225*a^5*x^5*arccos(a*x)^2 - 18*a^5*x^5 - 40*a^3*x^3 - 30*(3*a^4*x^4
 + 4*a^2*x^2 + 8)*sqrt(-a^2*x^2 + 1)*arccos(a*x) - 240*a*x)/a^5
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01

$$\int x^4 \arccos(ax)^2\,dx$$
$$= \begin{cases} \frac{x^5 \operatorname{acos}^2{(ax)}}{5} - \frac{2x^5}{125} - \frac{2x^4\sqrt{-a^2x^2+1}\operatorname{acos}{(ax)}}{25a} - \frac{8x^3}{225a^2} - \frac{8x^2\sqrt{-a^2x^2+1}\operatorname{acos}{(ax)}}{75a^3} - \frac{16x}{75a^4} - \frac{16\sqrt{-a^2x^2+1}\operatorname{acos}{(ax)}}{75a^5} & \text{for } a \neq 0 \\ \frac{\pi^2x^5}{20} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**4*acos(a*x)**2,x)
```

```
[Out] Piecewise((x**5*acos(a*x)**2/5 - 2*x**5/125 - 2*x**4*sqrt(-a**2*x**2 + 1)*a
cos(a*x)/(25*a) - 8*x**3/(225*a**2) - 8*x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)
/(75*a**3) - 16*x/(75*a**4) - 16*sqrt(-a**2*x**2 + 1)*acos(a*x)/(75*a**5),
Ne(a, 0)), (pi**2*x**5/20, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.85

$$\int x^4 \arccos(ax)^2\,dx$$
$$= \frac{1}{5}\,x^5 \arccos{(ax)}^2 - \frac{2}{75}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2} + \frac{4\sqrt{-a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{-a^2x^2+1}}{a^6}\right)a \arccos{(ax)} - \frac{2\,(9\,a^4x^5 + 20\,a^2x^3 + 120\,x)}{1125\,a^4}$$

```
[In] integrate(x^4*arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccos(a*x)^2 - 2/75*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^
2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a*arccos(a*x) - 2/1125*(9*a^4*x^
5 + 20*a^2*x^3 + 120*x)/a^4
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int x^4 \arccos(ax)^2\, dx = \frac{1}{5}\, x^5 \arccos{(ax)}^2 - \frac{2}{125}\, x^5 - \frac{2\,\sqrt{-a^2x^2+1}x^4 \arccos{(ax)}}{25\,a} - \frac{8\,x^3}{225\,a^2} - \frac{8\,\sqrt{-a^2x^2+1}x^2 \arccos{(ax)}}{75\,a^3} - \frac{16\,x}{75\,a^4} - \frac{16\,\sqrt{-a^2x^2+1}\arccos{(ax)}}{75\,a^5}$$

```
[In] integrate(x^4*arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/5*x^5*arccos(a*x)^2 - 2/125*x^5 - 2/25*sqrt(-a^2*x^2 + 1)*x^4*arccos(a*x)
/a - 8/225*x^3/a^2 - 8/75*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)/a^3 - 16/75*x/
a^4 - 16/75*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^2\, dx = \int x^4\, \mathrm{acos}(a\,x)^2\, dx$$

```
[In] int(x^4*acos(a*x)^2,x)
```

```
[Out] int(x^4*acos(a*x)^2, x)
```

3.13 $\int x^3 \arccos(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 98

$$\int x^3 \arccos(ax)^2 \, dx = -\frac{3x^2}{32a^2} - \frac{x^4}{32} - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{16a^3} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{8a} - \frac{3\arccos(ax)^2}{32a^4} + \frac{1}{4}x^4\arccos(ax)^2$$

```
[Out] -3/32*x^2/a^2-1/32*x^4-3/32*arccos(a*x)^2/a^4+1/4*x^4*arccos(a*x)^2-3/16*x*
arccos(a*x)*(-a^2*x^2+1)^(1/2)/a^3-1/8*x^3*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4724, 4796, 4738, 30}

$$\int x^3 \arccos(ax)^2 \, dx = -\frac{3\arccos(ax)^2}{32a^4} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{8a} - \frac{3x^2}{32a^2} - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{16a^3} + \frac{1}{4}x^4\arccos(ax)^2 - \frac{x^4}{32}$$

```
[In] Int[x^3*ArcCos[a*x]^2,x]
```

```
[Out] (-3*x^2)/(32*a^2) - x^4/32 - (3*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(16*a^3) -
 (x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(8*a) - (3*ArcCos[a*x]^2)/(32*a^4) + (
x^4*ArcCos[a*x]^2)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\arccos(ax)^2 + \frac{1}{2}a\int\frac{x^4\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{8a} + \frac{1}{4}x^4\arccos(ax)^2 - \frac{\int x^3\,dx}{8} + \frac{3\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{8a} \\
&= -\frac{x^4}{32} - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{16a^3} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{8a} \\
&\quad + \frac{1}{4}x^4\arccos(ax)^2 + \frac{3\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{16a^3} - \frac{3\int x\,dx}{16a^2} \\
&= -\frac{3x^2}{32a^2} - \frac{x^4}{32} - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{16a^3} \\
&\quad - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)}{8a} - \frac{3\arccos(ax)^2}{32a^4} + \frac{1}{4}x^4\arccos(ax)^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.76

$$\int x^3 \arccos(ax)^2\,dx = \frac{-a^2x^2(3+a^2x^2) - 2ax\sqrt{1-a^2x^2}(3+2a^2x^2)\arccos(ax) + (-3+8a^4x^4)\arccos(ax)^2}{32a^4}$$

```
[In] Integrate[x^3*ArcCos[a*x]^2,x]
```

```
[Out] (-(a^2*x^2*(3 + a^2*x^2)) - 2*a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)*ArcCos[
a*x] + (-3 + 8*a^4*x^4)*ArcCos[a*x]^2)/(32*a^4)
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{\frac{a^4x^4\arccos(ax)^2}{4}-\frac{\arccos(ax)\left(2a^3x^3\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}+3\arccos(ax)\right)}{16}+\frac{3\arccos(ax)^2}{32}-\frac{\left(2a^2x^2+3\right)^2}{128}}{a^4}$	91
default	$\frac{\frac{a^4x^4\arccos(ax)^2}{4}-\frac{\arccos(ax)\left(2a^3x^3\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}+3\arccos(ax)\right)}{16}+\frac{3\arccos(ax)^2}{32}-\frac{\left(2a^2x^2+3\right)^2}{128}}{a^4}$	91

```
[In] int(x^3*arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccos(a*x)^2-1/16*arccos(a*x)*(2*a^3*x^3*(-a^2*x^2+1)^(
1/2)+3*a*x*(-a^2*x^2+1)^(1/2)+3*arccos(a*x))+3/32*arccos(a*x)^2-1/128*(2*a^
2*x^2+3)^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.71

$$\int x^3 \arccos(ax)^2\,dx = -\frac{a^4x^4 + 3\,a^2x^2 - (8\,a^4x^4 - 3)\arccos{(ax)}^2 + 2\,(2\,a^3x^3 + 3\,ax)\sqrt{-a^2x^2+1}\arccos{(ax)}}{32\,a^4}$$

```
[In] integrate(x^3*arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(a^4*x^4 + 3*a^2*x^2 - (8*a^4*x^4 - 3)*arccos(a*x)^2 + 2*(2*a^3*x^3 +
 3*a*x)*sqrt(-a^2*x^2 + 1)*arccos(a*x))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99

$$\int x^3 \arccos(ax)^2 \, dx = \begin{cases} \frac{x^4 \operatorname{acos}^2(ax)}{4} - \frac{x^4}{32} - \frac{x^3\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{8a} - \frac{3x^2}{32a^2} - \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{16a^3} - \frac{3\operatorname{acos}^2(ax)}{32a^4} & \text{for } a \neq 0 \\ \frac{\pi^2 x^4}{16} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**3*acos(a*x)**2,x)
```

```
[Out] Piecewise((x**4*acos(a*x)**2/4 - x**4/32 - x**3*sqrt(-a**2*x**2 + 1)*acos(a
*x)/(8*a) - 3*x**2/(32*a**2) - 3*x*sqrt(-a**2*x**2 + 1)*acos(a*x)/(16*a**3)
 - 3*acos(a*x)**2/(32*a**4), Ne(a, 0)), (pi**2*x**4/16, True))
```

Maxima [F]

$$\int x^3 \arccos(ax)^2 \, dx = \int x^3 \arccos(ax)^2 \, dx$$

```
[In] integrate(x^3*arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2 - a*integrate(1/2*sqrt
(a*x + 1)*sqrt(-a*x + 1)*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)/(a^
2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\int x^3 \arccos(ax)^2 \, dx = \frac{1}{4} x^4 \arccos(ax)^2 - \frac{1}{32} x^4 - \frac{\sqrt{-a^2x^2+1}x^3 \arccos(ax)}{8a} - \frac{3x^2}{32a^2} - \frac{3\sqrt{-a^2x^2+1}x \arccos(ax)}{16a^3} - \frac{3\arccos(ax)^2}{32a^4} + \frac{15}{256a^4}$$

```
[In] integrate(x^3*arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arccos(a*x)^2 - 1/32*x^4 - 1/8*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)/a
 - 3/32*x^2/a^2 - 3/16*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)/a^3 - 3/32*arccos(a
*x)^2/a^4 + 15/256/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^2 \, dx = \int x^3 \operatorname{acos}(a\,x)^2 \, dx$$

```
[In] int(x^3*acos(a*x)^2,x)
[Out] int(x^3*acos(a*x)^2, x)
```

3.14 $\int x^2 \arccos(ax)^2 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int x^2 \arccos(ax)^2 \, dx = -\frac{4x}{9a^2} - \frac{2x^3}{27} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)}{9a^3} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)}{9a} + \frac{1}{3}x^3\arccos(ax)^2$$

```
[Out] -4/9*x/a^2-2/27*x^3+1/3*x^3*arccos(a*x)^2-4/9*arccos(a*x)*(-a^2*x^2+1)^(1/2
)/a^3-2/9*x^2*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4724, 4796, 4768, 8, 30}

$$\int x^2 \arccos(ax)^2 \, dx = -\frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)}{9a} - \frac{4x}{9a^2} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)}{9a^3} + \frac{1}{3}x^3\arccos(ax)^2 - \frac{2x^3}{27}$$

```
[In] Int[x^2*ArcCos[a*x]^2,x]
```

```
[Out] (-4*x)/(9*a^2) - (2*x^3)/27 - (4*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(9*a^3) - (
2*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(9*a) + (x^3*ArcCos[a*x]^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arccos(ax)^2 + \frac{1}{3}(2a)\int \frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)}{9a} + \frac{1}{3}x^3\arccos(ax)^2 - \frac{2\int x^2\,dx}{9} + \frac{4\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{9a} \\
&= -\frac{2x^3}{27} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)}{9a^3} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)}{9a} + \frac{1}{3}x^3\arccos(ax)^2 - \frac{4\int 1\,dx}{9a^2} \\
&= -\frac{4x}{9a^2} - \frac{2x^3}{27} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)}{9a^3} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)}{9a} + \frac{1}{3}x^3\arccos(ax)^2
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.77

$$\int x^2 \arccos(ax)^2 \, dx = -\frac{4x}{9a^2} - \frac{2x^3}{27} - \frac{2\sqrt{1-a^2x^2}(2+a^2x^2)\arccos(ax)}{9a^3} + \frac{1}{3}x^3\arccos(ax)^2$$

```
[In] Integrate[x^2*ArcCos[a*x]^2,x]
```

```
[Out] (-4*x)/(9*a^2) - (2*x^3)/27 - (2*Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcCos[a*x
])/(9*a^3) + (x^3*ArcCos[a*x]^2)/3
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{\frac{\arccos(ax)^2 a^3 x^3}{3} - \frac{2\arccos(ax)\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{9} - \frac{2a^3x^3}{27} - \frac{4ax}{9}}{a^3}$	59
default	$\frac{\frac{\arccos(ax)^2 a^3 x^3}{3} - \frac{2\arccos(ax)\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{9} - \frac{2a^3x^3}{27} - \frac{4ax}{9}}{a^3}$	59

```
[In] int(x^2*arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*arccos(a*x)^2*a^3*x^3-2/9*arccos(a*x)*(a^2*x^2+2)*(-a^2*x^2+1)^(
1/2)-2/27*a^3*x^3-4/9*a*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

$$\int x^2 \arccos(ax)^2 \, dx = \frac{9\,a^3x^3\arccos(ax)^2 - 2\,a^3x^3 - 6\,(a^2x^2+2)\sqrt{-a^2x^2+1}\arccos(ax) - 12\,ax}{27\,a^3}$$

```
[In] integrate(x^2*arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*arccos(a*x)^2 - 2*a^3*x^3 - 6*(a^2*x^2 + 2)*sqrt(-a^2*x^2 +
 1)*arccos(a*x) - 12*a*x)/a^3
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.01

$$\int x^2 \arccos(ax)^2\,dx = \begin{cases} \frac{x^3 \operatorname{acos}^2(ax)}{3} - \frac{2x^3}{27} - \frac{2x^2\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{9a} - \frac{4x}{9a^2} - \frac{4\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{9a^3} & \text{for } a \neq 0 \\ \frac{\pi^2x^3}{12} & \text{otherwise} \end{cases}$$

```
[In] integrate(x**2*acos(a*x)**2,x)
```

```
[Out] Piecewise((x**3*acos(a*x)**2/3 - 2*x**3/27 - 2*x**2*sqrt(-a**2*x**2 + 1)*ac
os(a*x)/(9*a) - 4*x/(9*a**2) - 4*sqrt(-a**2*x**2 + 1)*acos(a*x)/(9*a**3), N
e(a, 0)), (pi**2*x**3/12, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int x^2 \arccos(ax)^2\,dx = \frac{1}{3}\,x^3 \arccos(ax)^2 - \frac{2}{9}\,a\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\arccos(ax) - \frac{2\,(a^2x^3+6\,x)}{27\,a^2}$$

```
[In] integrate(x^2*arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(a*x)^2 - 2/9*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2
 + 1)/a^4)*arccos(a*x) - 2/27*(a^2*x^3 + 6*x)/a^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83

$$\int x^2 \arccos(ax)^2\,dx = \frac{1}{3}\,x^3 \arccos(ax)^2 - \frac{2}{27}\,x^3 - \frac{2\sqrt{-a^2x^2+1}x^2 \arccos(ax)}{9\,a} - \frac{4\,x}{9\,a^2} - \frac{4\sqrt{-a^2x^2+1}\arccos(ax)}{9\,a^3}$$

```
[In] integrate(x^2*arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(a*x)^2 - 2/27*x^3 - 2/9*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)/a
 - 4/9*x/a^2 - 4/9*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^2 \, dx = \int x^2 \operatorname{acos}(a\,x)^2 \, dx$$

```
[In] int(x^2*acos(a*x)^2,x)
[Out] int(x^2*acos(a*x)^2, x)
```

3.15 $\int x \arccos(ax)^2 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 60

$$\int x \arccos(ax)^2 \, dx = -\frac{x^2}{4} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a} - \frac{\arccos(ax)^2}{4a^2} + \frac{1}{2}x^2\arccos(ax)^2$$

```
[Out] -1/4*x^2-1/4*arccos(a*x)^2/a^2+1/2*x^2*arccos(a*x)^2-1/2*x*arccos(a*x)*(-a^
2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4724, 4796, 4738, 30}

$$\int x \arccos(ax)^2 \, dx = -\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a} - \frac{\arccos(ax)^2}{4a^2} + \frac{1}{2}x^2\arccos(ax)^2 - \frac{x^2}{4}$$

```
[In] Int[x*ArcCos[a*x]^2,x]
```

```
[Out] -1/4*x^2 - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a) - ArcCos[a*x]^2/(4*a^2)
+ (x^2*ArcCos[a*x]^2)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
```

```
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arccos(ax)^2 + a\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a} + \frac{1}{2}x^2\arccos(ax)^2 - \frac{\int x\,dx}{2} + \frac{\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{2a}\\
&= -\frac{x^2}{4} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a} - \frac{\arccos(ax)^2}{4a^2} + \frac{1}{2}x^2\arccos(ax)^2
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

$$\int x\arccos(ax)^2\,dx = -\frac{x^2}{4} - \frac{x\sqrt{1-a^2x^2}\arccos(ax)}{2a} + \frac{(-1+2a^2x^2)\arccos(ax)^2}{4a^2}$$

```
[In] Integrate[x*ArcCos[a*x]^2,x]
[Out] -1/4*x^2 - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a) + ((-1 + 2*a^2*x^2)*ArcC
os[a*x]^2)/(4*a^2)
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result	size
derivativedivides	$\frac{\frac{a^{2}x^{2}\arccos(ax)^{2}}{2}-\frac{\arccos(ax)\left(ax\sqrt{-a^{2}x^{2}+1}+\arccos(ax)\right)}{2}+\frac{\arccos(ax)^{2}}{4}-\frac{a^{2}x^{2}}{4}+\frac{1}{4}}{a^{2}}$	63
default	$\frac{\frac{a^{2}x^{2}\arccos(ax)^{2}}{2}-\frac{\arccos(ax)\left(ax\sqrt{-a^{2}x^{2}+1}+\arccos(ax)\right)}{2}+\frac{\arccos(ax)^{2}}{4}-\frac{a^{2}x^{2}}{4}+\frac{1}{4}}{a^{2}}$	63

```
[In] int(x*arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*a^2*x^2*arccos(a*x)^2-1/2*arccos(a*x)*(a*x*(-a^2*x^2+1)^(1/2)+ar
ccos(a*x))+1/4*arccos(a*x)^2-1/4*a^2*x^2+1/4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.85

$$\int x\arccos(ax)^{2}\,dx=-\frac{a^{2}x^{2}+2\sqrt{-a^{2}x^{2}+1}ax\arccos(ax)-(2a^{2}x^{2}-1)\arccos(ax)^{2}}{4a^{2}}$$

```
[In] integrate(x*arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*x^2 + 2*sqrt(-a^2*x^2 + 1)*a*x*arccos(a*x) - (2*a^2*x^2 - 1)*arcc
os(a*x)^2)/a^2
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97

$$\int x\arccos(ax)^{2}\,dx=\begin{cases}\frac{x^{2}\operatorname{acos}^{2}(ax)}{2}-\frac{x^{2}}{4}-\frac{x\sqrt{-a^{2}x^{2}+1}\operatorname{acos}(ax)}{2a}-\frac{\operatorname{acos}^{2}(ax)}{4a^{2}} & \text{for } a\neq 0\\ \frac{\pi^{2}x^{2}}{8} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*acos(a*x)**2,x)
```

```
[Out] Piecewise((x**2*acos(a*x)**2/2 - x**2/4 - x*sqrt(-a**2*x**2 + 1)*acos(a*x)/
(2*a) - acos(a*x)**2/(4*a**2), Ne(a, 0)), (pi**2*x**2/8, True))
```

Maxima [F]

$$\int x \arccos(ax)^2 \, dx = \int x \arccos{(ax)}^2 \, dx$$

```
[In] integrate(x*arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2 - a*integrate(sqrt(a*x
 + 1)*sqrt(-a*x + 1)*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)/(a^2*x^
2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int x \arccos(ax)^2 \, dx = \frac{1}{2} x^2 \arccos{(ax)}^2 - \frac{1}{4} x^2 - \frac{\sqrt{-a^2x^2+1}x \arccos{(ax)}}{2a} - \frac{\arccos{(ax)}^2}{4a^2} + \frac{1}{8a^2}$$

```
[In] integrate(x*arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] 1/2*x^2*arccos(a*x)^2 - 1/4*x^2 - 1/2*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)/a -
1/4*arccos(a*x)^2/a^2 + 1/8/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x \arccos(ax)^2 \, dx = \int x \operatorname{acos}(a\,x)^2 \, dx$$

```
[In] int(x*acos(a*x)^2,x)
```

```
[Out] int(x*acos(a*x)^2, x)
```

3.16 $\int \arccos(ax)^2 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \arccos(ax)^2 \, dx = -2x - \frac{2\sqrt{1-a^2x^2}\arccos(ax)}{a} + x\arccos(ax)^2$$

```
[Out] -2*x+x*arccos(a*x)^2-2*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4716, 4768, 8}

$$\int \arccos(ax)^2 \, dx = -\frac{2\sqrt{1-a^2x^2}\arccos(ax)}{a} + x\arccos(ax)^2 - 2x$$

```
[In] Int[ArcCos[a*x]^2,x]
```

```
[Out] -2*x - (2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a + x*ArcCos[a*x]^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= x\arccos(ax)^2 + (2a)\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx\\ &= -\frac{2\sqrt{1-a^2x^2}\arccos(ax)}{a} + x\arccos(ax)^2 - 2\int 1\,dx\\ &= -2x - \frac{2\sqrt{1-a^2x^2}\arccos(ax)}{a} + x\arccos(ax)^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \arccos(ax)^2\,dx = -2x - \frac{2\sqrt{1-a^2x^2}\arccos(ax)}{a} + x\arccos(ax)^2$$

```
[In] Integrate[ArcCos[a*x]^2,x]
[Out] -2*x - (2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a + x*ArcCos[a*x]^2
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\arccos(ax)^2ax-2ax-2\arccos(ax)\sqrt{-a^2x^2+1}}{a}$	37
default	$\frac{\arccos(ax)^2ax-2ax-2\arccos(ax)\sqrt{-a^2x^2+1}}{a}$	37

```
[In] int(arccos(a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/a*(arccos(a*x)^2*a*x-2*a*x-2*arccos(a*x)*(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \arccos(ax)^2\,dx = \frac{ax\arccos\left(ax\right)^2 - 2\,ax - 2\,\sqrt{-a^2x^2+1}\arccos\left(ax\right)}{a}$$

```
[In] integrate(arccos(a*x)^2,x, algorithm="fricas")
[Out] (a*x*arccos(a*x)^2 - 2*a*x - 2*sqrt(-a^2*x^2 + 1)*arccos(a*x))/a
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.06

$$\int \arccos(ax)^2\,dx = \begin{cases} x\operatorname{acos}^2\left(ax\right) - 2x - \frac{2\sqrt{-a^2x^2+1}\operatorname{acos}\left(ax\right)}{a} & \text{for}\ a \neq 0 \\ \frac{\pi^2 x}{4} & \text{otherwise} \end{cases}$$

```
[In] integrate(acos(a*x)**2,x)
[Out] Piecewise((x*acos(a*x)**2 - 2*x - 2*sqrt(-a**2*x**2 + 1)*acos(a*x)/a, Ne(a,
 0)), (pi**2*x/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \arccos(ax)^2\,dx = x\arccos\left(ax\right)^2 - 2\,x - \frac{2\,\sqrt{-a^2x^2+1}\arccos\left(ax\right)}{a}$$

```
[In] integrate(arccos(a*x)^2,x, algorithm="maxima")
[Out] x*arccos(a*x)^2 - 2*x - 2*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \arccos(ax)^2\, dx = x \arccos{(ax)}^2 - 2\,x - \frac{2\,\sqrt{-a^2x^2+1}\arccos{(ax)}}{a}$$

```
[In] integrate(arccos(a*x)^2,x, algorithm="giac")
[Out] x*arccos(a*x)^2 - 2*x - 2*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \arccos(ax)^2\, dx = \begin{cases} \frac{x\,\pi^2}{4} & \text{if } a=0 \\ x\,\left(\mathrm{acos}(a\,x)^2-2\right)-\frac{2\,\mathrm{acos}(a\,x)\,\sqrt{1-a^2\,x^2}}{a} & \text{if } a\neq 0 \end{cases}$$

```
[In] int(acos(a*x)^2,x)
[Out] piecewise(a == 0, (x*pi^2)/4, a ~= 0, x*(acos(a*x)^2 - 2) - (2*acos(a*x)*(-
 a^2*x^2 + 1)^(1/2))/a)
```

3.17 $\int \frac{\arccos(ax)^2}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 73

$$\int \frac{\arccos(ax)^2}{x}\,dx = -\frac{1}{3}i\arccos(ax)^3 + \arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - i\arccos(ax)\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + \frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)$$

```
[Out] -1/3*I*arccos(a*x)^3+arccos(a*x)^2*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-I*arc
cos(a*x)*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(a*x+I*(-a
^2*x^2+1)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4722, 3800, 2221, 2611, 2320, 6724}

$$\int \frac{\arccos(ax)^2}{x}\,dx = -i\arccos(ax)\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + \frac{1}{2}\mathrm{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) - \frac{1}{3}i\arccos(ax)^3 + \arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right)$$

```
[In] Int[ArcCos[a*x]^2/x,x]
```

```
[Out] (-1/3*I)*ArcCos[a*x]^3 + ArcCos[a*x]^2*Log[1 + E^((2*I)*ArcCos[a*x])] - I*A
rcCos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + PolyLog[3, -E^((2*I)*ArcCos
[a*x])]/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int x^2\tan(x)\,dx, x, \arccos(ax)\right)$$

$$= -\frac{1}{3}i\arccos(ax)^3 + 2i\text{Subst}\left(\int \frac{e^{2ix}x^2}{1+e^{2ix}}\,dx, x, \arccos(ax)\right)$$

$$= -\frac{1}{3}i\arccos(ax)^3 + \arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - 2\text{Subst}\left(\int x\log\left(1+e^{2ix}\right)\,dx, x, \arccos(ax)\right)$$

$$= -\frac{1}{3}i\arccos(ax)^3 + \arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - i\arccos(ax)\text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right) + i\text{Subst}\left(\int \text{PolyLog}\left(2, -e^{2ix}\right)\,dx, x, \arccos(ax)\right)$$

$$= -\frac{1}{3}i\arccos(ax)^3 + \arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - i\arccos(ax)\text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{2i\arccos(ax)}\right)$$

$$= -\frac{1}{3}i\arccos(ax)^3 + \arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - i\arccos(ax)\text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right) + \frac{1}{2}\text{PolyLog}\left(3, -e^{2i\arccos(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^2}{x}\,dx = -\frac{1}{3}i\arccos(ax)^3 + \arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - i\arccos(ax)\text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right) + \frac{1}{2}\text{PolyLog}\left(3, -e^{2i\arccos(ax)}\right)$$

```
[In] Integrate[ArcCos[a*x]^2/x,x]
[Out] (-1/3*I)*ArcCos[a*x]^3 + ArcCos[a*x]^2*Log[1 + E^((2*I)*ArcCos[a*x])] - I*A
rcCos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + PolyLog[3, -E^((2*I)*ArcCos
[a*x])]/2
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.38

method	result
derivativedivides	$-\frac{i \arccos(ax)^3}{3}+\arccos\left(ax\right)^2\ln\left(1+\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right)-i\arccos\left(ax\right)\mathrm{polylog}\left(2,\right.$
default	$-\frac{i \arccos(ax)^3}{3}+\arccos\left(ax\right)^2\ln\left(1+\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right)-i\arccos\left(ax\right)\mathrm{polylog}\left(2,\right.$

```
[In] int(arccos(a*x)^2/x,x,method=_RETURNVERBOSE)
[Out] -1/3*I*arccos(a*x)^3+arccos(a*x)^2*ln(1+(I*(-a^2*x^2+1)^(1/2)+a*x)^2)-I*arc
cos(a*x)*polylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+1/2*polylog(3,-(I*(-a^2*x
^2+1)^(1/2)+a*x)^2)
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{x}\,dx=\int \frac{\arccos\left(ax\right)^2}{x}\,dx$$

```
[In] integrate(arccos(a*x)^2/x,x, algorithm="fricas")
[Out] integral(arccos(a*x)^2/x, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x}\,dx=\int \frac{\operatorname{acos}^2\left(ax\right)}{x}\,dx$$

```
[In] integrate(acos(a*x)**2/x,x)
[Out] Integral(acos(a*x)**2/x, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{x} \, dx = \int \frac{\arccos{(ax)}^2}{x} \, dx$$

```
[In] integrate(arccos(a*x)^2/x,x, algorithm="maxima")
[Out] integrate(arccos(a*x)^2/x, x)
```

Giac [F]

$$\int \frac{\arccos(ax)^2}{x} \, dx = \int \frac{\arccos{(ax)}^2}{x} \, dx$$

```
[In] integrate(arccos(a*x)^2/x,x, algorithm="giac")
[Out] integrate(arccos(a*x)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x} \, dx = \int \frac{\mathrm{acos}(a\,x)^2}{x} \, dx$$

```
[In] int(acos(a*x)^2/x,x)
[Out] int(acos(a*x)^2/x, x)
```

3.18 $\int \frac{\arccos(ax)^2}{x^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 74

$$\int \frac{\arccos(ax)^2}{x^2} \, dx = -\frac{\arccos(ax)^2}{x} - 4ia \arccos(ax) \arctan\left(e^{i \arccos(ax)}\right) + 2ia \operatorname{PolyLog}\left(2, -ie^{i \arccos(ax)}\right) - 2ia \operatorname{PolyLog}\left(2, ie^{i \arccos(ax)}\right)$$

```
[Out] -arccos(a*x)^2/x-4*I*a*arccos(a*x)*arctan(a*x+I*(-a^2*x^2+1)^(1/2))+2*I*a*p
olylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-2*I*a*polylog(2,I*(a*x+I*(-a^2*x^2+
1)^(1/2)))
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4724, 4804, 4266, 2317, 2438}

$$\int \frac{\arccos(ax)^2}{x^2} \, dx = -4ia \arccos(ax) \arctan\left(e^{i \arccos(ax)}\right) + 2ia \operatorname{PolyLog}\left(2, -ie^{i \arccos(ax)}\right) - 2ia \operatorname{PolyLog}\left(2, ie^{i \arccos(ax)}\right) - \frac{\arccos(ax)^2}{x}$$

```
[In] Int[ArcCos[a*x]^2/x^2,x]
```

```
[Out] -(ArcCos[a*x]^2/x) - (4*I)*a*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] + (2*I)*
a*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (2*I)*a*PolyLog[2, I*E^(I*ArcCos[a*x
])]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)^2}{x} - (2a)\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\arccos(ax)^2}{x} + (2a)\text{Subst}\left(\int x\sec(x)\,dx, x, \arccos(ax)\right) \\
&= -\frac{\arccos(ax)^2}{x} - 4ia\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) \\
&\quad - (2a)\text{Subst}\left(\int \log\left(1-ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&\quad + (2a)\text{Subst}\left(\int \log\left(1+ie^{ix}\right)\,dx, x, \arccos(ax)\right)
\end{aligned}
$$

$$= -\frac{\arccos(ax)^2}{x} - 4ia\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) + (2ia)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i\arccos(ax)}\right) - (2ia)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{i\arccos(ax)}\right)$$

$$= -\frac{\arccos(ax)^2}{x} - 4ia\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) + 2ia\,\text{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) - 2ia\,\text{PolyLog}\left(2, ie^{i\arccos(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.32

$$\int \frac{\arccos(ax)^2}{x^2}\,dx = -\frac{\arccos(ax)\left(\arccos(ax) + 2ax\left(-\log\left(1-ie^{i\arccos(ax)}\right) + \log\left(1+ie^{i\arccos(ax)}\right)\right)\right)}{x} + 2ia\,\text{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) - 2ia\,\text{PolyLog}\left(2, ie^{i\arccos(ax)}\right)$$

```
[In] Integrate[ArcCos[a*x]^2/x^2,x]
[Out] -((ArcCos[a*x]*(ArcCos[a*x] + 2*a*x*(-Log[1 - I*E^(I*ArcCos[a*x])] + Log[1
+ I*E^(I*ArcCos[a*x])])))/x) + (2*I)*a*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] -
 (2*I)*a*PolyLog[2, I*E^(I*ArcCos[a*x])]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.84

method	result
derivativedivides	$a\left(-\frac{\arccos(ax)^2}{ax} - 2\arccos(ax)\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right) + 2\arccos(ax)\ln\left(1-i\left(i\right.\right.\right.$
default	$a\left(-\frac{\arccos(ax)^2}{ax} - 2\arccos(ax)\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right) + 2\arccos(ax)\ln\left(1-i\left(i\right.\right.\right.$

```
[In] int(arccos(a*x)^2/x^2,x,method=_RETURNVERBOSE)
[Out] a*(-arccos(a*x)^2/a/x-2*arccos(a*x)*ln(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))+2*ar
ccos(a*x)*ln(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x))+2*I*dilog(1+I*(I*(-a^2*x^2+1)^
(1/2)+a*x))-2*I*dilog(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x)))
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{x^2}\,dx = \int \frac{\arccos\left(ax\right)^2}{x^2}\,dx$$

```
[In] integrate(arccos(a*x)^2/x^2,x, algorithm="fricas")
[Out] integral(arccos(a*x)^2/x^2, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x^2}\,dx = \int \frac{\operatorname{acos}^2\left(ax\right)}{x^2}\,dx$$

```
[In] integrate(acos(a*x)**2/x**2,x)
[Out] Integral(acos(a*x)**2/x**2, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{x^2}\,dx = \int \frac{\arccos\left(ax\right)^2}{x^2}\,dx$$

```
[In] integrate(arccos(a*x)^2/x^2,x, algorithm="maxima")
[Out] (2*a*x*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a
*x + 1), a*x)/(a^2*x^3 - x), x) - arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x
)^2)/x
```

Giac [F]

$$\int \frac{\arccos(ax)^2}{x^2}\,dx = \int \frac{\arccos\left(ax\right)^2}{x^2}\,dx$$

```
[In] integrate(arccos(a*x)^2/x^2,x, algorithm="giac")
[Out] integrate(arccos(a*x)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x^2}\,dx = \int \frac{\operatorname{acos}(a\,x)^2}{x^2}\,dx$$

```
[In] int(acos(a*x)^2/x^2,x)
```

```
[Out] int(acos(a*x)^2/x^2, x)
```

3.19 $\int \frac{\arccos(ax)^2}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{x} - \frac{\arccos(ax)^2}{2x^2} + a^2\log(x)$$

```
[Out] -1/2*arccos(a*x)^2/x^2+a^2*ln(x)+a*arccos(a*x)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4724, 4772, 29}

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{x} + a^2\log(x) - \frac{\arccos(ax)^2}{2x^2}$$

```
[In] Int[ArcCos[a*x]^2/x^3,x]
[Out] (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/x - ArcCos[a*x]^2/(2*x^2) + a^2*Log[x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4772

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*Ar
cCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2
*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)^2}{2x^2} - a\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{x} - \frac{\arccos(ax)^2}{2x^2} + a^2\int \frac{1}{x}\,dx \\
&= \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{x} - \frac{\arccos(ax)^2}{2x^2} + a^2\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{x} - \frac{\arccos(ax)^2}{2x^2} + a^2\log(x)$$

```
[In] Integrate[ArcCos[a*x]^2/x^3,x]
[Out] (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/x - ArcCos[a*x]^2/(2*x^2) + a^2*Log[x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$a^2\left(-\frac{\arccos(ax)^2}{2a^2x^2}+\frac{\arccos(ax)\sqrt{-a^2x^2+1}}{xa}+\ln(ax)\right)$	47
default	$a^2\left(-\frac{\arccos(ax)^2}{2a^2x^2}+\frac{\arccos(ax)\sqrt{-a^2x^2+1}}{xa}+\ln(ax)\right)$	47

```
[In] int(arccos(a*x)^2/x^3,x,method=_RETURNVERBOSE)
[Out] a^2*(-1/2*arccos(a*x)^2/a^2/x^2+arccos(a*x)/x/a*(-a^2*x^2+1)^(1/2)+ln(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \frac{2\,a^2x^2\log\left(x\right) + 2\,\sqrt{-a^2x^2+1}ax\arccos\left(ax\right) - \arccos\left(ax\right)^2}{2\,x^2}$$

```
[In] integrate(arccos(a*x)^2/x^3,x, algorithm="fricas")
[Out] 1/2*(2*a^2*x^2*log(x) + 2*sqrt(-a^2*x^2 + 1)*a*x*arccos(a*x) - arccos(a*x)^
2)/x^2
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \int \frac{\operatorname{acos}^2\left(ax\right)}{x^3}\,dx$$

```
[In] integrate(acos(a*x)**2/x**3,x)
[Out] Integral(acos(a*x)**2/x**3, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.91

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = a^2\log\left(x\right) + \frac{\sqrt{-a^2x^2+1}a\arccos\left(ax\right)}{x} - \frac{\arccos\left(ax\right)^2}{2\,x^2}$$

```
[In] integrate(arccos(a*x)^2/x^3,x, algorithm="maxima")
[Out] a^2*log(x) + sqrt(-a^2*x^2 + 1)*a*arccos(a*x)/x - 1/2*arccos(a*x)^2/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(39) = 78.

Time = 0.32 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.91

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = -\frac{1}{2}\left(\left(\frac{a^4x}{\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{\sqrt{-a^2x^2+1}|a|+a}{x|a|}\right)\arccos\left(ax\right) - 2\,a\log\left(|x|\right)\right)a - \frac{\arccos\left(ax\right)^2}{2\,x^2}$$

```
[In] integrate(arccos(a*x)^2/x^3,x, algorithm="giac")
```

```
[Out] -1/2*((a^4*x/((sqrt(-a^2*x^2 + 1)*abs(a) + a)*abs(a)) - (sqrt(-a^2*x^2 + 1)
*abs(a) + a)/(x*abs(a)))*arccos(a*x) - 2*a*log(abs(x)))*a - 1/2*arccos(a*x)
^2/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x^3}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{x^3}\,dx$$

```
[In] int(acos(a*x)^2/x^3,x)
```

```
[Out] int(acos(a*x)^2/x^3, x)
```

3.20 $\int \frac{\arccos(ax)^2}{x^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 124

$$\int \frac{\arccos(ax)^2}{x^4} dx = -\frac{a^2}{3x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{3x^2} - \frac{\arccos(ax)^2}{3x^3} - \frac{2}{3}ia^3\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) + \frac{1}{3}ia^3\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - \frac{1}{3}ia^3\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)$$

```
[Out] -1/3*a^2/x-1/3*arccos(a*x)^2/x^3-2/3*I*a^3*arccos(a*x)*arctan(a*x+I*(-a^2*x
^2+1)^(1/2))+1/3*I*a^3*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-1/3*I*a^3*p
olylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))+1/3*a*arccos(a*x)*(-a^2*x^2+1)^(1/2)
/x^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4724, 4790, 4804, 4266, 2317, 2438, 30}

$$\int \frac{\arccos(ax)^2}{x^4} dx = -\frac{2}{3}ia^3\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) + \frac{1}{3}ia^3\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - \frac{1}{3}ia^3\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right) + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{3x^2} - \frac{a^2}{3x} - \frac{\arccos(ax)^2}{3x^3}$$

```
[In] Int[ArcCos[a*x]^2/x^4,x]
```

```
[Out] -1/3*a^2/x + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(3*x^2) - ArcCos[a*x]^2/(3*x
^3) - ((2*I)/3)*a^3*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] + (I/3)*a^3*PolyL
og[2, (-I)*E^(I*ArcCos[a*x])] - (I/3)*a^3*PolyLog[2, I*E^(I*ArcCos[a*x])]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4790

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
- c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)^2}{3x^3} - \frac{1}{3}(2a)\int \frac{\arccos(ax)}{x^3\sqrt{1-a^2x^2}}\,dx \\
&= \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{3x^2} - \frac{\arccos(ax)^2}{3x^3} + \frac{1}{3}a^2\int \frac{1}{x^2}\,dx - \frac{1}{3}a^3\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{a^2}{3x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{3x^2} - \frac{\arccos(ax)^2}{3x^3} + \frac{1}{3}a^3\text{Subst}\left(\int x\sec(x)\,dx, x, \arccos(ax)\right) \\
&= -\frac{a^2}{3x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{3x^2} - \frac{\arccos(ax)^2}{3x^3} - \frac{2}{3}ia^3\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) \\
&\quad - \frac{1}{3}a^3\text{Subst}\left(\int \log\left(1-ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&\quad + \frac{1}{3}a^3\text{Subst}\left(\int \log\left(1+ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&= -\frac{a^2}{3x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{3x^2} - \frac{\arccos(ax)^2}{3x^3} - \frac{2}{3}ia^3\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) \\
&\quad + \frac{1}{3}\left(ia^3\right)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{i\arccos(ax)}\right) \\
&\quad - \frac{1}{3}\left(ia^3\right)\text{Subst}\left(\int \frac{\log(1+ix)}{x}\,dx, x, e^{i\arccos(ax)}\right) \\
&= -\frac{a^2}{3x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{3x^2} - \frac{\arccos(ax)^2}{3x^3} - \frac{2}{3}ia^3\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right) \\
&\quad + \frac{1}{3}ia^3\,\text{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) - \frac{1}{3}ia^3\,\text{PolyLog}\left(2, ie^{i\arccos(ax)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23

$$\int \frac{\arccos(ax)^2}{x^4}\,dx =$$
$$-\frac{a^2x^2 - ax\sqrt{1-a^2x^2}\arccos(ax) + \arccos(ax)^2 - a^3x^3\arccos(ax)\log\left(1-ie^{i\arccos(ax)}\right) + a^3x^3\arccos(ax)}{3x^3}$$

```
[In] Integrate[ArcCos[a*x]^2/x^4,x]
```

```
[Out] -1/3*(a^2*x^2 - a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x] + ArcCos[a*x]^2 - a^3*x^3
*ArcCos[a*x]*Log[1 - I*E^(I*ArcCos[a*x])] + a^3*x^3*ArcCos[a*x]*Log[1 + I*E
^(I*ArcCos[a*x])] - I*a^3*x^3*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] + I*a^3*x^
3*PolyLog[2, I*E^(I*ArcCos[a*x])])/x^3
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.34

method	result
derivativedivides	$a^3\left(-\frac{-\sqrt{-a^2x^2+1}\,\arccos(ax)ax+\arccos(ax)^2+a^2x^2}{3a^3x^3}-\frac{\arccos(ax)\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right)}{3}+\frac{\arccos(ax)\ln\left(\right.}{}\right.$
default	$a^3\left(-\frac{-\sqrt{-a^2x^2+1}\,\arccos(ax)ax+\arccos(ax)^2+a^2x^2}{3a^3x^3}-\frac{\arccos(ax)\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right)}{3}+\frac{\arccos(ax)\ln\left(\right.}{}\right.$

```
[In] int(arccos(a*x)^2/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3*(-(-a^2*x^2+1)^(1/2)*arccos(a*x)*a*x+arccos(a*x)^2+a^2*x^2)/a^3/x
^3-1/3*arccos(a*x)*ln(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))+1/3*arccos(a*x)*ln(1-
I*(I*(-a^2*x^2+1)^(1/2)+a*x))+1/3*I*dilog(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))-1
/3*I*dilog(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x)))
```

Fricas [F]

$$\int \frac{\arccos(ax)^2}{x^4}\,dx = \int \frac{\arccos{(ax)}^2}{x^4}\,dx$$

```
[In] integrate(arccos(a*x)^2/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^2/x^4, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x^4}\,dx = \int \frac{\operatorname{acos}^2{(ax)}}{x^4}\,dx$$

```
[In] integrate(acos(a*x)**2/x**4,x)
```

```
[Out] Integral(acos(a*x)**2/x**4, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^2}{x^4}\,dx = \int \frac{\arccos{(ax)}^2}{x^4}\,dx$$

```
[In] integrate(arccos(a*x)^2/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(6*a*x^3*integrate(1/3*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x +
1)*sqrt(-a*x + 1), a*x)/(a^2*x^5 - x^3), x) - arctan2(sqrt(a*x + 1)*sqrt(-a
*x + 1), a*x)^2)/x^3
```

Giac [F]

$$\int \frac{\arccos(ax)^2}{x^4}\,dx = \int \frac{\arccos{(ax)}^2}{x^4}\,dx$$

```
[In] integrate(arccos(a*x)^2/x^4,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^2/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x^4}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{x^4}\,dx$$

```
[In] int(acos(a*x)^2/x^4,x)
```

```
[Out] int(acos(a*x)^2/x^4, x)
```

3.21 $\int \frac{\arccos(ax)^2}{x^5}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 87

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = -\frac{a^2}{12x^2} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{6x^3} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)}{3x} - \frac{\arccos(ax)^2}{4x^4} + \frac{1}{3}a^4\log(x)$$

```
[Out] -1/12*a^2/x^2-1/4*arccos(a*x)^2/x^4+1/3*a^4*ln(x)+1/6*a*arccos(a*x)*(-a^2*x
^2+1)^(1/2)/x^3+1/3*a^3*arccos(a*x)*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4724, 4790, 4772, 29, 30}

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = \frac{1}{3}a^4\log(x) + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{6x^3} - \frac{a^2}{12x^2} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)}{3x} - \frac{\arccos(ax)^2}{4x^4}$$

```
[In] Int[ArcCos[a*x]^2/x^5,x]
```

```
[Out] -1/12*a^2/x^2 + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(6*x^3) + (a^3*Sqrt[1 - a
^2*x^2]*ArcCos[a*x])/(3*x) - ArcCos[a*x]^2/(4*x^4) + (a^4*Log[x])/3
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4772

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*Ar
cCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2
*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 4790

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
- c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)^2}{4x^4} - \frac{1}{2}a\int \frac{\arccos(ax)}{x^4\sqrt{1-a^2x^2}}\,dx \\
&= \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{6x^3} - \frac{\arccos(ax)^2}{4x^4} + \frac{1}{6}a^2\int \frac{1}{x^3}\,dx - \frac{1}{3}a^3\int \frac{\arccos(ax)}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{a^2}{12x^2} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{6x^3} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)}{3x} - \frac{\arccos(ax)^2}{4x^4} + \frac{1}{3}a^4\int \frac{1}{x}\,dx \\
&= -\frac{a^2}{12x^2} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)}{6x^3} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)}{3x} - \frac{\arccos(ax)^2}{4x^4} + \frac{1}{3}a^4\log(x)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.79

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = -\frac{a^2}{12x^2} + \frac{a\sqrt{1-a^2x^2}(1+2a^2x^2)\arccos(ax)}{6x^3} - \frac{\arccos(ax)^2}{4x^4} + \frac{1}{3}a^4\log(x)$$

```
[In] Integrate[ArcCos[a*x]^2/x^5,x]
```

```
[Out] -1/12*a^2/x^2 + (a*Sqrt[1 - a^2*x^2]*(1 + 2*a^2*x^2)*ArcCos[a*x])/(6*x^3) -
 ArcCos[a*x]^2/(4*x^4) + (a^4*Log[x])/3
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$a^4\left(-\frac{\arccos(ax)^2}{4a^4x^4} + \frac{\arccos(ax)\sqrt{-a^2x^2+1}}{6a^3x^3} - \frac{1}{12a^2x^2} + \frac{\arccos(ax)\sqrt{-a^2x^2+1}}{3xa} + \frac{\ln(ax)}{3}\right)$	82
default	$a^4\left(-\frac{\arccos(ax)^2}{4a^4x^4} + \frac{\arccos(ax)\sqrt{-a^2x^2+1}}{6a^3x^3} - \frac{1}{12a^2x^2} + \frac{\arccos(ax)\sqrt{-a^2x^2+1}}{3xa} + \frac{\ln(ax)}{3}\right)$	82

```
[In] int(arccos(a*x)^2/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4*arccos(a*x)^2/a^4/x^4+1/6*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a^3/x^3-
1/12/a^2/x^2+1/3*arccos(a*x)/x/a*(-a^2*x^2+1)^(1/2)+1/3*ln(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.71

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = \frac{4\,a^4x^4\log\left(x\right) - a^2x^2 + 2\left(2\,a^3x^3 + ax\right)\sqrt{-a^2x^2+1}\arccos\left(ax\right) - 3\,\arccos\left(ax\right)^2}{12\,x^4}$$

```
[In] integrate(arccos(a*x)^2/x^5,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^4*x^4*log(x) - a^2*x^2 + 2*(2*a^3*x^3 + a*x)*sqrt(-a^2*x^2 + 1)*a
rccos(a*x) - 3*arccos(a*x)^2)/x^4
```

Sympy [F]

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = \int \frac{\operatorname{acos}^2(ax)}{x^5}\,dx$$

```
[In] integrate(acos(a*x)**2/x**5,x)
[Out] Integral(acos(a*x)**2/x**5, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.85

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = \frac{1}{12}\left(4\,a^2\log(x) - \frac{1}{x^2}\right)a^2 + \frac{1}{6}\left(\frac{2\sqrt{-a^2x^2+1}a^2}{x} + \frac{\sqrt{-a^2x^2+1}}{x^3}\right)a\arccos(ax) - \frac{\arccos(ax)^2}{4\,x^4}$$

```
[In] integrate(arccos(a*x)^2/x^5,x, algorithm="maxima")
[Out] 1/12*(4*a^2*log(x) - 1/x^2)*a^2 + 1/6*(2*sqrt(-a^2*x^2 + 1)*a^2/x + sqrt(-a
^2*x^2 + 1)/x^3)*a*arccos(a*x) - 1/4*arccos(a*x)^2/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. 2(73) = 146.

Time = 0.35 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.13

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = -\frac{1}{48}\left(\left(\left(\frac{\left(a^4 + \frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)^2}{x^2}\right)a^6x^3}{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3|a|} - \frac{\frac{9\left(\sqrt{-a^2x^2+1}|a|+a\right)a^4}{x} + \frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^3}{x^3}}{a^2|a|}\right)\arccos(ax) - \frac{4\left(2\,a^4\,\mathrm{lo}\right.}{} \right.\right. - \frac{\arccos(ax)^2}{4\,x^4}$$

```
[In] integrate(arccos(a*x)^2/x^5,x, algorithm="giac")
[Out] -1/48*(((a^4 + 9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/x^2)*a^6*x^3/((sqrt(-a^2
*x^2 + 1)*abs(a) + a)^3*abs(a)) - (9*(sqrt(-a^2*x^2 + 1)*abs(a) + a)*a^4/x
+ (sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/x^3)/(a^2*abs(a)))*arccos(a*x) - 4*(2*a
^4*log(a^2*x^2) - (2*(a^2*x^2 - 1)*a^4 + 3*a^4)/(a^2*x^2))/a)*a - 1/4*arcco
s(a*x)^2/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^2}{x^5}\,dx = \int \frac{\mathrm{acos}(a\,x)^2}{x^5}\,dx$$

```
[In] int(acos(a*x)^2/x^5,x)
[Out] int(acos(a*x)^2/x^5, x)
```

3.22 $\int x^4 \arccos(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 201

$$\int x^4 \arccos(ax)^3 \, dx = \frac{298\sqrt{1-a^2x^2}}{375a^5} - \frac{76(1-a^2x^2)^{3/2}}{1125a^5} + \frac{6(1-a^2x^2)^{5/2}}{625a^5} - \frac{16x\arccos(ax)}{25a^4} - \frac{8x^3\arccos(ax)}{75a^2} - \frac{6}{125}x^5\arccos(ax) - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^5} - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{25a} + \frac{1}{5}x^5\arccos(ax)^3$$

```
[Out] -76/1125*(-a^2*x^2+1)^(3/2)/a^5+6/625*(-a^2*x^2+1)^(5/2)/a^5-16/25*x*arccos
(a*x)/a^4-8/75*x^3*arccos(a*x)/a^2-6/125*x^5*arccos(a*x)+1/5*x^5*arccos(a*x
)^3+298/375*(-a^2*x^2+1)^(1/2)/a^5-8/25*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)/a^
5-4/25*x^2*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)/a^3-3/25*x^4*arccos(a*x)^2*(-a^
2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used

= {4724, 4796, 4768, 4716, 267, 272, 45}

$$\int x^4 \arccos(ax)^3\,dx = -\frac{16x\arccos(ax)}{25a^4} - \frac{8x^3\arccos(ax)}{75a^2} - \frac{3x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{25a} - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^5} + \frac{6(1-a^2x^2)^{5/2}}{625a^5} - \frac{76(1-a^2x^2)^{3/2}}{1125a^5} + \frac{298\sqrt{1-a^2x^2}}{375a^5} - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^3} + \frac{1}{5}x^5\arccos(ax)^3 - \frac{6}{125}x^5\arccos(ax)$$

```
[In] Int[x^4*ArcCos[a*x]^3,x]
```

```
[Out] (298*Sqrt[1 - a^2*x^2])/(375*a^5) - (76*(1 - a^2*x^2)^(3/2))/(1125*a^5) + (
6*(1 - a^2*x^2)^(5/2))/(625*a^5) - (16*x*ArcCos[a*x])/(25*a^4) - (8*x^3*Arc
Cos[a*x])/(75*a^2) - (6*x^5*ArcCos[a*x])/125 - (8*Sqrt[1 - a^2*x^2]*ArcCos[
a*x]^2)/(25*a^5) - (4*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(25*a^3) - (3*x^
4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(25*a) + (x^5*ArcCos[a*x]^3)/5
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
```

```
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\arccos(ax)^3+\frac{1}{5}(3a)\int\frac{x^5\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{3x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{25a}+\frac{1}{5}x^5\arccos(ax)^3\\
&\quad-\frac{6}{25}\int x^4\arccos(ax)\,dx+\frac{12\int\frac{x^3\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{25a}\\
&= -\frac{6}{125}x^5\arccos(ax)-\frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^3}-\frac{3x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{25a}\\
&\quad+\frac{1}{5}x^5\arccos(ax)^3+\frac{8\int\frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{25a^3}-\frac{8\int x^2\arccos(ax)\,dx}{25a^2}-\frac{1}{125}(6a)\int\frac{x^5}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{8x^3\arccos(ax)}{75a^2}-\frac{6}{125}x^5\arccos(ax)-\frac{8\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^5}\\
&\quad-\frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^3}-\frac{3x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{25a}+\frac{1}{5}x^5\arccos(ax)^3\\
&\quad-\frac{16\int\arccos(ax)\,dx}{25a^4}-\frac{8\int\frac{x^3}{\sqrt{1-a^2x^2}}\,dx}{75a}-\frac{1}{125}(3a)\text{Subst}\left(\int\frac{x^2}{\sqrt{1-a^2x}}\,dx,x,x^2\right)
\end{aligned}$$

$$= -\frac{16x\arccos(ax)}{25a^4} - \frac{8x^3\arccos(ax)}{75a^2} - \frac{6}{125}x^5\arccos(ax) - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^5} - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{25a} + \frac{1}{5}x^5\arccos(ax)^3 - \frac{16\int\frac{x}{\sqrt{1-a^2x^2}}dx}{25a^3} - \frac{4\text{Subst}\left(\int\frac{x}{\sqrt{1-a^2x}}dx,x,x^2\right)}{75a} - \frac{1}{125}(3a)\text{Subst}\left(\int\left(\frac{1}{a^4\sqrt{1-a^2x}} - \frac{2\sqrt{1-a^2x}}{a^4} + \frac{(1-a^2x)^{3/2}}{a^4}\right)dx,x,x^2\right)$$

$$= \frac{86\sqrt{1-a^2x^2}}{125a^5} - \frac{4(1-a^2x^2)^{3/2}}{125a^5} + \frac{6(1-a^2x^2)^{5/2}}{625a^5} - \frac{16x\arccos(ax)}{25a^4} - \frac{8x^3\arccos(ax)}{75a^2} - \frac{6}{125}x^5\arccos(ax) - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^5} - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{25a} + \frac{1}{5}x^5\arccos(ax)^3 - \frac{4\text{Subst}\left(\int\left(\frac{1}{a^2\sqrt{1-a^2x}} - \frac{\sqrt{1-a^2x}}{a^2}\right)dx,x,x^2\right)}{75a}$$

$$= \frac{298\sqrt{1-a^2x^2}}{375a^5} - \frac{76(1-a^2x^2)^{3/2}}{1125a^5} + \frac{6(1-a^2x^2)^{5/2}}{625a^5} - \frac{16x\arccos(ax)}{25a^4} - \frac{8x^3\arccos(ax)}{75a^2} - \frac{6}{125}x^5\arccos(ax) - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^5} - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\arccos(ax)^2}{25a} + \frac{1}{5}x^5\arccos(ax)^3$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.61

$$\int x^4\arccos(ax)^3\,dx = \frac{2\sqrt{1-a^2x^2}(2072+136a^2x^2+27a^4x^4) - 30ax(120+20a^2x^2+9a^4x^4)\arccos(ax) - 225\sqrt{1-a^2x^2}(8+4}{5625a^5}$$

```
[In] Integrate[x^4*ArcCos[a*x]^3,x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2]*(2072 + 136*a^2*x^2 + 27*a^4*x^4) - 30*a*x*(120 + 20*a
^2*x^2 + 9*a^4*x^4)*ArcCos[a*x] - 225*Sqrt[1 - a^2*x^2]*(8 + 4*a^2*x^2 + 3*
a^4*x^4)*ArcCos[a*x]^2 + 1125*a^5*x^5*ArcCos[a*x]^3)/(5625*a^5)
```

Maple [A] (verified)

Time = 2.25 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\frac{\arccos(ax)^3 a^5 x^5}{5}-\frac{\arccos(ax)^2\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{25}-\frac{6a^5x^5\arccos(ax)}{125}+\frac{2\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{625}-\frac{8a^3x^3\arccos(}{75}}{a^5}$
default	$\frac{\frac{\arccos(ax)^3 a^5 x^5}{5}-\frac{\arccos(ax)^2\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{25}-\frac{6a^5x^5\arccos(ax)}{125}+\frac{2\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{625}-\frac{8a^3x^3\arccos(}{75}}{a^5}$

```
[In] int(x^4*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*arccos(a*x)^3*a^5*x^5-1/25*arccos(a*x)^2*(3*a^4*x^4+4*a^2*x^2+8)
*(-a^2*x^2+1)^(1/2)-6/125*a^5*x^5*arccos(a*x)+2/625*(3*a^4*x^4+4*a^2*x^2+8)
*(-a^2*x^2+1)^(1/2)-8/75*a^3*x^3*arccos(a*x)+8/225*(a^2*x^2+2)*(-a^2*x^2+1)
^(1/2)+16/25*(-a^2*x^2+1)^(1/2)-16/25*a*x*arccos(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.52

$$\int x^4 \arccos(ax)^3 \, dx$$
$$= \frac{1125\, a^5x^5 \arccos(ax)^3 - 30\,(9\, a^5x^5 + 20\, a^3x^3 + 120\, ax) \arccos(ax) + \left(54\, a^4x^4 + 272\, a^2x^2 - 225\,(3\, a^4x^4 + 4\right.}{5625\, a^5}$$

```
[In] integrate(x^4*arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/5625*(1125*a^5*x^5*arccos(a*x)^3 - 30*(9*a^5*x^5 + 20*a^3*x^3 + 120*a*x)*
arccos(a*x) + (54*a^4*x^4 + 272*a^2*x^2 - 225*(3*a^4*x^4 + 4*a^2*x^2 + 8)*a
rccos(a*x)^2 + 4144)*sqrt(-a^2*x^2 + 1))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int x^4 \arccos(ax)^3 \, dx$$
$$= \begin{cases} \frac{x^5 \operatorname{acos}^3(ax)}{5} - \frac{6x^5 \operatorname{acos}(ax)}{125} - \frac{3x^4\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{25a} + \frac{6x^4\sqrt{-a^2x^2+1}}{625a} - \frac{8x^3\operatorname{acos}(ax)}{75a^2} - \frac{4x^2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{25a^3} + \frac{272x^2\sqrt{-}}{562} \\ \frac{\pi^3x^5}{40} \end{cases}$$

```
[In] integrate(x**4*acos(a*x)**3,x)
```

```
[Out] Piecewise((x**5*acos(a*x)**3/5 - 6*x**5*acos(a*x)/125 - 3*x**4*sqrt(-a**2*x
**2 + 1)*acos(a*x)**2/(25*a) + 6*x**4*sqrt(-a**2*x**2 + 1)/(625*a) - 8*x**3
*acos(a*x)/(75*a**2) - 4*x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(25*a**3) +
 272*x**2*sqrt(-a**2*x**2 + 1)/(5625*a**3) - 16*x*acos(a*x)/(25*a**4) - 8*s
qrt(-a**2*x**2 + 1)*acos(a*x)**2/(25*a**5) + 4144*sqrt(-a**2*x**2 + 1)/(562
5*a**5), Ne(a, 0)), (pi**3*x**5/40, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.85

$$\int x^4 \arccos(ax)^3\,dx = \frac{1}{5}\,x^5\arccos\left(ax\right)^3 - \frac{1}{25}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2} + \frac{4\sqrt{-a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{-a^2x^2+1}}{a^6}\right)a\arccos\left(ax\right)^2 + \frac{2}{5625}\,a\left(\frac{27\sqrt{-a^2x^2+1}a^2x^4 + 136\sqrt{-a^2x^2+1}x^2 + \frac{2072\sqrt{-a^2x^2+1}}{a^2}}{a^4} - \frac{15\left(9\,a^4x^5 + 20\,a^2x^3 + 120\,x\right)\text{arcc}}{a^5}\right.$$

```
[In] integrate(x^4*arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccos(a*x)^3 - 1/25*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^
2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a*arccos(a*x)^2 + 2/5625*a*((27*
sqrt(-a^2*x^2 + 1)*a^2*x^4 + 136*sqrt(-a^2*x^2 + 1)*x^2 + 2072*sqrt(-a^2*x^
2 + 1)/a^2)/a^4 - 15*(9*a^4*x^5 + 20*a^2*x^3 + 120*x)*arccos(a*x)/a^5)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.87

$$\int x^4 \arccos(ax)^3\,dx = \frac{1}{5}\,x^5\arccos\left(ax\right)^3 - \frac{6}{125}\,x^5\arccos\left(ax\right) - \frac{3\sqrt{-a^2x^2+1}x^4\arccos\left(ax\right)^2}{25\,a} + \frac{6\sqrt{-a^2x^2+1}x^4}{625\,a} - \frac{8\,x^3\arccos\left(ax\right)}{75\,a^2} - \frac{4\sqrt{-a^2x^2+1}x^2\arccos\left(ax\right)^2}{25\,a^3} + \frac{272\sqrt{-a^2x^2+1}x^2}{5625\,a^3} - \frac{16\,x\arccos\left(ax\right)}{25\,a^4} - \frac{8\sqrt{-a^2x^2+1}\arccos\left(ax\right)^2}{25\,a^5} + \frac{4144\sqrt{-a^2x^2+1}}{5625\,a^5}$$

```
[In] integrate(x^4*arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/5*x^5*arccos(a*x)^3 - 6/125*x^5*arccos(a*x) - 3/25*sqrt(-a^2*x^2 + 1)*x^4
*arccos(a*x)^2/a + 6/625*sqrt(-a^2*x^2 + 1)*x^4/a - 8/75*x^3*arccos(a*x)/a^
2 - 4/25*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)^2/a^3 + 272/5625*sqrt(-a^2*x^2
+ 1)*x^2/a^3 - 16/25*x*arccos(a*x)/a^4 - 8/25*sqrt(-a^2*x^2 + 1)*arccos(a*x
)^2/a^5 + 4144/5625*sqrt(-a^2*x^2 + 1)/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^3\,dx = \int x^4\,\mathrm{acos}(a\,x)^3\,dx$$

```
[In] int(x^4*acos(a*x)^3,x)
```

```
[Out] int(x^4*acos(a*x)^3, x)
```

3.23 $\int x^3 \arccos(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 167

$$\int x^3 \arccos(ax)^3 \, dx = \frac{45x\sqrt{1-a^2x^2}}{256a^3} + \frac{3x^3\sqrt{1-a^2x^2}}{128a} - \frac{9x^2 \arccos(ax)}{32a^2} - \frac{3}{32}x^4 \arccos(ax) - \frac{9x\sqrt{1-a^2x^2}\arccos(ax)^2}{32a^3} - \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{16a} - \frac{3\arccos(ax)^3}{32a^4} + \frac{1}{4}x^4 \arccos(ax)^3 - \frac{45\arcsin(ax)}{256a^4}$$

```
[Out] -9/32*x^2*arccos(a*x)/a^2-3/32*x^4*arccos(a*x)-3/32*arccos(a*x)^3/a^4+1/4*x
^4*arccos(a*x)^3-45/256*arcsin(a*x)/a^4+45/256*x*(-a^2*x^2+1)^(1/2)/a^3+3/1
28*x^3*(-a^2*x^2+1)^(1/2)/a-9/32*x*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)/a^3-3/1
6*x^3*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4724, 4796, 4738, 327, 222}

$$\int x^3 \arccos(ax)^3 \, dx = -\frac{3\arccos(ax)^3}{32a^4} - \frac{45\arcsin(ax)}{256a^4} - \frac{9x^2\arccos(ax)}{32a^2} - \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{16a} + \frac{3x^3\sqrt{1-a^2x^2}}{128a} - \frac{9x\sqrt{1-a^2x^2}\arccos(ax)^2}{32a^3} + \frac{45x\sqrt{1-a^2x^2}}{256a^3} + \frac{1}{4}x^4\arccos(ax)^3 - \frac{3}{32}x^4\arccos(ax)$$

```
[In] Int[x^3*ArcCos[a*x]^3,x]
```

```
[Out] (45*x*Sqrt[1 - a^2*x^2])/(256*a^3) + (3*x^3*Sqrt[1 - a^2*x^2])/(128*a) - (9
*x^2*ArcCos[a*x])/(32*a^2) - (3*x^4*ArcCos[a*x])/32 - (9*x*Sqrt[1 - a^2*x^2
]*ArcCos[a*x]^2)/(32*a^3) - (3*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(16*a)
- (3*ArcCos[a*x]^3)/(32*a^4) + (x^4*ArcCos[a*x]^3)/4 - (45*ArcSin[a*x])/(25
6*a^4)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\arccos(ax)^3 + \frac{1}{4}(3a)\int \frac{x^4\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{16a} + \frac{1}{4}x^4\arccos(ax)^3 - \frac{3}{8}\int x^3\arccos(ax)\,dx + \frac{9\int \frac{x^2\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{16a} \\
&= -\frac{3}{32}x^4\arccos(ax) - \frac{9x\sqrt{1-a^2x^2}\arccos(ax)^2}{32a^3} - \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{16a} \\
&\quad + \frac{1}{4}x^4\arccos(ax)^3 + \frac{9\int \frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{32a^3} - \frac{9\int x\arccos(ax)\,dx}{16a^2} \\
&\quad - \frac{1}{32}(3a)\int \frac{x^4}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{3x^3\sqrt{1-a^2x^2}}{128a} - \frac{9x^2\arccos(ax)}{32a^2} - \frac{3}{32}x^4\arccos(ax) \\
&\quad - \frac{9x\sqrt{1-a^2x^2}\arccos(ax)^2}{32a^3} - \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{16a} \\
&\quad - \frac{3\arccos(ax)^3}{32a^4} + \frac{1}{4}x^4\arccos(ax)^3 - \frac{9\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx}{128a} - \frac{9\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx}{32a} \\
&= \frac{45x\sqrt{1-a^2x^2}}{256a^3} + \frac{3x^3\sqrt{1-a^2x^2}}{128a} - \frac{9x^2\arccos(ax)}{32a^2} - \frac{3}{32}x^4\arccos(ax) \\
&\quad - \frac{9x\sqrt{1-a^2x^2}\arccos(ax)^2}{32a^3} - \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{16a} \\
&\quad - \frac{3\arccos(ax)^3}{32a^4} + \frac{1}{4}x^4\arccos(ax)^3 - \frac{9\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{256a^3} - \frac{9\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{64a^3} \\
&= \frac{45x\sqrt{1-a^2x^2}}{256a^3} + \frac{3x^3\sqrt{1-a^2x^2}}{128a} - \frac{9x^2\arccos(ax)}{32a^2} - \frac{3}{32}x^4\arccos(ax) \\
&\quad - \frac{9x\sqrt{1-a^2x^2}\arccos(ax)^2}{32a^3} - \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)^2}{16a} \\
&\quad - \frac{3\arccos(ax)^3}{32a^4} + \frac{1}{4}x^4\arccos(ax)^3 - \frac{45\arcsin(ax)}{256a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.69

$$\int x^3 \arccos(ax)^3\,dx$$
$$= \frac{3ax\sqrt{1-a^2x^2}(15+2a^2x^2) - 24a^2x^2(3+a^2x^2)\arccos(ax) - 24ax\sqrt{1-a^2x^2}(3+2a^2x^2)\arccos(ax)^2 + 8(-}{256a^4}$$

```
[In] Integrate[x^3*ArcCos[a*x]^3,x]
```

```
[Out] (3*a*x*Sqrt[1 - a^2*x^2]*(15 + 2*a^2*x^2) - 24*a^2*x^2*(3 + a^2*x^2)*ArcCos
[a*x] - 24*a*x*Sqrt[1 - a^2*x^2]*(3 + 2*a^2*x^2)*ArcCos[a*x]^2 + 8*(-3 + 8*
a^4*x^4)*ArcCos[a*x]^3 - 45*ArcSin[a*x])/(256*a^4)
```

Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{\frac{a^4x^4\arccos(ax)^3}{4} - \frac{3\arccos(ax)^2\left(2a^3x^3\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}+3\arccos(ax)\right)}{32} - \frac{3a^4x^4\arccos(ax)}{32} + \frac{3ax\left(2a^2x^2+3\right)\sqrt{-a^2x^2+1}}{256}}{a^4}$
default	$\frac{\frac{a^4x^4\arccos(ax)^3}{4} - \frac{3\arccos(ax)^2\left(2a^3x^3\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}+3\arccos(ax)\right)}{32} - \frac{3a^4x^4\arccos(ax)}{32} + \frac{3ax\left(2a^2x^2+3\right)\sqrt{-a^2x^2+1}}{256}}{a^4}$

```
[In] int(x^3*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccos(a*x)^3-3/32*arccos(a*x)^2*(2*a^3*x^3*(-a^2*x^2+1)
^(1/2)+3*a*x*(-a^2*x^2+1)^(1/2)+3*arccos(a*x))-3/32*a^4*x^4*arccos(a*x)+3/2
56*a*x*(2*a^2*x^2+3)*(-a^2*x^2+1)^(1/2)+45/256*arccos(a*x)-9/32*a^2*x^2*arc
cos(a*x)+9/64*a*x*(-a^2*x^2+1)^(1/2)+3/16*arccos(a*x)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.57

$$\int x^3 \arccos(ax)^3\,dx$$
$$= \frac{8\,(8\,a^4x^4-3)\arccos\,(ax)^3 - 3\,(8\,a^4x^4+24\,a^2x^2-15)\arccos\,(ax) + 3\,\left(2\,a^3x^3 - 8\,(2\,a^3x^3+3\,ax)\arccos\,(a}{256\,a^4}$$

```
[In] integrate(x^3*arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(8*(8*a^4*x^4 - 3)*arccos(a*x)^3 - 3*(8*a^4*x^4 + 24*a^2*x^2 - 15)*ar
ccos(a*x) + 3*(2*a^3*x^3 - 8*(2*a^3*x^3 + 3*a*x)*arccos(a*x)^2 + 15*a*x)*sq
rt(-a^2*x^2 + 1))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00

$$\int x^3 \arccos(ax)^3\,dx$$
$$= \begin{cases} \frac{x^4 \operatorname{acos}^3(ax)}{4} - \frac{3x^4 \operatorname{acos}(ax)}{32} - \frac{3x^3\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{16a} + \frac{3x^3\sqrt{-a^2x^2+1}}{128a} - \frac{9x^2\operatorname{acos}(ax)}{32a^2} - \frac{9x\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{32a^3} + \frac{45x\sqrt{-\cdots}}{256\cdots} \\ \frac{\pi^3 x^4}{32} \end{cases}$$

```
[In] integrate(x**3*acos(a*x)**3,x)
```

```
[Out] Piecewise((x**4*acos(a*x)**3/4 - 3*x**4*acos(a*x)/32 - 3*x**3*sqrt(-a**2*x*
*2 + 1)*acos(a*x)**2/(16*a) + 3*x**3*sqrt(-a**2*x**2 + 1)/(128*a) - 9*x**2*
acos(a*x)/(32*a**2) - 9*x*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(32*a**3) + 45*
x*sqrt(-a**2*x**2 + 1)/(256*a**3) - 3*acos(a*x)**3/(32*a**4) + 45*acos(a*x)
/(256*a**4), Ne(a, 0)), (pi**3*x**4/32, True))
```

Maxima [F]

$$\int x^3 \arccos(ax)^3\,dx = \int x^3 \arccos(ax)^3\,dx$$

```
[In] integrate(x^3*arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 - 3*a*integrate(1/4*sq
rt(a*x + 1)*sqrt(-a*x + 1)*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2
/(a^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.84

$$\int x^3 \arccos(ax)^3\,dx = \frac{1}{4}x^4\arccos(ax)^3 - \frac{3}{32}x^4\arccos(ax) - \frac{3\sqrt{-a^2x^2+1}x^3\arccos(ax)^2}{16a} + \frac{3\sqrt{-a^2x^2+1}x^3}{128a} - \frac{9x^2\arccos(ax)}{32a^2} - \frac{9\sqrt{-a^2x^2+1}x\arccos(ax)^2}{32a^3} - \frac{3\arccos(ax)^3}{32a^4} + \frac{45\sqrt{-a^2x^2+1}x}{256a^3} + \frac{45\arccos(ax)}{256a^4}$$

```
[In] integrate(x^3*arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arccos(a*x)^3 - 3/32*x^4*arccos(a*x) - 3/16*sqrt(-a^2*x^2 + 1)*x^3*
arccos(a*x)^2/a + 3/128*sqrt(-a^2*x^2 + 1)*x^3/a - 9/32*x^2*arccos(a*x)/a^2
 - 9/32*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^2/a^3 - 3/32*arccos(a*x)^3/a^4 + 4
5/256*sqrt(-a^2*x^2 + 1)*x/a^3 + 45/256*arccos(a*x)/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^3 \, dx = \int x^3 \operatorname{acos}(a\,x)^3 \, dx$$

```
[In] int(x^3*acos(a*x)^3,x)
```

```
[Out] int(x^3*acos(a*x)^3, x)
```

3.24 $\int x^2 \arccos(ax)^3 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 136

$$\int x^2 \arccos(ax)^3 \, dx = \frac{14\sqrt{1-a^2x^2}}{9a^3} - \frac{2(1-a^2x^2)^{3/2}}{27a^3} - \frac{4x\arccos(ax)}{3a^2} - \frac{2}{9}x^3\arccos(ax) - \frac{2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a} + \frac{1}{3}x^3\arccos(ax)^3$$

```
[Out] -2/27*(-a^2*x^2+1)^(3/2)/a^3-4/3*x*arccos(a*x)/a^2-2/9*x^3*arccos(a*x)+1/3*
x^3*arccos(a*x)^3+14/9*(-a^2*x^2+1)^(1/2)/a^3-2/3*arccos(a*x)^2*(-a^2*x^2+1
)^(1/2)/a^3-1/3*x^2*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4724, 4796, 4768, 4716, 267, 272, 45}

$$\int x^2 \arccos(ax)^3 \, dx = -\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a} - \frac{4x\arccos(ax)}{3a^2} - \frac{2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^3} - \frac{2(1-a^2x^2)^{3/2}}{27a^3} + \frac{14\sqrt{1-a^2x^2}}{9a^3} + \frac{1}{3}x^3\arccos(ax)^3 - \frac{2}{9}x^3\arccos(ax)$$

```
[In] Int[x^2*ArcCos[a*x]^3,x]
```

```
[Out] (14*Sqrt[1 - a^2*x^2])/(9*a^3) - (2*(1 - a^2*x^2)^(3/2))/(27*a^3) - (4*x*Ar
cCos[a*x])/(3*a^2) - (2*x^3*ArcCos[a*x])/9 - (2*Sqrt[1 - a^2*x^2]*ArcCos[a*
```

```
x]^2)/(3*a^3) - (x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(3*a) + (x^3*ArcCos[a
*x]^3)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
```

```
b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arccos(ax)^3 + a\int\frac{x^3\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a} + \frac{1}{3}x^3\arccos(ax)^3 - \frac{2}{3}\int x^2\arccos(ax)\,dx + \frac{2\int\frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{3a}\\
&= -\frac{2}{9}x^3\arccos(ax) - \frac{2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a}\\
&\quad + \frac{1}{3}x^3\arccos(ax)^3 - \frac{4\int\arccos(ax)\,dx}{3a^2} - \frac{1}{9}(2a)\int\frac{x^3}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{4x\arccos(ax)}{3a^2} - \frac{2}{9}x^3\arccos(ax) - \frac{2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^3}\\
&\quad - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a} + \frac{1}{3}x^3\arccos(ax)^3\\
&\quad - \frac{4\int\frac{x}{\sqrt{1-a^2x^2}}\,dx}{3a} - \frac{1}{9}a\text{Subst}\left(\int\frac{x}{\sqrt{1-a^2x}}\,dx, x, x^2\right)\\
&= \frac{4\sqrt{1-a^2x^2}}{3a^3} - \frac{4x\arccos(ax)}{3a^2} - \frac{2}{9}x^3\arccos(ax) - \frac{2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^3}\\
&\quad - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a} + \frac{1}{3}x^3\arccos(ax)^3\\
&\quad - \frac{1}{9}a\text{Subst}\left(\int\left(\frac{1}{a^2\sqrt{1-a^2x}} - \frac{\sqrt{1-a^2x}}{a^2}\right)\,dx, x, x^2\right)\\
&= \frac{14\sqrt{1-a^2x^2}}{9a^3} - \frac{2(1-a^2x^2)^{3/2}}{27a^3} - \frac{4x\arccos(ax)}{3a^2} - \frac{2}{9}x^3\arccos(ax)\\
&\qquad - \frac{2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\arccos(ax)^2}{3a} + \frac{1}{3}x^3\arccos(ax)^3
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.70

$$\int x^2 \arccos(ax)^3 \, dx$$
$$= \frac{2\sqrt{1-a^2x^2}(20+a^2x^2) - 6ax(6+a^2x^2)\arccos(ax) - 9\sqrt{1-a^2x^2}(2+a^2x^2)\arccos(ax)^2 + 9a^3x^3\arccos(a}{27a^3}$$

```
[In] Integrate[x^2*ArcCos[a*x]^3,x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2]*(20 + a^2*x^2) - 6*a*x*(6 + a^2*x^2)*ArcCos[a*x] - 9*S
qrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcCos[a*x]^2 + 9*a^3*x^3*ArcCos[a*x]^3)/(27
*a^3)
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{a^3x^3\arccos(ax)^3}{3} - \frac{\arccos(ax)^2\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{3} + \frac{4\sqrt{-a^2x^2+1}}{3} - \frac{4ax\arccos(ax)}{3} - \frac{2a^3x^3\arccos(ax)}{9} + \frac{2\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{27}}{a^3}$
default	$\frac{\frac{a^3x^3\arccos(ax)^3}{3} - \frac{\arccos(ax)^2\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{3} + \frac{4\sqrt{-a^2x^2+1}}{3} - \frac{4ax\arccos(ax)}{3} - \frac{2a^3x^3\arccos(ax)}{9} + \frac{2\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{27}}{a^3}$

```
[In] int(x^2*arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arccos(a*x)^3-1/3*arccos(a*x)^2*(a^2*x^2+2)*(-a^2*x^2+1)
^(1/2)+4/3*(-a^2*x^2+1)^(1/2)-4/3*a*x*arccos(a*x)-2/9*a^3*x^3*arccos(a*x)+2
/27*(a^2*x^2+2)*(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int x^2 \arccos(ax)^3 \, dx$$
$$= \frac{9\,a^3x^3\arccos{(ax)}^3 - 6\left(a^3x^3+6\,ax\right)\arccos{(ax)} + \left(2\,a^2x^2 - 9\left(a^2x^2+2\right)\arccos{(ax)}^2 + 40\right)\sqrt{-a^2x^2+1}}{27\,a^3}$$

```
[In] integrate(x^2*arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*a^3*x^3*arccos(a*x)^3 - 6*(a^3*x^3 + 6*a*x)*arccos(a*x) + (2*a^2*x^
2 - 9*(a^2*x^2 + 2)*arccos(a*x)^2 + 40)*sqrt(-a^2*x^2 + 1))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int x^2 \arccos(ax)^3\,dx$$
$$= \begin{cases} \frac{x^3 \operatorname{acos}^3(ax)}{3} - \frac{2x^3 \operatorname{acos}(ax)}{9} - \frac{x^2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{3a} + \frac{2x^2\sqrt{-a^2x^2+1}}{27a} - \frac{4x\operatorname{acos}(ax)}{3a^2} - \frac{2\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{3a^3} + \frac{40\sqrt{-a^2x^2+1}}{27a^3} \\ \frac{\pi^3x^3}{24} \end{cases}$$

```
[In] integrate(x**2*acos(a*x)**3,x)
```

```
[Out] Piecewise((x**3*acos(a*x)**3/3 - 2*x**3*acos(a*x)/9 - x**2*sqrt(-a**2*x**2
+ 1)*acos(a*x)**2/(3*a) + 2*x**2*sqrt(-a**2*x**2 + 1)/(27*a) - 4*x*acos(a*x
)/(3*a**2) - 2*sqrt(-a**2*x**2 + 1)*acos(a*x)**2/(3*a**3) + 40*sqrt(-a**2*x
**2 + 1)/(27*a**3), Ne(a, 0)), (pi**3*x**3/24, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.88

$$\int x^2 \arccos(ax)^3\,dx = \frac{1}{3}x^3\arccos(ax)^3 - \frac{1}{3}a\left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\sqrt{-a^2x^2+1}}{a^4}\right)\arccos(ax)^2 + \frac{2}{27}a\left(\frac{\sqrt{-a^2x^2+1}x^2 + \frac{20\sqrt{-a^2x^2+1}}{a^2}}{a^2} - \frac{3(a^2x^3+6x)\arccos(ax)}{a^3}\right)$$

```
[In] integrate(x^2*arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(a*x)^3 - 1/3*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2
 + 1)/a^4)*arccos(a*x)^2 + 2/27*a*((sqrt(-a^2*x^2 + 1)*x^2 + 20*sqrt(-a^2*x
^2 + 1)/a^2)/a^2 - 3*(a^2*x^3 + 6*x)*arccos(a*x)/a^3)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.86

$$\int x^2 \arccos(ax)^3 \, dx = \frac{1}{3} x^3 \arccos(ax)^3 - \frac{2}{9} x^3 \arccos(ax) - \frac{\sqrt{-a^2x^2+1}x^2 \arccos(ax)^2}{3a} + \frac{2\sqrt{-a^2x^2+1}x^2}{27a} - \frac{4x \arccos(ax)}{3a^2} - \frac{2\sqrt{-a^2x^2+1}\arccos(ax)^2}{3a^3} + \frac{40\sqrt{-a^2x^2+1}}{27a^3}$$

```
[In] integrate(x^2*arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(a*x)^3 - 2/9*x^3*arccos(a*x) - 1/3*sqrt(-a^2*x^2 + 1)*x^2*ar
ccos(a*x)^2/a + 2/27*sqrt(-a^2*x^2 + 1)*x^2/a - 4/3*x*arccos(a*x)/a^2 - 2/3
*sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/a^3 + 40/27*sqrt(-a^2*x^2 + 1)/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^3 \, dx = \int x^2 \operatorname{acos}(a\,x)^3 \, dx$$

```
[In] int(x^2*acos(a*x)^3,x)
```

```
[Out] int(x^2*acos(a*x)^3, x)
```

3.25 $\int x \arccos(ax)^3 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 99

$$\int x \arccos(ax)^3 \, dx = \frac{3x\sqrt{1-a^2x^2}}{8a} - \frac{3}{4}x^2 \arccos(ax) - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^2}{4a} - \frac{\arccos(ax)^3}{4a^2} + \frac{1}{2}x^2 \arccos(ax)^3 - \frac{3\arcsin(ax)}{8a^2}$$

```
[Out] -3/4*x^2*arccos(a*x)-1/4*arccos(a*x)^3/a^2+1/2*x^2*arccos(a*x)^3-3/8*arcsin
(a*x)/a^2+3/8*x*(-a^2*x^2+1)^(1/2)/a-3/4*x*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)
/a
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4724, 4796, 4738, 327, 222}

$$\int x \arccos(ax)^3 \, dx = -\frac{3x\sqrt{1-a^2x^2}\arccos(ax)^2}{4a} - \frac{\arccos(ax)^3}{4a^2} - \frac{3\arcsin(ax)}{8a^2} + \frac{3x\sqrt{1-a^2x^2}}{8a} + \frac{1}{2}x^2 \arccos(ax)^3 - \frac{3}{4}x^2 \arccos(ax)$$

```
[In] Int[x*ArcCos[a*x]^3,x]
```

```
[Out] (3*x*Sqrt[1 - a^2*x^2])/(8*a) - (3*x^2*ArcCos[a*x])/4 - (3*x*Sqrt[1 - a^2*x
^2]*ArcCos[a*x]^2)/(4*a) - ArcCos[a*x]^3/(4*a^2) + (x^2*ArcCos[a*x]^3)/2 -
(3*ArcSin[a*x])/(8*a^2)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\arccos(ax)^3 + \frac{1}{2}(3a)\int\frac{x^2\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3x\sqrt{1-a^2x^2}\arccos(ax)^2}{4a} + \frac{1}{2}x^2\arccos(ax)^3 - \frac{3}{2}\int x\arccos(ax)\,dx + \frac{3\int\frac{\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx}{4a}$$

$$= -\frac{3}{4}x^2\arccos(ax) - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^2}{4a} - \frac{\arccos(ax)^3}{4a^2} + \frac{1}{2}x^2\arccos(ax)^3 - \frac{1}{4}(3a)\int \frac{x^2}{\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{3x\sqrt{1-a^2x^2}}{8a} - \frac{3}{4}x^2\arccos(ax) - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^2}{4a} - \frac{\arccos(ax)^3}{4a^2} + \frac{1}{2}x^2\arccos(ax)^3 - \frac{3\int \frac{1}{\sqrt{1-a^2x^2}}\,dx}{8a}$$

$$= \frac{3x\sqrt{1-a^2x^2}}{8a} - \frac{3}{4}x^2\arccos(ax) - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^2}{4a} - \frac{\arccos(ax)^3}{4a^2} + \frac{1}{2}x^2\arccos(ax)^3 - \frac{3\arcsin(ax)}{8a^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int x\arccos(ax)^3\,dx = \frac{3ax\sqrt{1-a^2x^2} - 6a^2x^2\arccos(ax) - 6ax\sqrt{1-a^2x^2}\arccos(ax)^2 + (-2+4a^2x^2)\arccos(ax)^3 - 3\arcsin(a}{8a^2}$$

```
[In] Integrate[x*ArcCos[a*x]^3,x]
[Out] (3*a*x*Sqrt[1 - a^2*x^2] - 6*a^2*x^2*ArcCos[a*x] - 6*a*x*Sqrt[1 - a^2*x^2]*
ArcCos[a*x]^2 + (-2 + 4*a^2*x^2)*ArcCos[a*x]^3 - 3*ArcSin[a*x])/(8*a^2)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{\frac{\arccos(ax)^3 a^2x^2}{2} - \frac{3\arccos(ax)^2\left(ax\sqrt{-a^2x^2+1}+\arccos(ax)\right)}{4} - \frac{3a^2x^2\arccos(ax)}{4} + \frac{3ax\sqrt{-a^2x^2+1}}{8} + \frac{3\arccos(ax)}{8} + \frac{\arccos(ax)^3}{2}}{a^2}$
default	$\frac{\frac{\arccos(ax)^3 a^2x^2}{2} - \frac{3\arccos(ax)^2\left(ax\sqrt{-a^2x^2+1}+\arccos(ax)\right)}{4} - \frac{3a^2x^2\arccos(ax)}{4} + \frac{3ax\sqrt{-a^2x^2+1}}{8} + \frac{3\arccos(ax)}{8} + \frac{\arccos(ax)^3}{2}}{a^2}$

```
[In] int(x*arccos(a*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/a^2*(1/2*arccos(a*x)^3*a^2*x^2-3/4*arccos(a*x)^2*(a*x*(-a^2*x^2+1)^(1/2)+
arccos(a*x))-3/4*a^2*x^2*arccos(a*x)+3/8*a*x*(-a^2*x^2+1)^(1/2)+3/8*arccos(
a*x)+1/2*arccos(a*x)^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

$$\int x \arccos(ax)^3 \, dx = \frac{2\,(2\,a^2x^2-1)\arccos{(ax)}^3 - 3\,(2\,a^2x^2-1)\arccos{(ax)} - 3\,\sqrt{-a^2x^2+1}\left(2\,ax\arccos{(ax)}^2 - ax\right)}{8\,a^2}$$

```
[In] integrate(x*arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2*x^2 - 1)*arccos(a*x)^3 - 3*(2*a^2*x^2 - 1)*arccos(a*x) - 3*sq
rt(-a^2*x^2 + 1)*(2*a*x*arccos(a*x)^2 - a*x))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00

$$\int x \arccos(ax)^3 \, dx = \begin{cases} \frac{x^2\operatorname{acos}^3{(ax)}}{2} - \frac{3x^2\operatorname{acos}{(ax)}}{4} - \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}^2{(ax)}}{4a} + \frac{3x\sqrt{-a^2x^2+1}}{8a} - \frac{\operatorname{acos}^3{(ax)}}{4a^2} + \frac{3\operatorname{acos}{(ax)}}{8a^2} & \text{for } a \neq 0 \\ \frac{\pi^3x^2}{16} & \text{otherwise} \end{cases}$$

```
[In] integrate(x*acos(a*x)**3,x)
```

```
[Out] Piecewise((x**2*acos(a*x)**3/2 - 3*x**2*acos(a*x)/4 - 3*x*sqrt(-a**2*x**2 +
 1)*acos(a*x)**2/(4*a) + 3*x*sqrt(-a**2*x**2 + 1)/(8*a) - acos(a*x)**3/(4*a
**2) + 3*acos(a*x)/(8*a**2), Ne(a, 0)), (pi**3*x**2/16, True))
```

Maxima [F]

$$\int x \arccos(ax)^3 \, dx = \int x \arccos{(ax)}^3 \, dx$$

```
[In] integrate(x*arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 - 3*a*integrate(1/2*sq
rt(a*x + 1)*sqrt(-a*x + 1)*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2
/(a^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.84

$$\int x \arccos(ax)^3 \, dx = \frac{1}{2} x^2 \arccos{(ax)}^3 - \frac{3}{4} x^2 \arccos{(ax)} - \frac{3 \sqrt{-a^2 x^2 + 1} x \arccos{(ax)}^2}{4 a} - \frac{\arccos{(ax)}^3}{4 a^2} + \frac{3 \sqrt{-a^2 x^2 + 1} x}{8 a} + \frac{3 \arccos{(ax)}}{8 a^2}$$

```
[In] integrate(x*arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/2*x^2*arccos(a*x)^3 - 3/4*x^2*arccos(a*x) - 3/4*sqrt(-a^2*x^2 + 1)*x*arcc
os(a*x)^2/a - 1/4*arccos(a*x)^3/a^2 + 3/8*sqrt(-a^2*x^2 + 1)*x/a + 3/8*arcc
os(a*x)/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x \arccos(ax)^3 \, dx = \int x \operatorname{acos}(a x)^3 \, dx$$

```
[In] int(x*acos(a*x)^3,x)
```

```
[Out] int(x*acos(a*x)^3, x)
```

3.26 $\int \arccos(ax)^3 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 60

$$\int \arccos(ax)^3 \, dx = \frac{6\sqrt{1-a^2x^2}}{a} - 6x\arccos(ax) - \frac{3\sqrt{1-a^2x^2}\arccos(ax)^2}{a} + x\arccos(ax)^3$$

```
[Out] -6*x*arccos(a*x)+x*arccos(a*x)^3+6*(-a^2*x^2+1)^(1/2)/a-3*arccos(a*x)^2*(-a
^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4716, 4768, 267}

$$\int \arccos(ax)^3 \, dx = -\frac{3\sqrt{1-a^2x^2}\arccos(ax)^2}{a} + \frac{6\sqrt{1-a^2x^2}}{a} + x\arccos(ax)^3 - 6x\arccos(ax)$$

```
[In] Int[ArcCos[a*x]^3,x]
```

```
[Out] (6*Sqrt[1 - a^2*x^2])/a - 6*x*ArcCos[a*x] - (3*Sqrt[1 - a^2*x^2]*ArcCos[a*x
]^2)/a + x*ArcCos[a*x]^3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arccos(ax)^3 + (3a)\int \frac{x\arccos(ax)^2}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{3\sqrt{1-a^2x^2}\arccos(ax)^2}{a} + x\arccos(ax)^3 - 6\int \arccos(ax)\,dx \\
&= -6x\arccos(ax) - \frac{3\sqrt{1-a^2x^2}\arccos(ax)^2}{a} + x\arccos(ax)^3 - (6a)\int \frac{x}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{6\sqrt{1-a^2x^2}}{a} - 6x\arccos(ax) - \frac{3\sqrt{1-a^2x^2}\arccos(ax)^2}{a} + x\arccos(ax)^3
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \arccos(ax)^3\,dx = \frac{6\sqrt{1-a^2x^2}}{a} - 6x\arccos(ax) - \frac{3\sqrt{1-a^2x^2}\arccos(ax)^2}{a} + x\arccos(ax)^3$$

```
[In] Integrate[ArcCos[a*x]^3,x]
[Out] (6*Sqrt[1 - a^2*x^2])/a - 6*x*ArcCos[a*x] - (3*Sqrt[1 - a^2*x^2]*ArcCos[a*x
]^2)/a + x*ArcCos[a*x]^3
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\arccos(ax)^3 ax-3\arccos(ax)^2\sqrt{-a^2x^2+1}+6\sqrt{-a^2x^2+1}-6ax\arccos(ax)}{a}$	57
default	$\frac{\arccos(ax)^3 ax-3\arccos(ax)^2\sqrt{-a^2x^2+1}+6\sqrt{-a^2x^2+1}-6ax\arccos(ax)}{a}$	57

```
[In] int(arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(arccos(a*x)^3*a*x-3*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)+6*(-a^2*x^2+1)^(1
/2)-6*a*x*arccos(a*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \arccos(ax)^3\,dx = \frac{ax\arccos(ax)^3 - 6\,ax\arccos(ax) - 3\sqrt{-a^2x^2+1}\left(\arccos(ax)^2 - 2\right)}{a}$$

```
[In] integrate(arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] (a*x*arccos(a*x)^3 - 6*a*x*arccos(a*x) - 3*sqrt(-a^2*x^2 + 1)*(arccos(a*x)^
2 - 2))/a
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int \arccos(ax)^3\,dx = \begin{cases} x\operatorname{acos}^3(ax) - 6x\operatorname{acos}(ax) - \frac{3\sqrt{-a^2x^2+1}\operatorname{acos}^2(ax)}{a} + \frac{6\sqrt{-a^2x^2+1}}{a} & \text{for } a \neq 0 \\ \frac{\pi^3x}{8} & \text{otherwise} \end{cases}$$

```
[In] integrate(acos(a*x)**3,x)
```

```
[Out] Piecewise((x*acos(a*x)**3 - 6*x*acos(a*x) - 3*sqrt(-a**2*x**2 + 1)*acos(a*x
)**2/a + 6*sqrt(-a**2*x**2 + 1)/a, Ne(a, 0)), (pi**3*x/8, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.31 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \arccos(ax)^3\,dx = x\arccos(ax)^3 - \frac{3\sqrt{-a^2x^2+1}\arccos(ax)^2}{a} - \frac{6\left(ax\arccos(ax) - \sqrt{-a^2x^2+1}\right)}{a}$$

```
[In] integrate(arccos(a*x)^3,x, algorithm="maxima")
[Out] x*arccos(a*x)^3 - 3*sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/a - 6*(a*x*arccos(a*x)
 - sqrt(-a^2*x^2 + 1))/a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \arccos(ax)^3\,dx = x\arccos(ax)^3 - 6\,x\arccos(ax) - \frac{3\sqrt{-a^2x^2+1}\arccos(ax)^2}{a} + \frac{6\sqrt{-a^2x^2+1}}{a}$$

```
[In] integrate(arccos(a*x)^3,x, algorithm="giac")
[Out] x*arccos(a*x)^3 - 6*x*arccos(a*x) - 3*sqrt(-a^2*x^2 + 1)*arccos(a*x)^2/a +
6*sqrt(-a^2*x^2 + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \arccos(ax)^3\,dx = \begin{cases} \frac{x\,\pi^3}{8} & \text{if } a=0 \\ -x\left(6\,\mathrm{acos}(a\,x) - \mathrm{acos}(a\,x)^3\right) - \frac{\sqrt{1-a^2\,x^2}\left(3\,\mathrm{acos}(a\,x)^2-6\right)}{a} & \text{if } a\neq 0 \end{cases}$$

```
[In] int(acos(a*x)^3,x)
[Out] piecewise(a == 0, (x*pi^3)/8, a ~= 0, - x*(6*acos(a*x) - acos(a*x)^3) - ((-
 a^2*x^2 + 1)^(1/2)*(3*acos(a*x)^2 - 6))/a)
```

3.27 $\int \frac{\arccos(ax)^3}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 101

$$\int \frac{\arccos(ax)^3}{x}\,dx = -\frac{1}{4}i\arccos(ax)^4 + \arccos(ax)^3\log\left(1+e^{2i\arccos(ax)}\right) - \frac{3}{2}i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + \frac{3}{2}\arccos(ax)\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) + \frac{3}{4}i\operatorname{PolyLog}\left(4,-e^{2i\arccos(ax)}\right)$$

```
[Out] -1/4*I*arccos(a*x)^4+arccos(a*x)^3*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-3/2*I
*arccos(a*x)^2*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3/2*arccos(a*x)*pol
ylog(3,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3/4*I*polylog(4,-(a*x+I*(-a^2*x^2+1)^
(1/2))^2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4722, 3800, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{\arccos(ax)^3}{x}\,dx = -\frac{3}{2}i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + \frac{3}{2}\arccos(ax)\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) + \frac{3}{4}i\operatorname{PolyLog}\left(4,-e^{2i\arccos(ax)}\right) - \frac{1}{4}i\arccos(ax)^4 + \arccos(ax)^3\log\left(1+e^{2i\arccos(ax)}\right)$$

```
[In] Int[ArcCos[a*x]^3/x,x]
```

```
[Out] (-1/4*I)*ArcCos[a*x]^4 + ArcCos[a*x]^3*Log[1 + E^((2*I)*ArcCos[a*x])] - ((3
*I)/2)*ArcCos[a*x]^2*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + (3*ArcCos[a*x]*Po
lyLog[3, -E^((2*I)*ArcCos[a*x])])/2 + ((3*I)/4)*PolyLog[4, -E^((2*I)*ArcCos
[a*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^3\tan(x)\,dx, x, \arccos(ax)\right)\\
&= -\frac{1}{4}i\arccos(ax)^4 + 2i\text{Subst}\left(\int \frac{e^{2ix}x^3}{1+e^{2ix}}\,dx, x, \arccos(ax)\right)\\
&= -\frac{1}{4}i\arccos(ax)^4 + \arccos(ax)^3\log\left(1+e^{2i\arccos(ax)}\right)\\
&\quad - 3\text{Subst}\left(\int x^2\log\left(1+e^{2ix}\right)\,dx, x, \arccos(ax)\right)\\
&= -\frac{1}{4}i\arccos(ax)^4 + \arccos(ax)^3\log\left(1+e^{2i\arccos(ax)}\right)\\
&\quad - \frac{3}{2}i\arccos(ax)^2\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)\\
&\quad + 3i\text{Subst}\left(\int x\,\text{PolyLog}\left(2,-e^{2ix}\right)\,dx, x, \arccos(ax)\right)\\
&= -\frac{1}{4}i\arccos(ax)^4 + \arccos(ax)^3\log\left(1+e^{2i\arccos(ax)}\right)\\
&\quad - \frac{3}{2}i\arccos(ax)^2\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)\\
&\quad + \frac{3}{2}\arccos(ax)\,\text{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)\\
&\quad - \frac{3}{2}\text{Subst}\left(\int \text{PolyLog}\left(3,-e^{2ix}\right)\,dx, x, \arccos(ax)\right)\\
&= -\frac{1}{4}i\arccos(ax)^4 + \arccos(ax)^3\log\left(1+e^{2i\arccos(ax)}\right)\\
&\quad - \frac{3}{2}i\arccos(ax)^2\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)\\
&\quad + \frac{3}{2}\arccos(ax)\,\text{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)\\
&\quad + \frac{3}{4}i\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x}\,dx, x, e^{2i\arccos(ax)}\right)
\end{aligned}
$$

$$= -\frac{1}{4}i\arccos(ax)^4 + \arccos(ax)^3\log\left(1+e^{2i\arccos(ax)}\right) - \frac{3}{2}i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + \frac{3}{2}\arccos(ax)\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) + \frac{3}{4}i\operatorname{PolyLog}\left(4,-e^{2i\arccos(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^3}{x}\,dx = -\frac{1}{4}i\arccos(ax)^4 + \arccos(ax)^3\log\left(1+e^{2i\arccos(ax)}\right) - \frac{3}{2}i\arccos(ax)^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + \frac{3}{2}\arccos(ax)\operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) + \frac{3}{4}i\operatorname{PolyLog}\left(4,-e^{2i\arccos(ax)}\right)$$

```
[In] Integrate[ArcCos[a*x]^3/x,x]
```

```
[Out] (-1/4*I)*ArcCos[a*x]^4 + ArcCos[a*x]^3*Log[1 + E^((2*I)*ArcCos[a*x])] - ((3
*I)/2)*ArcCos[a*x]^2*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + (3*ArcCos[a*x]*Po
lyLog[3, -E^((2*I)*ArcCos[a*x])])/2 + ((3*I)/4)*PolyLog[4, -E^((2*I)*ArcCos
[a*x])]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.34

method	result
derivativedivides	$-\frac{i\arccos(ax)^4}{4}+\arccos(ax)^3\ln\left(1+\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right)-\frac{3i\arccos(ax)^2\operatorname{polylog}\left(2,-\left(i\sqrt{-a\right.\right.}{2}$
default	$-\frac{i\arccos(ax)^4}{4}+\arccos(ax)^3\ln\left(1+\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right)-\frac{3i\arccos(ax)^2\operatorname{polylog}\left(2,-\left(i\sqrt{-a\right.\right.}{2}$

```
[In] int(arccos(a*x)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*arccos(a*x)^4+arccos(a*x)^3*ln(1+(I*(-a^2*x^2+1)^(1/2)+a*x)^2)-3/2*I
*arccos(a*x)^2*polylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+3/2*arccos(a*x)*pol
ylog(3,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+3/4*I*polylog(4,-(I*(-a^2*x^2+1)^(1/2
)+a*x)^2)
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x}\,dx = \int \frac{\arccos{(ax)}^3}{x}\,dx$$

```
[In] integrate(arccos(a*x)^3/x,x, algorithm="fricas")
[Out] integral(arccos(a*x)^3/x, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x}\,dx = \int \frac{\operatorname{acos}^3{(ax)}}{x}\,dx$$

```
[In] integrate(acos(a*x)**3/x,x)
[Out] Integral(acos(a*x)**3/x, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x}\,dx = \int \frac{\arccos{(ax)}^3}{x}\,dx$$

```
[In] integrate(arccos(a*x)^3/x,x, algorithm="maxima")
[Out] integrate(arccos(a*x)^3/x, x)
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{x}\,dx = \int \frac{\arccos{(ax)}^3}{x}\,dx$$

```
[In] integrate(arccos(a*x)^3/x,x, algorithm="giac")
[Out] integrate(arccos(a*x)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{x}\,dx$$

```
[In] int(acos(a*x)^3/x,x)
```

```
[Out] int(acos(a*x)^3/x, x)
```

3.28 $\int \frac{\arccos(ax)^3}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 122

$$\int \frac{\arccos(ax)^3}{x^2} dx = -\frac{\arccos(ax)^3}{x} - 6ia\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) + 6ia\arccos(ax)\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - 6ia\arccos(ax)\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right) - 6a\,\text{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) + 6a\,\text{PolyLog}\left(3,ie^{i\arccos(ax)}\right)$$

```
[Out] -arccos(a*x)^3/x-6*I*a*arccos(a*x)^2*arctan(a*x+I*(-a^2*x^2+1)^(1/2))+6*I*a
*arccos(a*x)*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-6*I*a*arccos(a*x)*pol
ylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))-6*a*polylog(3,-I*(a*x+I*(-a^2*x^2+1)^(
1/2)))+6*a*polylog(3,I*(a*x+I*(-a^2*x^2+1)^(1/2)))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {4724, 4804, 4266, 2611, 2320, 6724}

$$\int \frac{\arccos(ax)^3}{x^2} dx = -6ia\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) + 6ia\arccos(ax)\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - 6ia\arccos(ax)\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right) - 6a\,\text{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) + 6a\,\text{PolyLog}\left(3,ie^{i\arccos(ax)}\right) - \frac{\arccos(ax)^3}{x}$$

```
[In] Int[ArcCos[a*x]^3/x^2,x]
```

```
[Out] -(ArcCos[a*x]^3/x) - (6*I)*a*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] + (6*I
)*a*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (6*I)*a*ArcCos[a*x]*Po
lyLog[2, I*E^(I*ArcCos[a*x])] - 6*a*PolyLog[3, (-I)*E^(I*ArcCos[a*x])] + 6*
a*PolyLog[3, I*E^(I*ArcCos[a*x])]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)^3}{x} - (3a)\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\arccos(ax)^3}{x} + (3a)\text{Subst}\left(\int x^2\sec(x)\,dx, x, \arccos(ax)\right) \\
&= -\frac{\arccos(ax)^3}{x} - 6ia\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) \\
&\quad - (6a)\text{Subst}\left(\int x\log\left(1-ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&\quad + (6a)\text{Subst}\left(\int x\log\left(1+ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&= -\frac{\arccos(ax)^3}{x} - 6ia\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) \\
&\quad + 6ia\arccos(ax)\,\text{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) \\
&\quad - 6ia\arccos(ax)\,\text{PolyLog}\left(2, ie^{i\arccos(ax)}\right) \\
&\quad - (6ia)\text{Subst}\left(\int \text{PolyLog}\left(2, -ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&\quad + (6ia)\text{Subst}\left(\int \text{PolyLog}\left(2, ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&= -\frac{\arccos(ax)^3}{x} - 6ia\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) \\
&\quad + 6ia\arccos(ax)\,\text{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) \\
&\quad - 6ia\arccos(ax)\,\text{PolyLog}\left(2, ie^{i\arccos(ax)}\right) \\
&\quad - (6a)\text{Subst}\left(\int \frac{\text{PolyLog}(2, -ix)}{x}\,dx, x, e^{i\arccos(ax)}\right) \\
&\quad + (6a)\text{Subst}\left(\int \frac{\text{PolyLog}(2, ix)}{x}\,dx, x, e^{i\arccos(ax)}\right) \\
&= -\frac{\arccos(ax)^3}{x} - 6ia\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) \\
&\quad + 6ia\arccos(ax)\,\text{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) \\
&\quad - 6ia\arccos(ax)\,\text{PolyLog}\left(2, ie^{i\arccos(ax)}\right) \\
&\quad - 6a\,\text{PolyLog}\left(3, -ie^{i\arccos(ax)}\right) + 6a\,\text{PolyLog}\left(3, ie^{i\arccos(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.14

$$\int \frac{\arccos(ax)^3}{x^2} dx = -\frac{\arccos(ax)^3}{x} + 3a\left(\arccos(ax)^2 \left(\log\left(1 - ie^{i\arccos(ax)}\right) - \log\left(1 + ie^{i\arccos(ax)}\right)\right) + 2i\arccos(ax)\left(\text{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) - \text{PolyLog}\left(2, ie^{i\arccos(ax)}\right)\right) - 2\,\text{PolyLog}\left(3, -ie^{i\arccos(ax)}\right) + 2\,\text{PolyLog}\left(3, ie^{i\arccos(ax)}\right)\right)$$

```
[In] Integrate[ArcCos[a*x]^3/x^2,x]
[Out] -(ArcCos[a*x]^3/x) + 3*a*(ArcCos[a*x]^2*(Log[1 - I*E^(I*ArcCos[a*x])] - Log
[1 + I*E^(I*ArcCos[a*x])]) + (2*I)*ArcCos[a*x]*(PolyLog[2, (-I)*E^(I*ArcCos
[a*x])] - PolyLog[2, I*E^(I*ArcCos[a*x])]) - 2*PolyLog[3, (-I)*E^(I*ArcCos[
a*x])] + 2*PolyLog[3, I*E^(I*ArcCos[a*x])])
```

Maple [F]

$$\int \frac{\arccos(ax)^3}{x^2} dx$$

```
[In] int(arccos(a*x)^3/x^2,x)
[Out] int(arccos(a*x)^3/x^2,x)
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x^2} dx = \int \frac{\arccos(ax)^3}{x^2} dx$$

```
[In] integrate(arccos(a*x)^3/x^2,x, algorithm="fricas")
[Out] integral(arccos(a*x)^3/x^2, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x^2} \, dx = \int \frac{\operatorname{acos}^3(ax)}{x^2} \, dx$$

```
[In] integrate(acos(a*x)**3/x**2,x)
[Out] Integral(acos(a*x)**3/x**2, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x^2} \, dx = \int \frac{\arccos(ax)^3}{x^2} \, dx$$

```
[In] integrate(arccos(a*x)^3/x^2,x, algorithm="maxima")
[Out] -(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 - 3*a*x*integrate(sqrt(a*x +
 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2/(a^2*x^3 -
x), x))/x
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{x^2} \, dx = \int \frac{\arccos(ax)^3}{x^2} \, dx$$

```
[In] integrate(arccos(a*x)^3/x^2,x, algorithm="giac")
[Out] integrate(arccos(a*x)^3/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x^2} \, dx = \int \frac{\operatorname{acos}(a\,x)^3}{x^2} \, dx$$

```
[In] int(acos(a*x)^3/x^2,x)
[Out] int(acos(a*x)^3/x^2, x)
```

3.29 $\int \frac{\arccos(ax)^3}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 102

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = -\frac{3}{2}ia^2\arccos(ax)^2 + \frac{3a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{2x^2} + 3a^2\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - \frac{3}{2}ia^2\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)$$

```
[Out] -3/2*I*a^2*arccos(a*x)^2-1/2*arccos(a*x)^3/x^2+3*a^2*arccos(a*x)*ln(1+(a*x+
I*(-a^2*x^2+1)^(1/2))^2)-3/2*I*a^2*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)
+3/2*a*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4724, 4772, 4722, 3800, 2221, 2317, 2438}

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = -\frac{3}{2}ia^2\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + \frac{3a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{3}{2}ia^2\arccos(ax)^2 + 3a^2\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - \frac{\arccos(ax)^3}{2x^2}$$

```
[In] Int[ArcCos[a*x]^3/x^3,x]
```

```
[Out] ((-3*I)/2)*a^2*ArcCos[a*x]^2 + (3*a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(2*x)
- ArcCos[a*x]^3/(2*x^2) + 3*a^2*ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])]
- ((3*I)/2)*a^2*PolyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4772

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*Ar
cCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2
*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{\arccos(ax)^3}{2x^2} - \frac{1}{2}(3a)\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx$$
$$= \frac{3a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{2x^2} + (3a^2)\int \frac{\arccos(ax)}{x}\,dx$$
$$= \frac{3a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{2x^2} - (3a^2)\,\text{Subst}\left(\int x\tan(x)\,dx, x, \arccos(ax)\right)$$
$$= -\frac{3}{2}ia^2\arccos(ax)^2 + \frac{3a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{2x^2} + (6ia^2)\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \arccos(ax)\right)$$
$$= -\frac{3}{2}ia^2\arccos(ax)^2 + \frac{3a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{2x^2} + 3a^2\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - (3a^2)\,\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \arccos(ax)\right)$$
$$= -\frac{3}{2}ia^2\arccos(ax)^2 + \frac{3a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{2x^2} + 3a^2\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) + \frac{1}{2}(3ia^2)\,\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\arccos(ax)}\right)$$
$$= -\frac{3}{2}ia^2\arccos(ax)^2 + \frac{3a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{2x^2} + 3a^2\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - \frac{3}{2}ia^2\,\text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.90

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = \frac{1}{2}\left(\frac{3a\left(-iax+\sqrt{1-a^2x^2}\right)\arccos(ax)^2}{x} - \frac{\arccos(ax)^3}{x^2} + 6a^2\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - 3ia^2\,\text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right)\right)$$

```
[In] Integrate[ArcCos[a*x]^3/x^3,x]
[Out] ((3*a*((-I)*a*x + Sqrt[1 - a^2*x^2])*ArcCos[a*x]^2)/x - ArcCos[a*x]^3/x^2 +
 6*a^2*ArcCos[a*x]*Log[1 + E^((2*I)*ArcCos[a*x])] - (3*I)*a^2*PolyLog[2, -E
^((2*I)*ArcCos[a*x])])/2
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15

method	result
derivativedivides	$a^2\left(-\frac{\arccos(ax)^2\left(-3ia^2x^2-3ax\sqrt{-a^2x^2+1}+\arccos(ax)\right)}{2a^2x^2}-3i\arccos(ax)^2+3\arccos(ax)\ln\left(1+(i\right.\right.$
default	$a^2\left(-\frac{\arccos(ax)^2\left(-3ia^2x^2-3ax\sqrt{-a^2x^2+1}+\arccos(ax)\right)}{2a^2x^2}-3i\arccos(ax)^2+3\arccos(ax)\ln\left(1+(i\right.\right.$

```
[In] int(arccos(a*x)^3/x^3,x,method=_RETURNVERBOSE)
[Out] a^2*(-1/2*arccos(a*x)^2*(-3*I*a^2*x^2-3*a*x*(-a^2*x^2+1)^(1/2)+arccos(a*x))
/a^2/x^2-3*I*arccos(a*x)^2+3*arccos(a*x)*ln(1+(I*(-a^2*x^2+1)^(1/2)+a*x)^2)
-3/2*I*polylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2))
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = \int \frac{\arccos{(ax)}^3}{x^3}\,dx$$

```
[In] integrate(arccos(a*x)^3/x^3,x, algorithm="fricas")
[Out] integral(arccos(a*x)^3/x^3, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x^3}\,dx = \int \frac{\operatorname{acos}^3{(ax)}}{x^3}\,dx$$

```
[In] integrate(acos(a*x)**3/x**3,x)
[Out] Integral(acos(a*x)**3/x**3, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x^3}\, dx = \int \frac{\arccos\left(ax\right)^3}{x^3}\, dx$$

```
[In] integrate(arccos(a*x)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(6*a*x^2*integrate(1/2*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x +
1)*sqrt(-a*x + 1), a*x)^2/(a^2*x^4 - x^2), x) - arctan2(sqrt(a*x + 1)*sqrt(
-a*x + 1), a*x)^3)/x^2
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{x^3}\, dx = \int \frac{\arccos\left(ax\right)^3}{x^3}\, dx$$

```
[In] integrate(arccos(a*x)^3/x^3,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^3/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x^3}\, dx = \int \frac{\mathrm{acos}(a\,x)^3}{x^3}\, dx$$

```
[In] int(acos(a*x)^3/x^3,x)
```

```
[Out] int(acos(a*x)^3/x^3, x)
```

3.30 $\int \frac{\arccos(ax)^3}{x^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 192

$$\begin{aligned}\int \frac{\arccos(ax)^3}{x^4} dx = & -\frac{a^2 \arccos(ax)}{x} + \frac{a\sqrt{1-a^2x^2} \arccos(ax)^2}{2x^2} - \frac{\arccos(ax)^3}{3x^3} \\ & - ia^3 \arccos(ax)^2 \arctan\left(e^{i \arccos(ax)}\right) + a^3 \operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right) \\ & + ia^3 \arccos(ax) \operatorname{PolyLog}\left(2, -ie^{i \arccos(ax)}\right) \\ & - ia^3 \arccos(ax) \operatorname{PolyLog}\left(2, ie^{i \arccos(ax)}\right) \\ & - a^3 \operatorname{PolyLog}\left(3, -ie^{i \arccos(ax)}\right) + a^3 \operatorname{PolyLog}\left(3, ie^{i \arccos(ax)}\right)\end{aligned}$$

```
[Out] -a^2*arccos(a*x)/x-1/3*arccos(a*x)^3/x^3-I*a^3*arccos(a*x)^2*arctan(a*x+I*(
-a^2*x^2+1)^(1/2))+a^3*arctanh((-a^2*x^2+1)^(1/2))+I*a^3*arccos(a*x)*polylo
g(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-I*a^3*arccos(a*x)*polylog(2,I*(a*x+I*(-a
^2*x^2+1)^(1/2)))-a^3*polylog(3,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+a^3*polylog(
3,I*(a*x+I*(-a^2*x^2+1)^(1/2)))+1/2*a*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules

used = {4724, 4790, 4804, 4266, 2611, 2320, 6724, 272, 65, 214}

$$\int \frac{\arccos(ax)^3}{x^4}\,dx = -ia^3\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) + ia^3\arccos(ax)\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - ia^3\arccos(ax)\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right) - a^3\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) + a^3\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right) + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2} - \frac{a^2\arccos(ax)}{x} + a^3\operatorname{arctanh}\left(\sqrt{1-a^2x^2}\right) - \frac{\arccos(ax)^3}{3x^3}$$

```
[In] Int[ArcCos[a*x]^3/x^4,x]
```

```
[Out] -((a^2*ArcCos[a*x])/x) + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(2*x^2) - ArcC
os[a*x]^3/(3*x^3) - I*a^3*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] + a^3*Arc
Tanh[Sqrt[1 - a^2*x^2]] + I*a^3*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x
])] - I*a^3*ArcCos[a*x]*PolyLog[2, I*E^(I*ArcCos[a*x])] - a^3*PolyLog[3, (-
I)*E^(I*ArcCos[a*x])] + a^3*PolyLog[3, I*E^(I*ArcCos[a*x])]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4790

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
- c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)^3}{3x^3} - a\int \frac{\arccos(ax)^2}{x^3\sqrt{1-a^2x^2}}\,dx \\
&= \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2} - \frac{\arccos(ax)^3}{3x^3} + a^2\int \frac{\arccos(ax)}{x^2}\,dx - \frac{1}{2}a^3\int \frac{\arccos(ax)^2}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{a^2\arccos(ax)}{x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2} - \frac{\arccos(ax)^3}{3x^3} \\
&\quad + \frac{1}{2}a^3\text{Subst}\left(\int x^2\sec(x)\,dx, x, \arccos(ax)\right) - a^3\int \frac{1}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{a^2\arccos(ax)}{x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2} - \frac{\arccos(ax)^3}{3x^3} \\
&\quad - ia^3\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) - \frac{1}{2}a^3\text{Subst}\left(\int \frac{1}{x\sqrt{1-a^2x}}\,dx, x, x^2\right) \\
&\quad - a^3\text{Subst}\left(\int x\log\left(1-ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&\quad + a^3\text{Subst}\left(\int x\log\left(1+ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&= -\frac{a^2\arccos(ax)}{x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2} - \frac{\arccos(ax)^3}{3x^3} \\
&\quad - ia^3\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) + ia^3\arccos(ax)\,\text{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) \\
&\quad - ia^3\arccos(ax)\,\text{PolyLog}\left(2, ie^{i\arccos(ax)}\right) + a\text{Subst}\left(\int \frac{1}{\frac{1}{a^2}-\frac{x^2}{a^2}}\,dx, x, \sqrt{1-a^2x^2}\right) \\
&\quad - \left(ia^3\right)\text{Subst}\left(\int \text{PolyLog}\left(2, -ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&\quad + \left(ia^3\right)\text{Subst}\left(\int \text{PolyLog}\left(2, ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&= -\frac{a^2\arccos(ax)}{x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2} - \frac{\arccos(ax)^3}{3x^3} \\
&\quad - ia^3\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) + a^3\text{arctanh}\left(\sqrt{1-a^2x^2}\right) \\
&\quad + ia^3\arccos(ax)\,\text{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) \\
&\quad - ia^3\arccos(ax)\,\text{PolyLog}\left(2, ie^{i\arccos(ax)}\right) \\
&\quad - a^3\text{Subst}\left(\int \frac{\text{PolyLog}(2, -ix)}{x}\,dx, x, e^{i\arccos(ax)}\right) \\
&\quad + a^3\text{Subst}\left(\int \frac{\text{PolyLog}(2, ix)}{x}\,dx, x, e^{i\arccos(ax)}\right)
\end{aligned}$$

$$= -\frac{a^2 \arccos(ax)}{x} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{2x^2} - \frac{\arccos(ax)^3}{3x^3} - ia^3\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) + a^3\text{arctanh}\left(\sqrt{1-a^2x^2}\right) + ia^3\arccos(ax)\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - ia^3\arccos(ax)\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right) - a^3\,\text{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) + a^3\,\text{PolyLog}\left(3,ie^{i\arccos(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.86

$$\int \frac{\arccos(ax)^3}{x^4}\,dx = a^3\Big(-i\arccos(ax)^2\arctan\left(e^{i\arccos(ax)}\right) + \text{arctanh}\left(\sqrt{1-a^2x^2}\right) + i\arccos(ax)\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right) - i\arccos(ax)\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right) - \text{PolyLog}\left(3,-ie^{i\arccos(ax)}\right) + \text{PolyLog}\left(3,ie^{i\arccos(ax)}\right)\Big) - \frac{\arccos(ax)\left(12a^2x^2+4\arccos(ax)^2-3\arccos(ax)\sin(2\arccos(ax))\right)}{12x^3}$$

```
[In] Integrate[ArcCos[a*x]^3/x^4,x]
```

```
[Out] a^3*((-I)*ArcCos[a*x]^2*ArcTan[E^(I*ArcCos[a*x])] + ArcTanh[Sqrt[1 - a^2*x^
2]] + I*ArcCos[a*x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - I*ArcCos[a*x]*Poly
Log[2, I*E^(I*ArcCos[a*x])] - PolyLog[3, (-I)*E^(I*ArcCos[a*x])] + PolyLog[
3, I*E^(I*ArcCos[a*x])]) - (ArcCos[a*x]*(12*a^2*x^2 + 4*ArcCos[a*x]^2 - 3*A
rcCos[a*x]*Sin[2*ArcCos[a*x]]))/(12*x^3)
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.33

method	result
derivativedivides	$a^3\left(-\frac{\arccos(ax)\left(-3\sqrt{-a^2x^2+1}\,\arccos(ax)ax+2\arccos(ax)^2+6a^2x^2\right)}{6a^3x^3} - \frac{\arccos(ax)^2\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right)}{2} -\right.$
default	$a^3\left(-\frac{\arccos(ax)\left(-3\sqrt{-a^2x^2+1}\,\arccos(ax)ax+2\arccos(ax)^2+6a^2x^2\right)}{6a^3x^3} - \frac{\arccos(ax)^2\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right)}{2} -\right.$

```
[In] int(arccos(a*x)^3/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/6/a^3/x^3*arccos(a*x)*(-3*(-a^2*x^2+1)^(1/2)*arccos(a*x)*a*x+2*arcc
os(a*x)^2+6*a^2*x^2)-1/2*arccos(a*x)^2*ln(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))+I
*arccos(a*x)*polylog(2,-I*(I*(-a^2*x^2+1)^(1/2)+a*x))-polylog(3,-I*(I*(-a^2
```

```
*x^2+1)^(1/2)+a*x))+1/2*arccos(a*x)^2*ln(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x))-I*
arccos(a*x)*polylog(2,I*(I*(-a^2*x^2+1)^(1/2)+a*x))+polylog(3,I*(I*(-a^2*x^
2+1)^(1/2)+a*x))-2*I*arctan(I*(-a^2*x^2+1)^(1/2)+a*x))
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x^4}\,dx = \int \frac{\arccos{(ax)}^3}{x^4}\,dx$$

```
[In] integrate(arccos(a*x)^3/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^3/x^4, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x^4}\,dx = \int \frac{\operatorname{acos}^3{(ax)}}{x^4}\,dx$$

```
[In] integrate(acos(a*x)**3/x**4,x)
```

```
[Out] Integral(acos(a*x)**3/x**4, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x^4}\,dx = \int \frac{\arccos{(ax)}^3}{x^4}\,dx$$

```
[In] integrate(arccos(a*x)^3/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(3*a*x^3*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*s
qrt(-a*x + 1), a*x)^2/(a^2*x^5 - x^3), x) - arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x)^3)/x^3
```

Giac [F]

$$\int \frac{\arccos(ax)^3}{x^4}\,dx = \int \frac{\arccos{(ax)}^3}{x^4}\,dx$$

```
[In] integrate(arccos(a*x)^3/x^4,x, algorithm="giac")
[Out] integrate(arccos(a*x)^3/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x^4}\,dx = \int \frac{\mathrm{acos}(a\,x)^3}{x^4}\,dx$$

```
[In] int(acos(a*x)^3/x^4,x)
[Out] int(acos(a*x)^3/x^4, x)
```

3.31 $\int \frac{\arccos(ax)^3}{x^5} dx$

Optimal result

Integrand size = 10, antiderivative size = 169

$$\int \frac{\arccos(ax)^3}{x^5} dx = \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\arccos(ax)}{4x^2} - \frac{1}{2}ia^4\arccos(ax)^2 + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{4x^4} + a^4\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - \frac{1}{2}ia^4\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)$$

```
[Out] -1/4*a^2*arccos(a*x)/x^2-1/2*I*a^4*arccos(a*x)^2-1/4*arccos(a*x)^3/x^4+a^4*
arccos(a*x)*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-1/2*I*a^4*polylog(2,-(a*x+I*
(-a^2*x^2+1)^(1/2))^2)+1/4*a^3*(-a^2*x^2+1)^(1/2)/x+1/4*a*arccos(a*x)^2*(-a
^2*x^2+1)^(1/2)/x^3+1/2*a^3*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {4724, 4790, 4772, 4722, 3800, 2221, 2317, 2438, 270}

$$\int \frac{\arccos(ax)^3}{x^5} dx = -\frac{1}{2}ia^4\operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) - \frac{1}{2}ia^4\arccos(ax)^2 + a^4\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - \frac{a^2\arccos(ax)}{4x^2} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} + \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{\arccos(ax)^3}{4x^4}$$

```
[In] Int[ArcCos[a*x]^3/x^5,x]
```

```
[Out] (a^3*Sqrt[1 - a^2*x^2])/(4*x) - (a^2*ArcCos[a*x])/(4*x^2) - (I/2)*a^4*ArcCo
s[a*x]^2 + (a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2)/(4*x^3) + (a^3*Sqrt[1 - a^2*
x^2]*ArcCos[a*x]^2)/(2*x) - ArcCos[a*x]^3/(4*x^4) + a^4*ArcCos[a*x]*Log[1 +
 E^((2*I)*ArcCos[a*x])] - (I/2)*a^4*PolyLog[2, -E^((2*I)*ArcCos[a*x])]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
```

```
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4772

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*Ar
cCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2
*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 4790

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
- c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)^3}{4x^4} - \frac{1}{4}(3a)\int \frac{\arccos(ax)^2}{x^4\sqrt{1-a^2x^2}}\,dx \\
&= \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{4x^3} - \frac{\arccos(ax)^3}{4x^4} + \frac{1}{2}a^2\int \frac{\arccos(ax)}{x^3}\,dx - \frac{1}{2}a^3\int \frac{\arccos(ax)^2}{x^2\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{a^2\arccos(ax)}{4x^2} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} \\
&\quad - \frac{\arccos(ax)^3}{4x^4} - \frac{1}{4}a^3\int \frac{1}{x^2\sqrt{1-a^2x^2}}\,dx + a^4\int \frac{\arccos(ax)}{x}\,dx \\
&= \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\arccos(ax)}{4x^2} + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{4x^3} \\
&\quad + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{4x^4} - a^4\text{Subst}\left(\int x\tan(x)\,dx, x, \arccos(ax)\right) \\
&= \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\arccos(ax)}{4x^2} - \frac{1}{2}ia^4\arccos(ax)^2 + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{4x^3} \\
&\quad + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{4x^4} + (2ia^4)\,\text{Subst}\left(\int \frac{e^{2ix}x}{1+e^{2ix}}\,dx, x, \arccos(ax)\right)
\end{aligned}$$

$$= \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\arccos(ax)}{4x^2} - \frac{1}{2}ia^4\arccos(ax)^2 + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{4x^4} + a^4\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - a^4\text{Subst}\left(\int\log\left(1+e^{2ix}\right)\,dx, x, \arccos(ax)\right)$$

$$= \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\arccos(ax)}{4x^2} - \frac{1}{2}ia^4\arccos(ax)^2 + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{4x^4} + a^4\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) + \frac{1}{2}\left(ia^4\right)\text{Subst}\left(\int\frac{\log(1+x)}{x}\,dx, x, e^{2i\arccos(ax)}\right)$$

$$= \frac{a^3\sqrt{1-a^2x^2}}{4x} - \frac{a^2\arccos(ax)}{4x^2} - \frac{1}{2}ia^4\arccos(ax)^2 + \frac{a\sqrt{1-a^2x^2}\arccos(ax)^2}{4x^3} + \frac{a^3\sqrt{1-a^2x^2}\arccos(ax)^2}{2x} - \frac{\arccos(ax)^3}{4x^4} + a^4\arccos(ax)\log\left(1+e^{2i\arccos(ax)}\right) - \frac{1}{2}ia^4\,\text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.89

$$\int\frac{\arccos(ax)^3}{x^5}\,dx = \frac{a^3x^3\sqrt{1-a^2x^2} + ax\left(-2ia^3x^3 + \sqrt{1-a^2x^2} + 2a^2x^2\sqrt{1-a^2x^2}\right)\arccos(ax)^2 - \arccos(ax)^3 + a^2x^2\arccos(a}{4x^4}$$

```
[In] Integrate[ArcCos[a*x]^3/x^5,x]
```

```
[Out] (a^3*x^3*Sqrt[1 - a^2*x^2] + a*x*((-2*I)*a^3*x^3 + Sqrt[1 - a^2*x^2] + 2*a^
2*x^2*Sqrt[1 - a^2*x^2])*ArcCos[a*x]^2 - ArcCos[a*x]^3 + a^2*x^2*ArcCos[a*x
]*(-1 + 4*a^2*x^2*Log[1 + E^((2*I)*ArcCos[a*x])]) - (2*I)*a^4*x^4*PolyLog[2
, -E^((2*I)*ArcCos[a*x])])/(4*x^4)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.12

method	result
derivativedivides	$a^4\left(-\frac{-2i\arccos(ax)^2a^4x^4-2\sqrt{-a^2x^2+1}\,\arccos(ax)^2a^3x^3-ia^4x^4-\sqrt{-a^2x^2+1}\,\arccos(ax)^2ax-a^3x^3\sqrt{-a^2x^2+1}+a}{4a^4x^4}\right.$
default	$a^4\left(-\frac{-2i\arccos(ax)^2a^4x^4-2\sqrt{-a^2x^2+1}\,\arccos(ax)^2a^3x^3-ia^4x^4-\sqrt{-a^2x^2+1}\,\arccos(ax)^2ax-a^3x^3\sqrt{-a^2x^2+1}+a}{4a^4x^4}\right.$

```
[In] int(arccos(a*x)^3/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^4*(-1/4*(-2*I*arccos(a*x)^2*a^4*x^4-2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*a^
3*x^3-I*a^4*x^4-(-a^2*x^2+1)^(1/2)*arccos(a*x)^2*a*x-a^3*x^3*(-a^2*x^2+1)^(
1/2)+arccos(a*x)^3+a^2*x^2*arccos(a*x))/a^4/x^4-I*arccos(a*x)^2+arccos(a*x)
*ln(1+(I*(-a^2*x^2+1)^(1/2)+a*x)^2)-1/2*I*polylog(2,-(I*(-a^2*x^2+1)^(1/2)+
a*x)^2))
```

Fricas [F]

$$\int \frac{\arccos(ax)^3}{x^5}\,dx = \int \frac{\arccos\left(ax\right)^3}{x^5}\,dx$$

```
[In] integrate(arccos(a*x)^3/x^5,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^3/x^5, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^3}{x^5}\,dx = \int \frac{\operatorname{acos}^3\left(ax\right)}{x^5}\,dx$$

```
[In] integrate(acos(a*x)**3/x**5,x)
```

```
[Out] Integral(acos(a*x)**3/x**5, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^3}{x^5} dx = \int \frac{\arccos(ax)^3}{x^5} dx$$

[In] integrate(arccos(a*x)^3/x^5,x, algorithm="maxima")

[Out] 1/4*(12*a*x^4*integrate(1/4*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2/(a^2*x^6 - x^4), x) - arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)/x^4

Giac [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^3}{x^5} dx = \text{Exception raised: TypeError}$$

[In] integrate(arccos(a*x)^3/x^5,x, algorithm="giac")

[Out] Exception raised: TypeError >> an error occurred running a Giac command:INPUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vecteur & l) Error: Bad Argument Value

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^3}{x^5} dx = \int \frac{\mathrm{acos}(a\,x)^3}{x^5} dx$$

[In] int(acos(a*x)^3/x^5,x)

[Out] int(acos(a*x)^3/x^5, x)

3.32 $\int x^5 \arccos(ax)^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 282

$$\begin{aligned}\int x^5 \arccos(ax)^4 \, dx = {} & \frac{245x^2}{1152a^4} + \frac{65x^4}{3456a^2} + \frac{x^6}{324} + \frac{245x\sqrt{1-a^2x^2}\arccos(ax)}{576a^5} \\ & + \frac{65x^3\sqrt{1-a^2x^2}\arccos(ax)}{864a^3} + \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)}{54a} \\ & + \frac{245\arccos(ax)^2}{1152a^6} - \frac{5x^2\arccos(ax)^2}{16a^4} - \frac{5x^4\arccos(ax)^2}{48a^2} \\ & - \frac{1}{18}x^6\arccos(ax)^2 - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^3}{24a^5} \\ & - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{36a^3} - \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} \\ & - \frac{5\arccos(ax)^4}{96a^6} + \frac{1}{6}x^6\arccos(ax)^4\end{aligned}$$

```
[Out] 245/1152*x^2/a^4+65/3456*x^4/a^2+1/324*x^6+245/1152*arccos(a*x)^2/a^6-5/16*
x^2*arccos(a*x)^2/a^4-5/48*x^4*arccos(a*x)^2/a^2-1/18*x^6*arccos(a*x)^2-5/9
6*arccos(a*x)^4/a^6+1/6*x^6*arccos(a*x)^4+245/576*x*arccos(a*x)*(-a^2*x^2+1
)^(1/2)/a^5+65/864*x^3*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a^3+1/54*x^5*arccos(a
*x)*(-a^2*x^2+1)^(1/2)/a-5/24*x*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)/a^5-5/36*x
^3*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)/a^3-1/9*x^5*arccos(a*x)^3*(-a^2*x^2+1)^
(1/2)/a
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4724, 4796, 4738, 30}

$$\int x^5 \arccos(ax)^4 \, dx = -\frac{5 \arccos(ax)^4}{96a^6} + \frac{245 \arccos(ax)^2}{1152a^6} - \frac{5x^2 \arccos(ax)^2}{16a^4} + \frac{245x^2}{1152a^4} - \frac{5x^4 \arccos(ax)^2}{48a^2} - \frac{x^5\sqrt{1-a^2x^2} \arccos(ax)^3}{9a} + \frac{x^5\sqrt{1-a^2x^2} \arccos(ax)}{54a} + \frac{65x^4}{3456a^2} - \frac{5x\sqrt{1-a^2x^2} \arccos(ax)^3}{24a^5} + \frac{245x\sqrt{1-a^2x^2} \arccos(ax)}{576a^5} - \frac{5x^3\sqrt{1-a^2x^2} \arccos(ax)^3}{36a^3} + \frac{65x^3\sqrt{1-a^2x^2} \arccos(ax)}{864a^3} + \frac{1}{6}x^6 \arccos(ax)^4 - \frac{1}{18}x^6 \arccos(ax)^2 + \frac{x^6}{324}$$

```
[In] Int[x^5*ArcCos[a*x]^4,x]
```

```
[Out] (245*x^2)/(1152*a^4) + (65*x^4)/(3456*a^2) + x^6/324 + (245*x*Sqrt[1 - a^2*
x^2]*ArcCos[a*x])/(576*a^5) + (65*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(864*a
^3) + (x^5*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(54*a) + (245*ArcCos[a*x]^2)/(115
2*a^6) - (5*x^2*ArcCos[a*x]^2)/(16*a^4) - (5*x^4*ArcCos[a*x]^2)/(48*a^2) -
(x^6*ArcCos[a*x]^2)/18 - (5*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(24*a^5) - (
5*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(36*a^3) - (x^5*Sqrt[1 - a^2*x^2]*Ar
cCos[a*x]^3)/(9*a) - (5*ArcCos[a*x]^4)/(96*a^6) + (x^6*ArcCos[a*x]^4)/6
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
```

```
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{6}x^6\arccos(ax)^4 + \frac{1}{3}(2a)\int\frac{x^6\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^5\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} + \frac{1}{6}x^6\arccos(ax)^4 - \frac{1}{3}\int x^5\arccos(ax)^2\,dx + \frac{5\int\frac{x^4\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{9a}$$

$$= -\frac{1}{18}x^6\arccos(ax)^2 - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{36a^3} - \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} + \frac{1}{6}x^6\arccos(ax)^4 + \frac{5\int\frac{x^2\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{12a^3} - \frac{5\int x^3\arccos(ax)^2\,dx}{12a^2} - \frac{1}{9}a\int\frac{x^6\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)}{54a} - \frac{5x^4\arccos(ax)^2}{48a^2} - \frac{1}{18}x^6\arccos(ax)^2 - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^3}{24a^5} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{36a^3} - \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} + \frac{1}{6}x^6\arccos(ax)^4 + \frac{\int x^5\,dx}{54} + \frac{5\int\frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{24a^5} - \frac{5\int x\arccos(ax)^2\,dx}{8a^4} - \frac{5\int\frac{x^4\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{54a} - \frac{5\int\frac{x^4\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{24a}$$

$$= \frac{x^6}{324} + \frac{65x^3\sqrt{1-a^2x^2}\arccos(ax)}{864a^3} + \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)}{54a} - \frac{5x^2\arccos(ax)^2}{16a^4} - \frac{5x^4\arccos(ax)^2}{48a^2} - \frac{1}{18}x^6\arccos(ax)^2 - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^3}{24a^5} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{36a^3} - \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} - \frac{5\arccos(ax)^4}{96a^6} + \frac{1}{6}x^6\arccos(ax)^4 - \frac{5\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{72a^3} - \frac{5\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{32a^3} - \frac{5\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{8a^3} + \frac{5\int x^3\,dx}{216a^2} + \frac{5\int x^3\,dx}{96a^2}$$

$$= \frac{65x^4}{3456a^2} + \frac{x^6}{324} + \frac{245x\sqrt{1-a^2x^2}\arccos(ax)}{576a^5} + \frac{65x^3\sqrt{1-a^2x^2}\arccos(ax)}{864a^3} + \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)}{54a} - \frac{5x^2\arccos(ax)^2}{16a^4} - \frac{5x^4\arccos(ax)^2}{48a^2} - \frac{1}{18}x^6\arccos(ax)^2 - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^3}{24a^5} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{36a^3} - \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} - \frac{5\arccos(ax)^4}{96a^6} + \frac{1}{6}x^6\arccos(ax)^4 - \frac{5\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{144a^5} - \frac{5\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{64a^5} - \frac{5\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{16a^5} + \frac{5\int x\,dx}{144a^4} + \frac{5\int x\,dx}{64a^4} + \frac{5\int x\,dx}{16a^4}$$

$$= \frac{245x^2}{1152a^4} + \frac{65x^4}{3456a^2} + \frac{x^6}{324} + \frac{245x\sqrt{1-a^2x^2}\arccos(ax)}{576a^5} + \frac{65x^3\sqrt{1-a^2x^2}\arccos(ax)}{864a^3} + \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)}{54a} + \frac{245\arccos(ax)^2}{1152a^6} - \frac{5x^2\arccos(ax)^2}{16a^4} - \frac{5x^4\arccos(ax)^2}{48a^2} - \frac{1}{18}x^6\arccos(ax)^2 - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^3}{24a^5} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{36a^3} - \frac{x^5\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} - \frac{5\arccos(ax)^4}{96a^6} + \frac{1}{6}x^6\arccos(ax)^4$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.59

$$\int x^5\arccos(ax)^4\,dx$$

$$= \frac{a^2x^2(2205+195a^2x^2+32a^4x^4)+6ax\sqrt{1-a^2x^2}(735+130a^2x^2+32a^4x^4)\arccos(ax)-9(-245+360a^2x$$

```
[In] Integrate[x^5*ArcCos[a*x]^4,x]
```

```
[Out] (a^2*x^2*(2205 + 195*a^2*x^2 + 32*a^4*x^4) + 6*a*x*Sqrt[1 - a^2*x^2]*(735 +
 130*a^2*x^2 + 32*a^4*x^4)*ArcCos[a*x] - 9*(-245 + 360*a^2*x^2 + 120*a^4*x^
4 + 64*a^6*x^6)*ArcCos[a*x]^2 - 144*a*x*Sqrt[1 - a^2*x^2]*(15 + 10*a^2*x^2
+ 8*a^4*x^4)*ArcCos[a*x]^3 + 108*(-5 + 16*a^6*x^6)*ArcCos[a*x]^4)/(10368*a^
6)
```

Maple [A] (verified)

Time = 2.31 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{\arccos(ax)^4 a^6 x^6}{6}-\frac{\arccos(ax)^3\left(8\sqrt{-a^2x^2+1}\,a^5x^5+10a^3x^3\sqrt{-a^2x^2+1}+15ax\sqrt{-a^2x^2+1}+15\arccos(ax)\right)}{72}-\frac{\arccos(ax)^2 a^6 x^6}{18}+$
default	$\frac{\arccos(ax)^4 a^6 x^6}{6}-\frac{\arccos(ax)^3\left(8\sqrt{-a^2x^2+1}\,a^5x^5+10a^3x^3\sqrt{-a^2x^2+1}+15ax\sqrt{-a^2x^2+1}+15\arccos(ax)\right)}{72}-\frac{\arccos(ax)^2 a^6 x^6}{18}+$

```
[In] int(x^5*arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^6*(1/6*arccos(a*x)^4*a^6*x^6-1/72*arccos(a*x)^3*(8*(-a^2*x^2+1)^(1/2)*a
^5*x^5+10*a^3*x^3*(-a^2*x^2+1)^(1/2)+15*a*x*(-a^2*x^2+1)^(1/2)+15*arccos(a*
x))-1/18*arccos(a*x)^2*a^6*x^6+1/432*arccos(a*x)*(8*(-a^2*x^2+1)^(1/2)*a^5*
x^5+10*a^3*x^3*(-a^2*x^2+1)^(1/2)+15*a*x*(-a^2*x^2+1)^(1/2)+15*arccos(a*x))
-245/1152*arccos(a*x)^2+1/324*a^6*x^6+5/864*a^4*x^4+25/144*a^2*x^2-5/48*a^4
*x^4*arccos(a*x)^2+5/192*arccos(a*x)*(2*a^3*x^3*(-a^2*x^2+1)^(1/2)+3*a*x*(-
a^2*x^2+1)^(1/2)+3*arccos(a*x))+5/1536*(2*a^2*x^2+3)^2-5/16*a^2*x^2*arccos(
a*x)^2+5/16*arccos(a*x)*(a*x*(-a^2*x^2+1)^(1/2)+arccos(a*x))-5/32+5/32*arcc
os(a*x)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.54

$$\int x^5 \arccos(ax)^4\,dx$$
$$= \frac{32\,a^6x^6 + 195\,a^4x^4 + 108\,(16\,a^6x^6 - 5)\arccos(ax)^4 + 2205\,a^2x^2 - 9\,(64\,a^6x^6 + 120\,a^4x^4 + 360\,a^2x^2 - 245}{}$$

```
[In] integrate(x^5*arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/10368*(32*a^6*x^6 + 195*a^4*x^4 + 108*(16*a^6*x^6 - 5)*arccos(a*x)^4 + 22
05*a^2*x^2 - 9*(64*a^6*x^6 + 120*a^4*x^4 + 360*a^2*x^2 - 245)*arccos(a*x)^2
 - 6*sqrt(-a^2*x^2 + 1)*(24*(8*a^5*x^5 + 10*a^3*x^3 + 15*a*x)*arccos(a*x)^3
 - (32*a^5*x^5 + 130*a^3*x^3 + 735*a*x)*arccos(a*x)))/a^6
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.98

$$\int x^5 \arccos(ax)^4 \, dx$$
$$= \begin{cases} \frac{x^6 \operatorname{acos}^4(ax)}{6} - \frac{x^6 \operatorname{acos}^2(ax)}{18} + \frac{x^6}{324} - \frac{x^5\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{9a} + \frac{x^5\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{54a} - \frac{5x^4\operatorname{acos}^2(ax)}{48a^2} + \frac{65x^4}{3456a^2} - \frac{5x^3\sqrt{-a^2x^2}}{3} \\ \frac{\pi^4 x^6}{96} \end{cases}$$

```
[In] integrate(x**5*acos(a*x)**4,x)
```

```
[Out] Piecewise((x**6*acos(a*x)**4/6 - x**6*acos(a*x)**2/18 + x**6/324 - x**5*sqr
t(-a**2*x**2 + 1)*acos(a*x)**3/(9*a) + x**5*sqrt(-a**2*x**2 + 1)*acos(a*x)/
(54*a) - 5*x**4*acos(a*x)**2/(48*a**2) + 65*x**4/(3456*a**2) - 5*x**3*sqrt(
-a**2*x**2 + 1)*acos(a*x)**3/(36*a**3) + 65*x**3*sqrt(-a**2*x**2 + 1)*acos(
a*x)/(864*a**3) - 5*x**2*acos(a*x)**2/(16*a**4) + 245*x**2/(1152*a**4) - 5*
x*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(24*a**5) + 245*x*sqrt(-a**2*x**2 + 1)*
acos(a*x)/(576*a**5) - 5*acos(a*x)**4/(96*a**6) + 245*acos(a*x)**2/(1152*a*
*6), Ne(a, 0)), (pi**4*x**6/96, True))
```

Maxima [F]

$$\int x^5 \arccos(ax)^4 \, dx = \int x^5 \arccos(ax)^4 \, dx$$

```
[In] integrate(x^5*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*x^6*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 2*a*integrate(1/3*sq
rt(a*x + 1)*sqrt(-a*x + 1)*x^6*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3
/(a^2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.87

$$\int x^5 \arccos(ax)^4\,dx = \frac{1}{6}x^6\arccos(ax)^4 - \frac{1}{18}x^6\arccos(ax)^2 - \frac{\sqrt{-a^2x^2+1}x^5\arccos(ax)^3}{9a} + \frac{1}{324}x^6 + \frac{\sqrt{-a^2x^2+1}x^5\arccos(ax)}{54a} - \frac{5x^4\arccos(ax)^2}{48a^2} - \frac{5\sqrt{-a^2x^2+1}x^3\arccos(ax)^3}{36a^3} + \frac{65x^4}{3456a^2} + \frac{65\sqrt{-a^2x^2+1}x^3\arccos(ax)}{864a^3} - \frac{5x^2\arccos(ax)^2}{16a^4} - \frac{5\sqrt{-a^2x^2+1}x\arccos(ax)^3}{24a^5} + \frac{245x^2}{1152a^4} - \frac{5\arccos(ax)^4}{96a^6} + \frac{245\sqrt{-a^2x^2+1}x\arccos(ax)}{576a^5} + \frac{245\arccos(ax)^2}{1152a^6} - \frac{9485}{82944a^6}$$

```
[In] integrate(x^5*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/6*x^6*arccos(a*x)^4 - 1/18*x^6*arccos(a*x)^2 - 1/9*sqrt(-a^2*x^2 + 1)*x^5
*arccos(a*x)^3/a + 1/324*x^6 + 1/54*sqrt(-a^2*x^2 + 1)*x^5*arccos(a*x)/a -
5/48*x^4*arccos(a*x)^2/a^2 - 5/36*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)^3/a^3
+ 65/3456*x^4/a^2 + 65/864*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)/a^3 - 5/16*x^
2*arccos(a*x)^2/a^4 - 5/24*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^3/a^5 + 245/115
2*x^2/a^4 - 5/96*arccos(a*x)^4/a^6 + 245/576*sqrt(-a^2*x^2 + 1)*x*arccos(a*
x)/a^5 + 245/1152*arccos(a*x)^2/a^6 - 9485/82944/a^6
```

Mupad [F(-1)]

Timed out.

$$\int x^5 \arccos(ax)^4\,dx = \int x^5\,\mathrm{acos}(a\,x)^4\,dx$$

```
[In] int(x^5*acos(a*x)^4,x)
```

```
[Out] int(x^5*acos(a*x)^4, x)
```

3.33 $\int x^4 \arccos(ax)^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 250

$$\int x^4 \arccos(ax)^4 \, dx = \frac{16576x}{5625a^4} + \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} + \frac{16576\sqrt{1-a^2x^2}\arccos(ax)}{5625a^5} + \frac{1088x^2\sqrt{1-a^2x^2}\arccos(ax)}{5625a^3} + \frac{24x^4\sqrt{1-a^2x^2}\arccos(ax)}{625a} - \frac{32x\arccos(ax)^2}{25a^4} - \frac{16x^3\arccos(ax)^2}{75a^2} - \frac{12}{125}x^5\arccos(ax)^2 - \frac{32\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^5} - \frac{16x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^3} - \frac{4x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{25a} + \frac{1}{5}x^5\arccos(ax)^4$$

```
[Out] 16576/5625*x/a^4+1088/16875*x^3/a^2+24/3125*x^5-32/25*x*arccos(a*x)^2/a^4-1
6/75*x^3*arccos(a*x)^2/a^2-12/125*x^5*arccos(a*x)^2+1/5*x^5*arccos(a*x)^4+1
6576/5625*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a^5+1088/5625*x^2*arccos(a*x)*(-a^
2*x^2+1)^(1/2)/a^3+24/625*x^4*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a-32/75*arccos
(a*x)^3*(-a^2*x^2+1)^(1/2)/a^5-16/75*x^2*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)/a
^3-4/25*x^4*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4724, 4796, 4768, 4716, 8, 30}

$$\begin{aligned}\int x^4 \arccos(ax)^4\,dx = &-\frac{32x\arccos(ax)^2}{25a^4}+\frac{16576x}{5625a^4}-\frac{16x^3\arccos(ax)^2}{75a^2}\\&-\frac{4x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{25a}+\frac{24x^4\sqrt{1-a^2x^2}\arccos(ax)}{625a}+\frac{1088x^3}{16875a^2}\\&-\frac{32\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^5}+\frac{16576\sqrt{1-a^2x^2}\arccos(ax)}{5625a^5}\\&-\frac{16x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^3}+\frac{1088x^2\sqrt{1-a^2x^2}\arccos(ax)}{5625a^3}\\&+\frac{1}{5}x^5\arccos(ax)^4-\frac{12}{125}x^5\arccos(ax)^2+\frac{24x^5}{3125}\end{aligned}$$

```
[In] Int[x^4*ArcCos[a*x]^4,x]
```

```
[Out] (16576*x)/(5625*a^4) + (1088*x^3)/(16875*a^2) + (24*x^5)/3125 + (16576*Sqrt
[1 - a^2*x^2]*ArcCos[a*x])/(5625*a^5) + (1088*x^2*Sqrt[1 - a^2*x^2]*ArcCos[
a*x])/(5625*a^3) + (24*x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(625*a) - (32*x*A
rcCos[a*x]^2)/(25*a^4) - (16*x^3*ArcCos[a*x]^2)/(75*a^2) - (12*x^5*ArcCos[a
*x]^2)/125 - (32*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(75*a^5) - (16*x^2*Sqrt[1
 - a^2*x^2]*ArcCos[a*x]^3)/(75*a^3) - (4*x^4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^
3)/(25*a) + (x^5*ArcCos[a*x]^4)/5
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
```

```
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\arccos(ax)^4+\frac{1}{5}(4a)\int\frac{x^5\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{4x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{25a}+\frac{1}{5}x^5\arccos(ax)^4\\
&\quad -\frac{12}{25}\int x^4\arccos(ax)^2\,dx+\frac{16\int\frac{x^3\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{25a}\\
&= -\frac{12}{125}x^5\arccos(ax)^2-\frac{16x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^3}\\
&\quad -\frac{4x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{25a}+\frac{1}{5}x^5\arccos(ax)^4+\frac{32\int\frac{x\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{75a^3}\\
&\quad -\frac{16\int x^2\arccos(ax)^2\,dx}{25a^2}-\frac{1}{125}(24a)\int\frac{x^5\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{24x^4\sqrt{1-a^2x^2}\arccos(ax)}{625a}-\frac{16x^3\arccos(ax)^2}{75a^2}-\frac{12}{125}x^5\arccos(ax)^2\\
&\quad -\frac{32\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^5}-\frac{16x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^3}\\
&\quad -\frac{4x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{25a}+\frac{1}{5}x^5\arccos(ax)^4+\frac{24\int x^4\,dx}{625}\\
&\quad -\frac{32\int\arccos(ax)^2\,dx}{25a^4}-\frac{96\int\frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{625a}-\frac{32\int\frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{75a}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{24x^5}{3125} + \frac{1088x^2\sqrt{1-a^2x^2}\arccos(ax)}{5625a^3} + \frac{24x^4\sqrt{1-a^2x^2}\arccos(ax)}{625a} \\
&\quad - \frac{32x\arccos(ax)^2}{25a^4} - \frac{16x^3\arccos(ax)^2}{75a^2} - \frac{12}{125}x^5\arccos(ax)^2 \\
&\quad - \frac{32\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^5} - \frac{16x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^3} \\
&\quad - \frac{4x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{25a} + \frac{1}{5}x^5\arccos(ax)^4 - \frac{64\int\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{625a^3} \\
&\quad - \frac{64\int\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{225a^3} - \frac{64\int\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{25a^3} + \frac{32\int x^2\,dx}{625a^2} + \frac{32\int x^2\,dx}{225a^2} \\
&= \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} + \frac{16576\sqrt{1-a^2x^2}\arccos(ax)}{5625a^5} + \frac{1088x^2\sqrt{1-a^2x^2}\arccos(ax)}{5625a^3} \\
&\quad + \frac{24x^4\sqrt{1-a^2x^2}\arccos(ax)}{625a} - \frac{32x\arccos(ax)^2}{25a^4} - \frac{16x^3\arccos(ax)^2}{75a^2} \\
&\quad - \frac{12}{125}x^5\arccos(ax)^2 - \frac{32\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^5} - \frac{16x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^3} \\
&\quad - \frac{4x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{25a} + \frac{1}{5}x^5\arccos(ax)^4 + \frac{64\int 1\,dx}{625a^4} + \frac{64\int 1\,dx}{225a^4} + \frac{64\int 1\,dx}{25a^4} \\
&= \frac{16576x}{5625a^4} + \frac{1088x^3}{16875a^2} + \frac{24x^5}{3125} + \frac{16576\sqrt{1-a^2x^2}\arccos(ax)}{5625a^5} \\
&\quad + \frac{1088x^2\sqrt{1-a^2x^2}\arccos(ax)}{5625a^3} + \frac{24x^4\sqrt{1-a^2x^2}\arccos(ax)}{625a} - \frac{32x\arccos(ax)^2}{25a^4} \\
&\quad - \frac{16x^3\arccos(ax)^2}{75a^2} - \frac{12}{125}x^5\arccos(ax)^2 - \frac{32\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^5} \\
&\quad - \frac{16x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{75a^3} - \frac{4x^4\sqrt{1-a^2x^2}\arccos(ax)^3}{25a} + \frac{1}{5}x^5\arccos(ax)^4
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.60

$$
\int x^4\arccos(ax)^4\,dx
$$

$$
= \frac{8ax(31080+680a^2x^2+81a^4x^4)+120\sqrt{1-a^2x^2}(2072+136a^2x^2+27a^4x^4)\arccos(ax)-900ax(120+2
}{8437
}
$$

```
[In] Integrate[x^4*ArcCos[a*x]^4,x]
```

```
[Out] (8*a*x*(31080 + 680*a^2*x^2 + 81*a^4*x^4) + 120*Sqrt[1 - a^2*x^2]*(2072 + 1
36*a^2*x^2 + 27*a^4*x^4)*ArcCos[a*x] - 900*a*x*(120 + 20*a^2*x^2 + 9*a^4*x^
4)*ArcCos[a*x]^2 - 4500*Sqrt[1 - a^2*x^2]*(8 + 4*a^2*x^2 + 3*a^4*x^4)*ArcCo
s[a*x]^3 + 16875*a^5*x^5*ArcCos[a*x]^4)/(84375*a^5)
```

Maple [A] (verified)

Time = 2.46 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.79

method	result
derivativedivides	$\frac{\arccos(ax)^4 a^5 x^5}{5} - \frac{4\arccos(ax)^3\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{75} - \frac{12\arccos(ax)^2 a^5x^5}{125} + \frac{8\arccos(ax)\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{625} +$
default	$\frac{\arccos(ax)^4 a^5 x^5}{5} - \frac{4\arccos(ax)^3\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{75} - \frac{12\arccos(ax)^2 a^5x^5}{125} + \frac{8\arccos(ax)\left(3a^4x^4+4a^2x^2+8\right)\sqrt{-a^2x^2+1}}{625} +$

```
[In] int(x^4*arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/5*arccos(a*x)^4*a^5*x^5-4/75*arccos(a*x)^3*(3*a^4*x^4+4*a^2*x^2+8)
*(-a^2*x^2+1)^(1/2)-12/125*arccos(a*x)^2*a^5*x^5+8/625*arccos(a*x)*(3*a^4*x
^4+4*a^2*x^2+8)*(-a^2*x^2+1)^(1/2)+24/3125*a^5*x^5+1088/16875*a^3*x^3+16576
/5625*a*x-16/75*arccos(a*x)^2*a^3*x^3+32/225*arccos(a*x)*(a^2*x^2+2)*(-a^2*
x^2+1)^(1/2)-32/25*arccos(a*x)^2*a*x+64/25*arccos(a*x)*(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.54

$$\int x^4 \arccos(ax)^4\,dx$$
$$= \frac{16875\,a^5x^5\arccos{(ax)}^4 + 648\,a^5x^5 + 5440\,a^3x^3 - 900\,(9\,a^5x^5 + 20\,a^3x^3 + 120\,ax)\arccos{(ax)}^2 + 248640\,ax}{84375\,a}$$

```
[In] integrate(x^4*arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/84375*(16875*a^5*x^5*arccos(a*x)^4 + 648*a^5*x^5 + 5440*a^3*x^3 - 900*(9*
a^5*x^5 + 20*a^3*x^3 + 120*a*x)*arccos(a*x)^2 + 248640*a*x - 60*sqrt(-a^2*x
^2 + 1)*(75*(3*a^4*x^4 + 4*a^2*x^2 + 8)*arccos(a*x)^3 - 2*(27*a^4*x^4 + 136
*a^2*x^2 + 2072)*arccos(a*x)))/a^5
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.99

$$\int x^4 \arccos(ax)^4\,dx$$
$$= \begin{cases} \frac{x^5\operatorname{acos}^4(ax)}{5} - \frac{12x^5\operatorname{acos}^2(ax)}{125} + \frac{24x^5}{3125} - \frac{4x^4\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{25a} + \frac{24x^4\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{625a} - \frac{16x^3\operatorname{acos}^2(ax)}{75a^2} + \frac{1088x^3}{16875a^2} - \\ \frac{\pi^4x^5}{80} \end{cases}$$

```
[In] integrate(x**4*acos(a*x)**4,x)
```

```
[Out] Piecewise((x**5*acos(a*x)**4/5 - 12*x**5*acos(a*x)**2/125 + 24*x**5/3125 -
4*x**4*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(25*a) + 24*x**4*sqrt(-a**2*x**2 +
 1)*acos(a*x)/(625*a) - 16*x**3*acos(a*x)**2/(75*a**2) + 1088*x**3/(16875*a
**2) - 16*x**2*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(75*a**3) + 1088*x**2*sqrt
(-a**2*x**2 + 1)*acos(a*x)/(5625*a**3) - 32*x*acos(a*x)**2/(25*a**4) + 1657
6*x/(5625*a**4) - 32*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(75*a**5) + 16576*sq
rt(-a**2*x**2 + 1)*acos(a*x)/(5625*a**5), Ne(a, 0)), (pi**4*x**5/80, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.82

$$\int x^4 \arccos(ax)^4\,dx = \frac{1}{5}\,x^5 \arccos\left(ax\right)^4 - \frac{4}{75}\left(\frac{3\sqrt{-a^2x^2+1}x^4}{a^2} + \frac{4\sqrt{-a^2x^2+1}x^2}{a^4} + \frac{8\sqrt{-a^2x^2+1}}{a^6}\right)a\arccos\left(ax\right)^3 + \frac{4}{84375}\left(2\,a\left(\frac{15\left(27\sqrt{-a^2x^2+1}a^2x^4 + 136\sqrt{-a^2x^2+1}x^2 + \frac{2072\sqrt{-a^2x^2+1}}{a^2}\right)\arccos\left(ax\right)}{a^5} + \frac{81\,a^4x^5 + 6}{}\right.\right.$$

```
[In] integrate(x^4*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/5*x^5*arccos(a*x)^4 - 4/75*(3*sqrt(-a^2*x^2 + 1)*x^4/a^2 + 4*sqrt(-a^2*x^
2 + 1)*x^2/a^4 + 8*sqrt(-a^2*x^2 + 1)/a^6)*a*arccos(a*x)^3 + 4/84375*(2*a*(
15*(27*sqrt(-a^2*x^2 + 1)*a^2*x^4 + 136*sqrt(-a^2*x^2 + 1)*x^2 + 2072*sqrt(
-a^2*x^2 + 1)/a^2)*arccos(a*x)/a^5 + (81*a^4*x^5 + 680*a^2*x^3 + 31080*x)/a
^6) - 225*(9*a^4*x^5 + 20*a^2*x^3 + 120*x)*arccos(a*x)^2/a^5)*a
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.85

$$\int x^4 \arccos(ax)^4 \, dx = \frac{1}{5} x^5 \arccos(ax)^4 - \frac{12}{125} x^5 \arccos(ax)^2 - \frac{4\sqrt{-a^2x^2+1}x^4 \arccos(ax)^3}{25a} + \frac{24}{3125} x^5 + \frac{24\sqrt{-a^2x^2+1}x^4 \arccos(ax)}{625a} - \frac{16x^3 \arccos(ax)^2}{75a^2} - \frac{16\sqrt{-a^2x^2+1}x^2 \arccos(ax)^3}{75a^3} + \frac{1088x^3}{16875a^2} + \frac{1088\sqrt{-a^2x^2+1}x^2 \arccos(ax)}{5625a^3} - \frac{32x \arccos(ax)^2}{25a^4} - \frac{32\sqrt{-a^2x^2+1} \arccos(ax)^3}{75a^5} + \frac{16576x}{5625a^4} + \frac{16576\sqrt{-a^2x^2+1} \arccos(ax)}{5625a^5}$$

```
[In] integrate(x^4*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/5*x^5*arccos(a*x)^4 - 12/125*x^5*arccos(a*x)^2 - 4/25*sqrt(-a^2*x^2 + 1)*
x^4*arccos(a*x)^3/a + 24/3125*x^5 + 24/625*sqrt(-a^2*x^2 + 1)*x^4*arccos(a*
x)/a - 16/75*x^3*arccos(a*x)^2/a^2 - 16/75*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*
x)^3/a^3 + 1088/16875*x^3/a^2 + 1088/5625*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x
)/a^3 - 32/25*x*arccos(a*x)^2/a^4 - 32/75*sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/
a^5 + 16576/5625*x/a^4 + 16576/5625*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^4 \, dx = \int x^4 \operatorname{acos}(a\,x)^4 \, dx$$

```
[In] int(x^4*acos(a*x)^4,x)
```

```
[Out] int(x^4*acos(a*x)^4, x)
```

3.34 $\int x^3 \arccos(ax)^4 \, dx$

Optimal result

Integrand size = 10, antiderivative size = 198

$$
\begin{aligned}
\int x^3 \arccos(ax)^4 \, dx = {} & \frac{45x^2}{128a^2} + \frac{3x^4}{128} + \frac{45x\sqrt{1-a^2x^2}\arccos(ax)}{64a^3} \\
& + \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)}{32a} + \frac{45\arccos(ax)^2}{128a^4} - \frac{9x^2\arccos(ax)^2}{16a^2} \\
& - \frac{3}{16}x^4\arccos(ax)^2 - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^3}{8a^3} \\
& - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a} - \frac{3\arccos(ax)^4}{32a^4} + \frac{1}{4}x^4\arccos(ax)^4
\end{aligned}
$$

```
[Out] 45/128*x^2/a^2+3/128*x^4+45/128*arccos(a*x)^2/a^4-9/16*x^2*arccos(a*x)^2/a^
2-3/16*x^4*arccos(a*x)^2-3/32*arccos(a*x)^4/a^4+1/4*x^4*arccos(a*x)^4+45/64
*x*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a^3+3/32*x^3*arccos(a*x)*(-a^2*x^2+1)^(1/
2)/a-3/8*x*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)/a^3-1/4*x^3*arccos(a*x)^3*(-a^2
*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used

$= \{4724, 4796, 4738, 30\}$

$$\int x^3 \arccos(ax)^4\,dx = -\frac{3\arccos(ax)^4}{32a^4} + \frac{45\arccos(ax)^2}{128a^4} - \frac{9x^2\arccos(ax)^2}{16a^2} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a} + \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)}{32a} + \frac{45x^2}{128a^2} - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^3}{8a^3} + \frac{45x\sqrt{1-a^2x^2}\arccos(ax)}{64a^3} + \frac{1}{4}x^4\arccos(ax)^4 - \frac{3}{16}x^4\arccos(ax)^2 + \frac{3x^4}{128}$$

```
[In] Int[x^3*ArcCos[a*x]^4,x]
```

```
[Out] (45*x^2)/(128*a^2) + (3*x^4)/128 + (45*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(64
*a^3) + (3*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(32*a) + (45*ArcCos[a*x]^2)/(
128*a^4) - (9*x^2*ArcCos[a*x]^2)/(16*a^2) - (3*x^4*ArcCos[a*x]^2)/16 - (3*x
*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(8*a^3) - (x^3*Sqrt[1 - a^2*x^2]*ArcCos[a
*x]^3)/(4*a) - (3*ArcCos[a*x]^4)/(32*a^4) + (x^4*ArcCos[a*x]^4)/4
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
```

```
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\arccos(ax)^4 + a\int \frac{x^4\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a} + \frac{1}{4}x^4\arccos(ax)^4 - \frac{3}{4}\int x^3\arccos(ax)^2\,dx + \frac{3\int \frac{x^2\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{4a} \\
&= -\frac{3}{16}x^4\arccos(ax)^2 - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^3}{8a^3} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a} \\
&\quad + \frac{1}{4}x^4\arccos(ax)^4 + \frac{3\int \frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{8a^3} - \frac{9\int x\arccos(ax)^2\,dx}{8a^2} \\
&\quad - \frac{1}{8}(3a)\int \frac{x^4\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx \\
&= \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)}{32a} - \frac{9x^2\arccos(ax)^2}{16a^2} - \frac{3}{16}x^4\arccos(ax)^2 \\
&\quad - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^3}{8a^3} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a} - \frac{3\arccos(ax)^4}{32a^4} \\
&\quad + \frac{1}{4}x^4\arccos(ax)^4 + \frac{3\int x^3\,dx}{32} - \frac{9\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{32a} - \frac{9\int \frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{8a} \\
&= \frac{3x^4}{128} + \frac{45x\sqrt{1-a^2x^2}\arccos(ax)}{64a^3} + \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)}{32a} - \frac{9x^2\arccos(ax)^2}{16a^2} \\
&\quad - \frac{3}{16}x^4\arccos(ax)^2 - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^3}{8a^3} - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a} \\
&\quad - \frac{3\arccos(ax)^4}{32a^4} + \frac{1}{4}x^4\arccos(ax)^4 - \frac{9\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{64a^3} - \frac{9\int \frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{16a^3} + \frac{9\int x\,dx}{64a^2} \\
&\quad + \frac{9\int x\,dx}{16a^2} \\
&= \frac{45x^2}{128a^2} + \frac{3x^4}{128} + \frac{45x\sqrt{1-a^2x^2}\arccos(ax)}{64a^3} + \frac{3x^3\sqrt{1-a^2x^2}\arccos(ax)}{32a} \\
&\quad + \frac{45\arccos(ax)^2}{128a^4} - \frac{9x^2\arccos(ax)^2}{16a^2} - \frac{3}{16}x^4\arccos(ax)^2 - \frac{3x\sqrt{1-a^2x^2}\arccos(ax)^3}{8a^3} \\
&\quad - \frac{x^3\sqrt{1-a^2x^2}\arccos(ax)^3}{4a} - \frac{3\arccos(ax)^4}{32a^4} + \frac{1}{4}x^4\arccos(ax)^4
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.68

$$\int x^3 \arccos(ax)^4 \, dx = \frac{3a^2x^2(15+a^2x^2)+6ax\sqrt{1-a^2x^2}(15+2a^2x^2)\arccos(ax)-3(-15+24a^2x^2+8a^4x^4)\arccos(ax)^2-16ax}{128a^4}$$

```
[In] Integrate[x^3*ArcCos[a*x]^4,x]
```

```
[Out] (3*a^2*x^2*(15 + a^2*x^2) + 6*a*x*Sqrt[1 - a^2*x^2]*(15 + 2*a^2*x^2)*ArcCos
[a*x] - 3*(-15 + 24*a^2*x^2 + 8*a^4*x^4)*ArcCos[a*x]^2 - 16*a*x*Sqrt[1 - a^
2*x^2]*(3 + 2*a^2*x^2)*ArcCos[a*x]^3 + 4*(-3 + 8*a^4*x^4)*ArcCos[a*x]^4)/(1
28*a^4)
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{a^4x^4\arccos(ax)^4}{4}-\frac{\arccos(ax)^3\left(2a^3x^3\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}+3\arccos(ax)\right)}{8}-\frac{3a^4x^4\arccos(ax)^2}{16}+\frac{3\arccos(ax)\left(2a^3x^3\sqrt{-a^2x\right.}{}$
default	$\frac{a^4x^4\arccos(ax)^4}{4}-\frac{\arccos(ax)^3\left(2a^3x^3\sqrt{-a^2x^2+1}+3ax\sqrt{-a^2x^2+1}+3\arccos(ax)\right)}{8}-\frac{3a^4x^4\arccos(ax)^2}{16}+\frac{3\arccos(ax)\left(2a^3x^3\sqrt{-a^2x\right.}{}$

```
[In] int(x^3*arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4*a^4*x^4*arccos(a*x)^4-1/8*arccos(a*x)^3*(2*a^3*x^3*(-a^2*x^2+1)^
(1/2)+3*a*x*(-a^2*x^2+1)^(1/2)+3*arccos(a*x))-3/16*a^4*x^4*arccos(a*x)^2+3/
64*arccos(a*x)*(2*a^3*x^3*(-a^2*x^2+1)^(1/2)+3*a*x*(-a^2*x^2+1)^(1/2)+3*arc
cos(a*x))-45/128*arccos(a*x)^2+3/512*(2*a^2*x^2+3)^2-9/16*a^2*x^2*arccos(a*
x)^2+9/16*arccos(a*x)*(a*x*(-a^2*x^2+1)^(1/2)+arccos(a*x))+9/32*a^2*x^2-9/3
2+9/32*arccos(a*x)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.61

$$\int x^3 \arccos(ax)^4 \, dx = \frac{3\,a^4x^4 + 4\,(8\,a^4x^4 - 3)\arccos{(ax)}^4 + 45\,a^2x^2 - 3\,(8\,a^4x^4 + 24\,a^2x^2 - 15)\arccos{(ax)}^2 - 2\,\sqrt{-a^2x^2 + 1}(8}{128\,a^4}$$

```
[In] integrate(x^3*arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(3*a^4*x^4 + 4*(8*a^4*x^4 - 3)*arccos(a*x)^4 + 45*a^2*x^2 - 3*(8*a^4*
x^4 + 24*a^2*x^2 - 15)*arccos(a*x)^2 - 2*sqrt(-a^2*x^2 + 1)*(8*(2*a^3*x^3 +
 3*a*x)*arccos(a*x)^3 - 3*(2*a^3*x^3 + 15*a*x)*arccos(a*x)))/a^4
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.99

$$\int x^3 \arccos(ax)^4 \, dx = \begin{cases} \frac{x^4 \operatorname{acos}^4(ax)}{4} - \frac{3x^4 \operatorname{acos}^2(ax)}{16} + \frac{3x^4}{128} - \frac{x^3\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{4a} + \frac{3x^3\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{32a} - \frac{9x^2\operatorname{acos}^2(ax)}{16a^2} + \frac{45x^2}{128a^2} - \frac{3x\sqrt{-a} \\ \frac{\pi^4x^4}{64} \end{cases}$$

```
[In] integrate(x**3*acos(a*x)**4,x)
```

```
[Out] Piecewise((x**4*acos(a*x)**4/4 - 3*x**4*acos(a*x)**2/16 + 3*x**4/128 - x**3
*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(4*a) + 3*x**3*sqrt(-a**2*x**2 + 1)*acos
(a*x)/(32*a) - 9*x**2*acos(a*x)**2/(16*a**2) + 45*x**2/(128*a**2) - 3*x*sqr
t(-a**2*x**2 + 1)*acos(a*x)**3/(8*a**3) + 45*x*sqrt(-a**2*x**2 + 1)*acos(a*
x)/(64*a**3) - 3*acos(a*x)**4/(32*a**4) + 45*acos(a*x)**2/(128*a**4), Ne(a,
 0)), (pi**4*x**4/64, True))
```

Maxima [F]

$$\int x^3 \arccos(ax)^4 \, dx = \int x^3 \arccos{(ax)}^4 \, dx$$

```
[In] integrate(x^3*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/4*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - a*integrate(sqrt(a*x
 + 1)*sqrt(-a*x + 1)*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(a^2*
x^2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.87

$$\int x^3 \arccos(ax)^4 \, dx = \frac{1}{4} x^4 \arccos(ax)^4 - \frac{3}{16} x^4 \arccos(ax)^2 - \frac{\sqrt{-a^2x^2+1}x^3 \arccos(ax)^3}{4a} + \frac{3}{128} x^4 + \frac{3\sqrt{-a^2x^2+1}x^3 \arccos(ax)}{32a} - \frac{9x^2 \arccos(ax)^2}{16a^2} - \frac{3\sqrt{-a^2x^2+1}x \arccos(ax)^3}{8a^3} + \frac{45x^2}{128a^2} - \frac{3\arccos(ax)^4}{32a^4} + \frac{45\sqrt{-a^2x^2+1}x \arccos(ax)}{64a^3} + \frac{45\arccos(ax)^2}{128a^4} - \frac{189}{1024a^4}$$

```
[In] integrate(x^3*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/4*x^4*arccos(a*x)^4 - 3/16*x^4*arccos(a*x)^2 - 1/4*sqrt(-a^2*x^2 + 1)*x^3
*arccos(a*x)^3/a + 3/128*x^4 + 3/32*sqrt(-a^2*x^2 + 1)*x^3*arccos(a*x)/a -
9/16*x^2*arccos(a*x)^2/a^2 - 3/8*sqrt(-a^2*x^2 + 1)*x*arccos(a*x)^3/a^3 + 4
5/128*x^2/a^2 - 3/32*arccos(a*x)^4/a^4 + 45/64*sqrt(-a^2*x^2 + 1)*x*arccos(
a*x)/a^3 + 45/128*arccos(a*x)^2/a^4 - 189/1024/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^4 \, dx = \int x^3 \operatorname{acos}(a\,x)^4 \, dx$$

```
[In] int(x^3*acos(a*x)^4,x)
```

```
[Out] int(x^3*acos(a*x)^4, x)
```

3.35 $\int x^2 \arccos(ax)^4\,dx$

Optimal result

Integrand size = 10, antiderivative size = 166

$$\int x^2 \arccos(ax)^4\,dx = \frac{160x}{27a^2} + \frac{8x^3}{81} + \frac{160\sqrt{1-a^2x^2}\arccos(ax)}{27a^3} + \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{27a} - \frac{8x\arccos(ax)^2}{3a^2} - \frac{4}{9}x^3\arccos(ax)^2 - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^3}{9a^3} - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} + \frac{1}{3}x^3\arccos(ax)^4$$

```
[Out] 160/27*x/a^2+8/81*x^3-8/3*x*arccos(a*x)^2/a^2-4/9*x^3*arccos(a*x)^2+1/3*x^3
*arccos(a*x)^4+160/27*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a^3+8/27*x^2*arccos(a*
x)*(-a^2*x^2+1)^(1/2)/a-8/9*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)/a^3-4/9*x^2*ar
ccos(a*x)^3*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4724, 4796, 4768, 4716, 8, 30}

$$\int x^2 \arccos(ax)^4\,dx = -\frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} + \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{27a} - \frac{8x\arccos(ax)^2}{3a^2} + \frac{160x}{27a^2} - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^3}{9a^3} + \frac{160\sqrt{1-a^2x^2}\arccos(ax)}{27a^3} + \frac{1}{3}x^3\arccos(ax)^4 - \frac{4}{9}x^3\arccos(ax)^2 + \frac{8x^3}{81}$$

```
[In] Int[x^2*ArcCos[a*x]^4,x]
```

```
[Out] (160*x)/(27*a^2) + (8*x^3)/81 + (160*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(27*a^3
) + (8*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(27*a) - (8*x*ArcCos[a*x]^2)/(3*a
^2) - (4*x^3*ArcCos[a*x]^2)/9 - (8*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(9*a^3)
 - (4*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(9*a) + (x^3*ArcCos[a*x]^4)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}x^3\arccos(ax)^4 + \frac{1}{3}(4a)\int\frac{x^3\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} + \frac{1}{3}x^3\arccos(ax)^4 - \frac{4}{3}\int x^2\arccos(ax)^2\,dx + \frac{8\int\frac{x\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{9a}$$

$$= -\frac{4}{9}x^3\arccos(ax)^2 - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^3}{9a^3} - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} + \frac{1}{3}x^3\arccos(ax)^4 - \frac{8\int\arccos(ax)^2\,dx}{3a^2} - \frac{1}{9}(8a)\int\frac{x^3\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{27a} - \frac{8x\arccos(ax)^2}{3a^2} - \frac{4}{9}x^3\arccos(ax)^2 - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^3}{9a^3} - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} + \frac{1}{3}x^3\arccos(ax)^4 + \frac{8\int x^2\,dx}{27} - \frac{16\int\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{27a} - \frac{16\int\frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{3a}$$

$$= \frac{8x^3}{81} + \frac{160\sqrt{1-a^2x^2}\arccos(ax)}{27a^3} + \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{27a} - \frac{8x\arccos(ax)^2}{3a^2} - \frac{4}{9}x^3\arccos(ax)^2 - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^3}{9a^3} - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} + \frac{1}{3}x^3\arccos(ax)^4 + \frac{16\int 1\,dx}{27a^2} + \frac{16\int 1\,dx}{3a^2}$$

$$= \frac{160x}{27a^2} + \frac{8x^3}{81} + \frac{160\sqrt{1-a^2x^2}\arccos(ax)}{27a^3} + \frac{8x^2\sqrt{1-a^2x^2}\arccos(ax)}{27a} - \frac{8x\arccos(ax)^2}{3a^2} - \frac{4}{9}x^3\arccos(ax)^2 - \frac{8\sqrt{1-a^2x^2}\arccos(ax)^3}{9a^3} - \frac{4x^2\sqrt{1-a^2x^2}\arccos(ax)^3}{9a} + \frac{1}{3}x^3\arccos(ax)^4$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.69

$$\int x^2\arccos(ax)^4\,dx$$

$$= \frac{8ax(60+a^2x^2) + 24\sqrt{1-a^2x^2}(20+a^2x^2)\arccos(ax) - 36ax(6+a^2x^2)\arccos(ax)^2 - 36\sqrt{1-a^2x^2}(2+}{81a^3}$$

```
[In] Integrate[x^2*ArcCos[a*x]^4,x]
```

```
[Out] (8*a*x*(60 + a^2*x^2) + 24*Sqrt[1 - a^2*x^2]*(20 + a^2*x^2)*ArcCos[a*x] - 3
6*a*x*(6 + a^2*x^2)*ArcCos[a*x]^2 - 36*Sqrt[1 - a^2*x^2]*(2 + a^2*x^2)*ArcC
os[a*x]^3 + 27*a^3*x^3*ArcCos[a*x]^4)/(81*a^3)
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\frac{a^3x^3\arccos(ax)^4}{3}-\frac{4\arccos(ax)^3\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{9}-\frac{8\arccos(ax)^2ax}{3}+\frac{160ax}{27}+\frac{16\arccos(ax)\sqrt{-a^2x^2+1}}{3}-\frac{4\arccos(ax)^2a^3x^3}{9}+}{a^3}$
default	$\frac{\frac{a^3x^3\arccos(ax)^4}{3}-\frac{4\arccos(ax)^3\left(a^2x^2+2\right)\sqrt{-a^2x^2+1}}{9}-\frac{8\arccos(ax)^2ax}{3}+\frac{160ax}{27}+\frac{16\arccos(ax)\sqrt{-a^2x^2+1}}{3}-\frac{4\arccos(ax)^2a^3x^3}{9}+}{a^3}$

```
[In] int(x^2*arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/3*a^3*x^3*arccos(a*x)^4-4/9*arccos(a*x)^3*(a^2*x^2+2)*(-a^2*x^2+1)
^(1/2)-8/3*arccos(a*x)^2*a*x+160/27*a*x+16/3*arccos(a*x)*(-a^2*x^2+1)^(1/2)
-4/9*arccos(a*x)^2*a^3*x^3+8/27*arccos(a*x)*(a^2*x^2+2)*(-a^2*x^2+1)^(1/2)+
8/81*a^3*x^3)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.60

$$\int x^2\arccos(ax)^4\,dx$$
$$=\frac{27\,a^3x^3\arccos{(ax)}^4+8\,a^3x^3-36\,(a^3x^3+6\,ax)\arccos{(ax)}^2+480\,ax-12\sqrt{-a^2x^2+1}\big(3\,(a^2x^2+2)\arccos}{81\,a^3}$$

```
[In] integrate(x^2*arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/81*(27*a^3*x^3*arccos(a*x)^4 + 8*a^3*x^3 - 36*(a^3*x^3 + 6*a*x)*arccos(a*
x)^2 + 480*a*x - 12*sqrt(-a^2*x^2 + 1)*(3*(a^2*x^2 + 2)*arccos(a*x)^3 - 2*(
a^2*x^2 + 20)*arccos(a*x)))/a^3
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int x^2\arccos(ax)^4\,dx$$
$$=\begin{cases}\frac{x^3\operatorname{acos}^4(ax)}{3}-\frac{4x^3\operatorname{acos}^2(ax)}{9}+\frac{8x^3}{81}-\frac{4x^2\sqrt{-a^2x^2+1}\operatorname{acos}^3(ax)}{9a}+\frac{8x^2\sqrt{-a^2x^2+1}\operatorname{acos}(ax)}{27a}-\frac{8x\operatorname{acos}^2(ax)}{3a^2}+\frac{160x}{27a^2}-\frac{8\sqrt{-a^2x^2+}}{9} \\ \frac{\pi^4x^3}{48}\end{cases}$$

```
[In] integrate(x**2*acos(a*x)**4,x)
```

```
[Out] Piecewise((x**3*acos(a*x)**4/3 - 4*x**3*acos(a*x)**2/9 + 8*x**3/81 - 4*x**2
*sqrt(-a**2*x**2 + 1)*acos(a*x)**3/(9*a) + 8*x**2*sqrt(-a**2*x**2 + 1)*acos
(a*x)/(27*a) - 8*x*acos(a*x)**2/(3*a**2) + 160*x/(27*a**2) - 8*sqrt(-a**2*x
**2 + 1)*acos(a*x)**3/(9*a**3) + 160*sqrt(-a**2*x**2 + 1)*acos(a*x)/(27*a**
3), Ne(a, 0)), (pi**4*x**3/48, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.88

$$\int x^2 \arccos(ax)^4 \, dx$$
$$= \frac{1}{3}\, x^3 \arccos\left(ax\right)^4 - \frac{4}{9}\, a \left(\frac{\sqrt{-a^2x^2+1}x^2}{a^2} + \frac{2\, \sqrt{-a^2x^2+1}}{a^4}\right) \arccos\left(ax\right)^3$$
$$+ \frac{4}{81}\, \left(2\, a \left(\frac{3\, \left(\sqrt{-a^2x^2+1}x^2 + \frac{20\, \sqrt{-a^2x^2+1}}{a^2}\right) \arccos\left(ax\right)}{a^3} + \frac{a^2x^3 + 60\, x}{a^4}\right) - \frac{9\, \left(a^2x^3 + 6\, x\right) \arccos\left(ax\right)^2}{a^3}\right)$$

```
[In] integrate(x^2*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/3*x^3*arccos(a*x)^4 - 4/9*a*(sqrt(-a^2*x^2 + 1)*x^2/a^2 + 2*sqrt(-a^2*x^2
 + 1)/a^4)*arccos(a*x)^3 + 4/81*(2*a*(3*(sqrt(-a^2*x^2 + 1)*x^2 + 20*sqrt(-
a^2*x^2 + 1)/a^2)*arccos(a*x)/a^3 + (a^2*x^3 + 60*x)/a^4) - 9*(a^2*x^3 + 6*
x)*arccos(a*x)^2/a^3)*a
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int x^2 \arccos(ax)^4 \, dx = \frac{1}{3}\, x^3 \arccos\left(ax\right)^4 - \frac{4}{9}\, x^3 \arccos\left(ax\right)^2 - \frac{4\, \sqrt{-a^2x^2+1}x^2 \arccos\left(ax\right)^3}{9\, a}$$
$$+ \frac{8}{81}\, x^3 + \frac{8\, \sqrt{-a^2x^2+1}x^2 \arccos\left(ax\right)}{27\, a} - \frac{8\, x \arccos\left(ax\right)^2}{3\, a^2}$$
$$- \frac{8\, \sqrt{-a^2x^2+1} \arccos\left(ax\right)^3}{9\, a^3} + \frac{160\, x}{27\, a^2} + \frac{160\, \sqrt{-a^2x^2+1} \arccos\left(ax\right)}{27\, a^3}$$

```
[In] integrate(x^2*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/3*x^3*arccos(a*x)^4 - 4/9*x^3*arccos(a*x)^2 - 4/9*sqrt(-a^2*x^2 + 1)*x^2*
arccos(a*x)^3/a + 8/81*x^3 + 8/27*sqrt(-a^2*x^2 + 1)*x^2*arccos(a*x)/a - 8/
3*x*arccos(a*x)^2/a^2 - 8/9*sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/a^3 + 160/27*x
/a^2 + 160/27*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^4 \, dx = \int x^2 \operatorname{acos}(a\, x)^4 \, dx$$

```
[In] int(x^2*acos(a*x)^4,x)
[Out] int(x^2*acos(a*x)^4, x)
```

3.36 $\int x \arccos(ax)^4 \, dx$

Optimal result

Integrand size = 8, antiderivative size = 112

$$\int x \arccos(ax)^4 \, dx = \frac{3x^2}{4} + \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{2a} + \frac{3\arccos(ax)^2}{4a^2} - \frac{3}{2}x^2\arccos(ax)^2 - \frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{a} - \frac{\arccos(ax)^4}{4a^2} + \frac{1}{2}x^2\arccos(ax)^4$$

```
[Out] 3/4*x^2+3/4*arccos(a*x)^2/a^2-3/2*x^2*arccos(a*x)^2-1/4*arccos(a*x)^4/a^2+1
/2*x^2*arccos(a*x)^4+3/2*x*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a-x*arccos(a*x)^3
*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4724, 4796, 4738, 30}

$$\int x \arccos(ax)^4 \, dx = -\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{a} + \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{2a} - \frac{\arccos(ax)^4}{4a^2} + \frac{3\arccos(ax)^2}{4a^2} + \frac{1}{2}x^2\arccos(ax)^4 - \frac{3}{2}x^2\arccos(ax)^2 + \frac{3x^2}{4}$$

```
[In] Int[x*ArcCos[a*x]^4,x]
```

```
[Out] (3*x^2)/4 + (3*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/(2*a) + (3*ArcCos[a*x]^2)/(
4*a^2) - (3*x^2*ArcCos[a*x]^2)/2 - (x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/a -
ArcCos[a*x]^4/(4*a^2) + (x^2*ArcCos[a*x]^4)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\arccos(ax)^4+(2a)\int\frac{x^2\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{a}+\frac{1}{2}x^2\arccos(ax)^4-3\int x\arccos(ax)^2\,dx+\frac{\int\frac{\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx}{a}\\
&= -\frac{3}{2}x^2\arccos(ax)^2-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{a}-\frac{\arccos(ax)^4}{4a^2}\\
&\quad+\frac{1}{2}x^2\arccos(ax)^4-(3a)\int\frac{x^2\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{2a}-\frac{3}{2}x^2\arccos(ax)^2-\frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{a}\\
&\quad-\frac{\arccos(ax)^4}{4a^2}+\frac{1}{2}x^2\arccos(ax)^4+\frac{3\int x\,dx}{2}-\frac{3\int\frac{\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{2a}
\end{aligned}
$$

$$= \frac{3x^2}{4} + \frac{3x\sqrt{1-a^2x^2}\arccos(ax)}{2a} + \frac{3\arccos(ax)^2}{4a^2} - \frac{3}{2}x^2\arccos(ax)^2 - \frac{x\sqrt{1-a^2x^2}\arccos(ax)^3}{a} - \frac{\arccos(ax)^4}{4a^2} + \frac{1}{2}x^2\arccos(ax)^4$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int x\arccos(ax)^4\,dx = \frac{3a^2x^2 + 6ax\sqrt{1-a^2x^2}\arccos(ax) + (3-6a^2x^2)\arccos(ax)^2 - 4ax\sqrt{1-a^2x^2}\arccos(ax)^3 + (-1+2a^2x^2}{4a^2}$$

```
[In] Integrate[x*ArcCos[a*x]^4,x]
```

```
[Out] (3*a^2*x^2 + 6*a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x] + (3 - 6*a^2*x^2)*ArcCos[a
*x]^2 - 4*a*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3 + (-1 + 2*a^2*x^2)*ArcCos[a*x
]^4)/(4*a^2)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.01

method	result
derivativedivides	$\frac{\frac{\arccos(ax)^4a^2x^2}{2} - \arccos(ax)^3\left(ax\sqrt{-a^2x^2+1}+\arccos(ax)\right) - \frac{3a^2x^2\arccos(ax)^2}{2} + \frac{3\arccos(ax)\left(ax\sqrt{-a^2x^2+1}+\arccos(ax)\right)}{2} -}{a^2}$
default	$\frac{\frac{\arccos(ax)^4a^2x^2}{2} - \arccos(ax)^3\left(ax\sqrt{-a^2x^2+1}+\arccos(ax)\right) - \frac{3a^2x^2\arccos(ax)^2}{2} + \frac{3\arccos(ax)\left(ax\sqrt{-a^2x^2+1}+\arccos(ax)\right)}{2} -}{a^2}$

```
[In] int(x*arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/2*arccos(a*x)^4*a^2*x^2-arccos(a*x)^3*(a*x*(-a^2*x^2+1)^(1/2)+arcc
os(a*x))-3/2*a^2*x^2*arccos(a*x)^2+3/2*arccos(a*x)*(a*x*(-a^2*x^2+1)^(1/2)+
arccos(a*x))-3/4*arccos(a*x)^2+3/4*a^2*x^2-3/4+3/4*arccos(a*x)^4)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.73

$$\int x \arccos(ax)^4 \, dx = \frac{(2\,a^2x^2-1)\arccos{(ax)}^4 + 3\,a^2x^2 - 3\,(2\,a^2x^2-1)\arccos{(ax)}^2 - 2\,\left(2\,ax\arccos{(ax)}^3 - 3\,ax\arccos{(ax)}\right)\sqrt{}}{4\,a^2}$$

```
[In] integrate(x*arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] 1/4*((2*a^2*x^2 - 1)*arccos(a*x)^4 + 3*a^2*x^2 - 3*(2*a^2*x^2 - 1)*arccos(a
*x)^2 - 2*(2*a*x*arccos(a*x)^3 - 3*a*x*arccos(a*x))*sqrt(-a^2*x^2 + 1))/a^2
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.98

$$\int x \arccos(ax)^4 \, dx = \begin{cases} \frac{x^2\operatorname{acos}^4{(ax)}}{2} - \frac{3x^2\operatorname{acos}^2{(ax)}}{2} + \frac{3x^2}{4} - \frac{x\sqrt{-a^2x^2+1}\operatorname{acos}^3{(ax)}}{a} + \frac{3x\sqrt{-a^2x^2+1}\operatorname{acos}{(ax)}}{2a} - \frac{\operatorname{acos}^4{(ax)}}{4a^2} + \frac{3\operatorname{acos}^2{(ax)}}{4a^2} & \text{for } a \neq \\ \frac{\pi^4x^2}{32} & \text{otherwis} \end{cases}$$

```
[In] integrate(x*acos(a*x)**4,x)
```

```
[Out] Piecewise((x**2*acos(a*x)**4/2 - 3*x**2*acos(a*x)**2/2 + 3*x**2/4 - x*sqrt(
-a**2*x**2 + 1)*acos(a*x)**3/a + 3*x*sqrt(-a**2*x**2 + 1)*acos(a*x)/(2*a) -
 acos(a*x)**4/(4*a**2) + 3*acos(a*x)**2/(4*a**2), Ne(a, 0)), (pi**4*x**2/32
, True))
```

Maxima [F]

$$\int x \arccos(ax)^4 \, dx = \int x \arccos{(ax)}^4 \, dx$$

```
[In] integrate(x*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 2*a*integrate(sqrt(a
*x + 1)*sqrt(-a*x + 1)*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(a^
2*x^2 - 1), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.90

$$\int x \arccos(ax)^4 \, dx = \frac{1}{2} x^2 \arccos(ax)^4 - \frac{3}{2} x^2 \arccos(ax)^2 - \frac{\sqrt{-a^2x^2+1}x \arccos(ax)^3}{a} + \frac{3}{4} x^2 - \frac{\arccos(ax)^4}{4a^2} + \frac{3\sqrt{-a^2x^2+1}x \arccos(ax)}{2a} + \frac{3 \arccos(ax)^2}{4a^2} - \frac{3}{8a^2}$$

```
[In] integrate(x*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/2*x^2*arccos(a*x)^4 - 3/2*x^2*arccos(a*x)^2 - sqrt(-a^2*x^2 + 1)*x*arccos
(a*x)^3/a + 3/4*x^2 - 1/4*arccos(a*x)^4/a^2 + 3/2*sqrt(-a^2*x^2 + 1)*x*arcc
os(a*x)/a + 3/4*arccos(a*x)^2/a^2 - 3/8/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x \arccos(ax)^4 \, dx = \int x \operatorname{acos}(a\,x)^4 \, dx$$

```
[In] int(x*acos(a*x)^4,x)
```

```
[Out] int(x*acos(a*x)^4, x)
```

3.37 $\int \arccos(ax)^4\,dx$

Optimal result

Integrand size = 6, antiderivative size = 69

$$\int \arccos(ax)^4\,dx = 24x + \frac{24\sqrt{1-a^2x^2}\arccos(ax)}{a} - 12x\arccos(ax)^2 - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^3}{a} + x\arccos(ax)^4$$

```
[Out] 24*x-12*x*arccos(a*x)^2+x*arccos(a*x)^4+24*arccos(a*x)*(-a^2*x^2+1)^(1/2)/a
-4*arccos(a*x)^3*(-a^2*x^2+1)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4716, 4768, 8}

$$\int \arccos(ax)^4\,dx = -\frac{4\sqrt{1-a^2x^2}\arccos(ax)^3}{a} + \frac{24\sqrt{1-a^2x^2}\arccos(ax)}{a} + x\arccos(ax)^4 - 12x\arccos(ax)^2 + 24x$$

```
[In] Int[ArcCos[a*x]^4,x]
```

```
[Out] 24*x + (24*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a - 12*x*ArcCos[a*x]^2 - (4*Sqrt[
1 - a^2*x^2]*ArcCos[a*x]^3)/a + x*ArcCos[a*x]^4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arccos(ax)^4 + (4a)\int \frac{x\arccos(ax)^3}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{4\sqrt{1-a^2x^2}\arccos(ax)^3}{a} + x\arccos(ax)^4 - 12\int \arccos(ax)^2\,dx\\
&= -12x\arccos(ax)^2 - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^3}{a} + x\arccos(ax)^4 - (24a)\int \frac{x\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx\\
&= \frac{24\sqrt{1-a^2x^2}\arccos(ax)}{a} - 12x\arccos(ax)^2\\
&\quad - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^3}{a} + x\arccos(ax)^4 + 24\int 1\,dx\\
&= 24x + \frac{24\sqrt{1-a^2x^2}\arccos(ax)}{a} - 12x\arccos(ax)^2 - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^3}{a} + x\arccos(ax)^4
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int \arccos(ax)^4\,dx = 24x + \frac{24\sqrt{1-a^2x^2}\arccos(ax)}{a} - 12x\arccos(ax)^2\\
- \frac{4\sqrt{1-a^2x^2}\arccos(ax)^3}{a} + x\arccos(ax)^4
\end{aligned}$$

```
[In] Integrate[ArcCos[a*x]^4,x]
```

```
[Out] 24*x + (24*Sqrt[1 - a^2*x^2]*ArcCos[a*x])/a - 12*x*ArcCos[a*x]^2 - (4*Sqrt[
1 - a^2*x^2]*ArcCos[a*x]^3)/a + x*ArcCos[a*x]^4
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{ax\arccos(ax)^{4}-4\sqrt{-a^{2}x^{2}+1}\,\arccos(ax)^{3}-12\arccos(ax)^{2}ax+24ax+24\arccos(ax)\sqrt{-a^{2}x^{2}+1}}{a}$	67
default	$\frac{ax\arccos(ax)^{4}-4\sqrt{-a^{2}x^{2}+1}\,\arccos(ax)^{3}-12\arccos(ax)^{2}ax+24ax+24\arccos(ax)\sqrt{-a^{2}x^{2}+1}}{a}$	67

```
[In] int(arccos(a*x)^4,x,method=_RETURNVERBOSE)
[Out] 1/a*(a*x*arccos(a*x)^4-4*(-a^2*x^2+1)^(1/2)*arccos(a*x)^3-12*arccos(a*x)^2*
a*x+24*a*x+24*arccos(a*x)*(-a^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \arccos(ax)^4\,dx = \frac{ax\arccos{(ax)}^4 - 12\,ax\arccos{(ax)}^2 + 24\,ax - 4\,\sqrt{-a^2x^2+1}\left(\arccos{(ax)}^3 - 6\,\arccos{(ax)}\right)}{a}$$

```
[In] integrate(arccos(a*x)^4,x, algorithm="fricas")
[Out] (a*x*arccos(a*x)^4 - 12*a*x*arccos(a*x)^2 + 24*a*x - 4*sqrt(-a^2*x^2 + 1)*(
arccos(a*x)^3 - 6*arccos(a*x)))/a
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01

$$\int \arccos(ax)^4\,dx = \begin{cases} x\operatorname{acos}^4{(ax)} - 12x\operatorname{acos}^2{(ax)} + 24x - \frac{4\sqrt{-a^2x^2+1}\operatorname{acos}^3{(ax)}}{a} + \frac{24\sqrt{-a^2x^2+1}\operatorname{acos}{(ax)}}{a} & \text{for } a \neq 0 \\ \frac{\pi^4x}{16} & \text{otherwise} \end{cases}$$

```
[In] integrate(acos(a*x)**4,x)
[Out] Piecewise((x*acos(a*x)**4 - 12*x*acos(a*x)**2 + 24*x - 4*sqrt(-a**2*x**2 +
1)*acos(a*x)**3/a + 24*sqrt(-a**2*x**2 + 1)*acos(a*x)/a, Ne(a, 0)), (pi**4*
x/16, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

$$\int \arccos(ax)^4\,dx = x\arccos\left(ax\right)^4 - \frac{4\sqrt{-a^2x^2+1}\arccos\left(ax\right)^3}{a} - 12\left(\frac{x\arccos\left(ax\right)^2}{a} - \frac{2\left(x+\frac{\sqrt{-a^2x^2+1}\arccos(ax)}{a}\right)}{a}\right)a$$

```
[In] integrate(arccos(a*x)^4,x, algorithm="maxima")
[Out] x*arccos(a*x)^4 - 4*sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/a - 12*(x*arccos(a*x)^
2/a - 2*(x + sqrt(-a^2*x^2 + 1)*arccos(a*x)/a)/a)*a
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.94

$$\int \arccos(ax)^4\,dx = x\arccos\left(ax\right)^4 - 12\,x\arccos\left(ax\right)^2 - \frac{4\sqrt{-a^2x^2+1}\arccos\left(ax\right)^3}{a} + 24\,x + \frac{24\sqrt{-a^2x^2+1}\arccos\left(ax\right)}{a}$$

```
[In] integrate(arccos(a*x)^4,x, algorithm="giac")
[Out] x*arccos(a*x)^4 - 12*x*arccos(a*x)^2 - 4*sqrt(-a^2*x^2 + 1)*arccos(a*x)^3/a
 + 24*x + 24*sqrt(-a^2*x^2 + 1)*arccos(a*x)/a
```

Mupad [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \arccos(ax)^4\,dx = \begin{cases} \frac{x\,\pi^4}{16} & \text{if}\ \ a=0 \\ x\,\left(\mathrm{acos}(a\,x)^4-12\,\mathrm{acos}(a\,x)^2+24\right)+\frac{\sqrt{1-a^2\,x^2}\,\left(24\,\mathrm{acos}(a\,x)-4\,\mathrm{acos}(a\,x)^3\right)}{a} & \text{if}\ \ a\neq 0 \end{cases}$$

```
[In] int(acos(a*x)^4,x)
[Out] piecewise(a == 0, (x*pi^4)/16, a ~= 0, x*(- 12*acos(a*x)^2 + acos(a*x)^4 +
24) + ((- a^2*x^2 + 1)^(1/2)*(24*acos(a*x) - 4*acos(a*x)^3))/a)
```

3.38 $\int \frac{\arccos(ax)^4}{x}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 119

$$\int \frac{\arccos(ax)^4}{x}\,dx = -\frac{1}{5}i\arccos(ax)^5+\arccos(ax)^4\log\left(1+e^{2i\arccos(ax)}\right)-2i\arccos(ax)^3\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)+3\arccos(ax)^2\,\mathrm{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)+3i\arccos(ax)\,\mathrm{PolyLog}\left(4,-e^{2i\arccos(ax)}\right)-\frac{3}{2}\,\mathrm{PolyLog}\left(5,-e^{2i\arccos(ax)}\right)$$

```
[Out] -1/5*I*arccos(a*x)^5+arccos(a*x)^4*ln(1+(a*x+I*(-a^2*x^2+1)^(1/2))^2)-2*I*a
rccos(a*x)^3*polylog(2,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3*arccos(a*x)^2*polyl
og(3,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+3*I*arccos(a*x)*polylog(4,-(a*x+I*(-a^2
*x^2+1)^(1/2))^2)-3/2*polylog(5,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4722, 3800, 2221, 2611, 6744, 2320, 6724}

$$\int \frac{\arccos(ax)^4}{x}\,dx = -2i\arccos(ax)^3\,\mathrm{PolyLog}\left(2,-e^{2i\arccos(ax)}\right)+3\arccos(ax)^2\,\mathrm{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)+3i\arccos(ax)\,\mathrm{PolyLog}\left(4,-e^{2i\arccos(ax)}\right)-\frac{3}{2}\,\mathrm{PolyLog}\left(5,-e^{2i\arccos(ax)}\right)-\frac{1}{5}i\arccos(ax)^5+\arccos(ax)^4\log\left(1+e^{2i\arccos(ax)}\right)$$

```
[In] Int[ArcCos[a*x]^4/x,x]
```

```
[Out] (-1/5*I)*ArcCos[a*x]^5 + ArcCos[a*x]^4*Log[1 + E^((2*I)*ArcCos[a*x])] - (2*
I)*ArcCos[a*x]^3*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + 3*ArcCos[a*x]^2*PolyL
og[3, -E^((2*I)*ArcCos[a*x])] + (3*I)*ArcCos[a*x]*PolyLog[4, -E^((2*I)*ArcC
os[a*x])] - (3*PolyLog[5, -E^((2*I)*ArcCos[a*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^4 \tan(x)\,dx, x, \arccos(ax)\right) \\
&= -\frac{1}{5} i \arccos(ax)^5 + 2i\text{Subst}\left(\int \frac{e^{2ix}x^4}{1+e^{2ix}}\,dx, x, \arccos(ax)\right) \\
&= -\frac{1}{5} i \arccos(ax)^5 + \arccos(ax)^4 \log\left(1+e^{2i\arccos(ax)}\right) \\
&\quad - 4\text{Subst}\left(\int x^3 \log\left(1+e^{2ix}\right)\,dx, x, \arccos(ax)\right) \\
&= -\frac{1}{5} i \arccos(ax)^5 + \arccos(ax)^4 \log\left(1+e^{2i\arccos(ax)}\right) \\
&\quad - 2i\arccos(ax)^3 \text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right) \\
&\quad + 6i\text{Subst}\left(\int x^2 \text{PolyLog}\left(2, -e^{2ix}\right)\,dx, x, \arccos(ax)\right) \\
&= -\frac{1}{5} i \arccos(ax)^5 + \arccos(ax)^4 \log\left(1+e^{2i\arccos(ax)}\right) \\
&\quad - 2i\arccos(ax)^3 \text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right) \\
&\quad + 3\arccos(ax)^2 \text{PolyLog}\left(3, -e^{2i\arccos(ax)}\right) \\
&\quad - 6\text{Subst}\left(\int x\,\text{PolyLog}\left(3, -e^{2ix}\right)\,dx, x, \arccos(ax)\right) \\
&= -\frac{1}{5} i \arccos(ax)^5 + \arccos(ax)^4 \log\left(1+e^{2i\arccos(ax)}\right) \\
&\quad - 2i\arccos(ax)^3 \text{PolyLog}\left(2, -e^{2i\arccos(ax)}\right) \\
&\quad + 3\arccos(ax)^2 \text{PolyLog}\left(3, -e^{2i\arccos(ax)}\right) \\
&\quad + 3i\arccos(ax)\,\text{PolyLog}\left(4, -e^{2i\arccos(ax)}\right) \\
&\quad - 3i\text{Subst}\left(\int \text{PolyLog}\left(4, -e^{2ix}\right)\,dx, x, \arccos(ax)\right)
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{1}{5}i\arccos(ax)^5 + \arccos(ax)^4\log\left(1+e^{2i\arccos(ax)}\right) \\
&\quad - 2i\arccos(ax)^3\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) \\
&\quad + 3\arccos(ax)^2\,\text{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) \\
&\quad + 3i\arccos(ax)\,\text{PolyLog}\left(4,-e^{2i\arccos(ax)}\right) \\
&\quad - \frac{3}{2}\text{Subst}\left(\int\frac{\text{PolyLog}(4,-x)}{x}\,dx,x,e^{2i\arccos(ax)}\right) \\
&= -\frac{1}{5}i\arccos(ax)^5 + \arccos(ax)^4\log\left(1+e^{2i\arccos(ax)}\right) \\
&\quad - 2i\arccos(ax)^3\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) \\
&\quad + 3\arccos(ax)^2\,\text{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) \\
&\quad + 3i\arccos(ax)\,\text{PolyLog}\left(4,-e^{2i\arccos(ax)}\right) - \frac{3}{2}\,\text{PolyLog}\left(5,-e^{2i\arccos(ax)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00

$$
\begin{aligned}
\int\frac{\arccos(ax)^4}{x}\,dx &= -\frac{1}{5}i\arccos(ax)^5 + \arccos(ax)^4\log\left(1+e^{2i\arccos(ax)}\right) \\
&\quad - 2i\arccos(ax)^3\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) \\
&\quad + 3\arccos(ax)^2\,\text{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) \\
&\quad + 3i\arccos(ax)\,\text{PolyLog}\left(4,-e^{2i\arccos(ax)}\right) - \frac{3}{2}\,\text{PolyLog}\left(5,-e^{2i\arccos(ax)}\right)
\end{aligned}
$$

```
[In] Integrate[ArcCos[a*x]^4/x,x]
```

```
[Out] (-1/5*I)*ArcCos[a*x]^5 + ArcCos[a*x]^4*Log[1 + E^((2*I)*ArcCos[a*x])] - (2*
I)*ArcCos[a*x]^3*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + 3*ArcCos[a*x]^2*PolyL
og[3, -E^((2*I)*ArcCos[a*x])] + (3*I)*ArcCos[a*x]*PolyLog[4, -E^((2*I)*ArcC
os[a*x])] - (3*PolyLog[5, -E^((2*I)*ArcCos[a*x])])/2
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.41

method	result
derivativedivides	$-\frac{i\arccos(ax)^5}{5} + \arccos(ax)^4\ln\left(1+\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right) - 2i\arccos(ax)^3\,\text{polylog}\left(2\right.$
default	$-\frac{i\arccos(ax)^5}{5} + \arccos(ax)^4\ln\left(1+\left(i\sqrt{-a^2x^2+1}+ax\right)^2\right) - 2i\arccos(ax)^3\,\text{polylog}\left(2\right.$

```
[In] int(arccos(a*x)^4/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*I*arccos(a*x)^5+arccos(a*x)^4*ln(1+(I*(-a^2*x^2+1)^(1/2)+a*x)^2)-2*I*a
rccos(a*x)^3*polylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+3*arccos(a*x)^2*polyl
og(3,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+3*I*arccos(a*x)*polylog(4,-(I*(-a^2*x^2
+1)^(1/2)+a*x)^2)-3/2*polylog(5,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)
```

Fricas [F]

$$\int \frac{\arccos(ax)^4}{x}\,dx = \int \frac{\arccos{(ax)}^4}{x}\,dx$$

```
[In] integrate(arccos(a*x)^4/x,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^4/x, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^4}{x}\,dx = \int \frac{\operatorname{acos}^4{(ax)}}{x}\,dx$$

```
[In] integrate(acos(a*x)**4/x,x)
```

```
[Out] Integral(acos(a*x)**4/x, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^4}{x}\,dx = \int \frac{\arccos{(ax)}^4}{x}\,dx$$

```
[In] integrate(arccos(a*x)^4/x,x, algorithm="maxima")
```

```
[Out] integrate(arccos(a*x)^4/x, x)
```

Giac [F]

$$\int \frac{\arccos(ax)^4}{x}\,dx = \int \frac{\arccos{(ax)}^4}{x}\,dx$$

```
[In] integrate(arccos(a*x)^4/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^4/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^4}{x}\,dx = \int \frac{\mathrm{acos}(a\,x)^4}{x}\,dx$$

```
[In] int(acos(a*x)^4/x,x)
```

```
[Out] int(acos(a*x)^4/x, x)
```

3.39 $\int \frac{\arccos(ax)^4}{x^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 176

$$\begin{aligned}\int \frac{\arccos(ax)^4}{x^2}\,dx = & -\frac{\arccos(ax)^4}{x} - 8ia\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)\\ & + 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)\\ & - 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\\ & - 24a\arccos(ax)\operatorname{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)\\ & + 24a\arccos(ax)\operatorname{PolyLog}\left(3,ie^{i\arccos(ax)}\right)\\ & - 24ia\operatorname{PolyLog}\left(4,-ie^{i\arccos(ax)}\right) + 24ia\operatorname{PolyLog}\left(4,ie^{i\arccos(ax)}\right)\end{aligned}$$

```
[Out] -arccos(a*x)^4/x-8*I*a*arccos(a*x)^3*arctan(a*x+I*(-a^2*x^2+1)^(1/2))+12*I*
a*arccos(a*x)^2*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-12*I*a*arccos(a*x)
^2*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2)))-24*a*arccos(a*x)*polylog(3,-I*(a
*x+I*(-a^2*x^2+1)^(1/2)))+24*a*arccos(a*x)*polylog(3,I*(a*x+I*(-a^2*x^2+1)^
(1/2)))-24*I*a*polylog(4,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+24*I*a*polylog(4,I*
(a*x+I*(-a^2*x^2+1)^(1/2)))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used

= {4724, 4804, 4266, 2611, 6744, 2320, 6724}

$$\int \frac{\arccos(ax)^4}{x^2} dx = -8ia \arccos(ax)^3 \arctan\left(e^{i \arccos(ax)}\right) + 12ia \arccos(ax)^2 \operatorname{PolyLog}\left(2, -ie^{i \arccos(ax)}\right) - 12ia \arccos(ax)^2 \operatorname{PolyLog}\left(2, ie^{i \arccos(ax)}\right) - 24a \arccos(ax) \operatorname{PolyLog}\left(3, -ie^{i \arccos(ax)}\right) + 24a \arccos(ax) \operatorname{PolyLog}\left(3, ie^{i \arccos(ax)}\right) - 24ia \operatorname{PolyLog}\left(4, -ie^{i \arccos(ax)}\right) + 24ia \operatorname{PolyLog}\left(4, ie^{i \arccos(ax)}\right) - \frac{\arccos(ax)^4}{x}$$

```
[In] Int[ArcCos[a*x]^4/x^2,x]
```

```
[Out] -(ArcCos[a*x]^4/x) - (8*I)*a*ArcCos[a*x]^3*ArcTan[E^(I*ArcCos[a*x])] + (12*
I)*a*ArcCos[a*x]^2*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (12*I)*a*ArcCos[a*x
]^2*PolyLog[2, I*E^(I*ArcCos[a*x])] - 24*a*ArcCos[a*x]*PolyLog[3, (-I)*E^(I
*ArcCos[a*x])] + 24*a*ArcCos[a*x]*PolyLog[3, I*E^(I*ArcCos[a*x])] - (24*I)*
a*PolyLog[4, (-I)*E^(I*ArcCos[a*x])] + (24*I)*a*PolyLog[4, I*E^(I*ArcCos[a*
x])]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)^4}{x} - (4a)\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{\arccos(ax)^4}{x} + (4a)\text{Subst}\left(\int x^3\sec(x)\,dx, x, \arccos(ax)\right) \\
&= -\frac{\arccos(ax)^4}{x} - 8ia\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right) \\
&\quad - (12a)\text{Subst}\left(\int x^2\log\left(1-ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&\quad + (12a)\text{Subst}\left(\int x^2\log\left(1+ie^{ix}\right)\,dx, x, \arccos(ax)\right)
\end{aligned}$$

$$= -\frac{\arccos(ax)^4}{x} - 8ia\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)$$
$$+ 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right)$$
$$- 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2, ie^{i\arccos(ax)}\right)$$
$$- (24ia)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(2, -ie^{ix}\right)\,dx, x, \arccos(ax)\right)$$
$$+ (24ia)\operatorname{Subst}\left(\int x\operatorname{PolyLog}\left(2, ie^{ix}\right)\,dx, x, \arccos(ax)\right)$$

$$= -\frac{\arccos(ax)^4}{x} - 8ia\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)$$
$$+ 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right)$$
$$- 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2, ie^{i\arccos(ax)}\right)$$
$$- 24a\arccos(ax)\operatorname{PolyLog}\left(3, -ie^{i\arccos(ax)}\right)$$
$$+ 24a\arccos(ax)\operatorname{PolyLog}\left(3, ie^{i\arccos(ax)}\right)$$
$$+ (24a)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3, -ie^{ix}\right)\,dx, x, \arccos(ax)\right)$$
$$- (24a)\operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3, ie^{ix}\right)\,dx, x, \arccos(ax)\right)$$

$$= -\frac{\arccos(ax)^4}{x} - 8ia\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)$$
$$+ 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right)$$
$$- 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2, ie^{i\arccos(ax)}\right)$$
$$- 24a\arccos(ax)\operatorname{PolyLog}\left(3, -ie^{i\arccos(ax)}\right)$$
$$+ 24a\arccos(ax)\operatorname{PolyLog}\left(3, ie^{i\arccos(ax)}\right)$$
$$- (24ia)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3, -ix)}{x}\,dx, x, e^{i\arccos(ax)}\right)$$
$$+ (24ia)\operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3, ix)}{x}\,dx, x, e^{i\arccos(ax)}\right)$$

$$= -\frac{\arccos(ax)^4}{x} - 8ia\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)$$
$$+ 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right)$$
$$- 12ia\arccos(ax)^2\operatorname{PolyLog}\left(2, ie^{i\arccos(ax)}\right)$$
$$- 24a\arccos(ax)\operatorname{PolyLog}\left(3, -ie^{i\arccos(ax)}\right)$$
$$+ 24a\arccos(ax)\operatorname{PolyLog}\left(3, ie^{i\arccos(ax)}\right)$$
$$- 24ia\operatorname{PolyLog}\left(4, -ie^{i\arccos(ax)}\right) + 24ia\operatorname{PolyLog}\left(4, ie^{i\arccos(ax)}\right)$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 549 vs. $2(176) = 352$.

Time = 0.74 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.12

$$\int \frac{\arccos(ax)^4}{x^2} dx = a\left(-\frac{7i\pi^4}{16} - \frac{1}{2}i\pi^3\arccos(ax) + \frac{3}{2}i\pi^2\arccos(ax)^2 - 2i\pi\arccos(ax)^3 + i\arccos(ax)^4 - \frac{\arccos(ax)^4}{ax} + 3\pi^2\arccos(ax)\log\left(1 - ie^{-i\arccos(ax)}\right) - 6\pi\arccos(ax)^2\log\left(1 - ie^{-i\arccos(ax)}\right) - \frac{1}{2}\pi^3\log\left(1 + ie^{-i\arccos(ax)}\right) + 4\arccos(ax)^3\log\left(1 + ie^{-i\arccos(ax)}\right) + \frac{1}{2}\pi^3\log\left(1 + ie^{i\arccos(ax)}\right) - 3\pi^2\arccos(ax)\log\left(1 + ie^{i\arccos(ax)}\right) + 6\pi\arccos(ax)^2\log\left(1 + ie^{i\arccos(ax)}\right) - 4\arccos(ax)^3\log\left(1 + ie^{i\arccos(ax)}\right) + \frac{1}{2}\pi^3\log\left(\tan\left(\frac{1}{4}(\pi + 2\arccos(ax))\right)\right) + 12i\arccos(ax)^2\operatorname{PolyLog}\left(2, -ie^{-i\arccos(ax)}\right) + 3i\pi(\pi - 4\arccos(ax))\operatorname{PolyLog}\left(2, ie^{-i\arccos(ax)}\right) + 3i\pi^2\operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) - 12i\pi\arccos(ax)\operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) + 12i\arccos(ax)^2\operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) + 24\arccos(ax)\operatorname{PolyLog}\left(3, -ie^{-i\arccos(ax)}\right) - 12\pi\operatorname{PolyLog}\left(3, ie^{-i\arccos(ax)}\right) + 12\pi\operatorname{PolyLog}\left(3, -ie^{i\arccos(ax)}\right) - 24\arccos(ax)\operatorname{PolyLog}\left(3, -ie^{i\arccos(ax)}\right) - 24i\operatorname{PolyLog}\left(4, -ie^{-i\arccos(ax)}\right) - 24i\operatorname{PolyLog}\left(4, -ie^{i\arccos(ax)}\right)\right)$$

```
[In] Integrate[ArcCos[a*x]^4/x^2,x]
```

```
[Out] a*(((-7*I)/16)*Pi^4 - (I/2)*Pi^3*ArcCos[a*x] + ((3*I)/2)*Pi^2*ArcCos[a*x]^2
 - (2*I)*Pi*ArcCos[a*x]^3 + I*ArcCos[a*x]^4 - ArcCos[a*x]^4/(a*x) + 3*Pi^2*
ArcCos[a*x]*Log[1 - I/E^(I*ArcCos[a*x])] - 6*Pi*ArcCos[a*x]^2*Log[1 - I/E^(
I*ArcCos[a*x])] - (Pi^3*Log[1 + I/E^(I*ArcCos[a*x])])/2 + 4*ArcCos[a*x]^3*L
og[1 + I/E^(I*ArcCos[a*x])] + (Pi^3*Log[1 + I*E^(I*ArcCos[a*x])])/2 - 3*Pi^
2*ArcCos[a*x]*Log[1 + I*E^(I*ArcCos[a*x])] + 6*Pi*ArcCos[a*x]^2*Log[1 + I*E
^(I*ArcCos[a*x])] - 4*ArcCos[a*x]^3*Log[1 + I*E^(I*ArcCos[a*x])] + (Pi^3*Lo
g[Tan[(Pi + 2*ArcCos[a*x])/4]])/2 + (12*I)*ArcCos[a*x]^2*PolyLog[2, (-I)/E^
(I*ArcCos[a*x])] + (3*I)*Pi*(Pi - 4*ArcCos[a*x])*PolyLog[2, I/E^(I*ArcCos[a
*x])] + (3*I)*Pi^2*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] - (12*I)*Pi*ArcCos[a*
```

```
x]*PolyLog[2, (-I)*E^(I*ArcCos[a*x])] + (12*I)*ArcCos[a*x]^2*PolyLog[2, (-I
)*E^(I*ArcCos[a*x])] + 24*ArcCos[a*x]*PolyLog[3, (-I)/E^(I*ArcCos[a*x])] -
12*Pi*PolyLog[3, I/E^(I*ArcCos[a*x])] + 12*Pi*PolyLog[3, (-I)*E^(I*ArcCos[a
*x])] - 24*ArcCos[a*x]*PolyLog[3, (-I)*E^(I*ArcCos[a*x])] - (24*I)*PolyLog[
4, (-I)/E^(I*ArcCos[a*x])] - (24*I)*PolyLog[4, (-I)*E^(I*ArcCos[a*x])])
```

Maple [F]

$$\int \frac{\arccos(ax)^4}{x^2}dx$$

[In] int(arccos(a*x)^4/x^2,x)

[Out] int(arccos(a*x)^4/x^2,x)

Fricas [F]

$$\int \frac{\arccos(ax)^4}{x^2} dx = \int \frac{\arccos(ax)^4}{x^2} dx$$

[In] integrate(arccos(a*x)^4/x^2,x, algorithm="fricas")

[Out] integral(arccos(a*x)^4/x^2, x)

Sympy [F]

$$\int \frac{\arccos(ax)^4}{x^2} dx = \int \frac{\operatorname{acos}^4(ax)}{x^2} dx$$

[In] integrate(acos(a*x)**4/x**2,x)

[Out] Integral(acos(a*x)**4/x**2, x)

Maxima [F]

$$\int \frac{\arccos(ax)^4}{x^2} dx = \int \frac{\arccos(ax)^4}{x^2} dx$$

[In] integrate(arccos(a*x)^4/x^2,x, algorithm="maxima")

[Out] -(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 4*a*x*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(a^2*x^3 - x), x))/x

Giac [F]

$$\int \frac{\arccos(ax)^4}{x^2}\,dx = \int \frac{\arccos\left(ax\right)^4}{x^2}\,dx$$

```
[In] integrate(arccos(a*x)^4/x^2,x, algorithm="giac")
[Out] integrate(arccos(a*x)^4/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^4}{x^2}\,dx = \int \frac{\mathrm{acos}(a\,x)^4}{x^2}\,dx$$

```
[In] int(acos(a*x)^4/x^2,x)
[Out] int(acos(a*x)^4/x^2, x)
```

3.40 $\int \frac{\arccos(ax)^4}{x^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 121

$$\int \frac{\arccos(ax)^4}{x^3} dx = -2ia^2 \arccos(ax)^3 + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{x} - \frac{\arccos(ax)^4}{2x^2} + 6a^2 \arccos(ax)^2 \log\left(1+e^{2i\arccos(ax)}\right) - 6ia^2 \arccos(ax) \operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + 3a^2 \operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)$$

```
[Out] -2*I*a^2*arccos(a*x)^3-1/2*arccos(a*x)^4/x^2+6*a^2*arccos(a*x)^2*ln(1+(a*x+
I*(-a^2*x^2+1)^(1/2))^2)-6*I*a^2*arccos(a*x)*polylog(2,-(a*x+I*(-a^2*x^2+1)
^(1/2))^2)+3*a^2*polylog(3,-(a*x+I*(-a^2*x^2+1)^(1/2))^2)+2*a*arccos(a*x)^3
*(-a^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {4724, 4772, 4722, 3800, 2221, 2611, 2320, 6724}

$$\int \frac{\arccos(ax)^4}{x^3} dx = -6ia^2 \arccos(ax) \operatorname{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + 3a^2 \operatorname{PolyLog}\left(3,-e^{2i\arccos(ax)}\right) + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{x} - 2ia^2 \arccos(ax)^3 + 6a^2 \arccos(ax)^2 \log\left(1+e^{2i\arccos(ax)}\right) - \frac{\arccos(ax)^4}{2x^2}$$

```
[In] Int[ArcCos[a*x]^4/x^3,x]
```

```
[Out] (-2*I)*a^2*ArcCos[a*x]^3 + (2*a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/x - ArcCos
[a*x]^4/(2*x^2) + 6*a^2*ArcCos[a*x]^2*Log[1 + E^((2*I)*ArcCos[a*x])] - (6*I
)*a^2*ArcCos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] + 3*a^2*PolyLog[3, -E^
((2*I)*ArcCos[a*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
```

```
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4772

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + Dist[b*c*(n/(f*(m + 1)))*Simp[(d + e*x
^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*Ar
cCos[c*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, p}, x] && EqQ[c^2
*d + e, 0] && GtQ[n, 0] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\frac{\arccos(ax)^4}{2x^2} - (2a)\int \frac{\arccos(ax)^3}{x^2\sqrt{1-a^2x^2}}\,dx$$

$$= \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{x} - \frac{\arccos(ax)^4}{2x^2} + (6a^2)\int \frac{\arccos(ax)^2}{x}\,dx$$

$$= \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{x} - \frac{\arccos(ax)^4}{2x^2} - (6a^2)\,\text{Subst}\left(\int x^2\tan(x)\,dx, x, \arccos(ax)\right)$$

$$= -2ia^2\arccos(ax)^3 + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{x} - \frac{\arccos(ax)^4}{2x^2} + (12ia^2)\,\text{Subst}\left(\int \frac{e^{2ix}x^2}{1+e^{2ix}}\,dx, x, \arccos(ax)\right)$$

$$= -2ia^2\arccos(ax)^3 + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{x} - \frac{\arccos(ax)^4}{2x^2} + 6a^2\arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - (12a^2)\,\text{Subst}\left(\int x\log\left(1+e^{2ix}\right)\,dx, x, \arccos(ax)\right)$$

$$= -2ia^2\arccos(ax)^3 + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{x} - \frac{\arccos(ax)^4}{2x^2} + 6a^2\arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - 6ia^2\arccos(ax)\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + (6ia^2)\,\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{2ix}\right)\,dx, x, \arccos(ax)\right)$$

$$= -2ia^2\arccos(ax)^3 + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{x} - \frac{\arccos(ax)^4}{2x^2} + 6a^2\arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - 6ia^2\arccos(ax)\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + (3a^2)\,\text{Subst}\left(\int\frac{\text{PolyLog}(2,-x)}{x}\,dx,x,e^{2i\arccos(ax)}\right)$$

$$= -2ia^2\arccos(ax)^3 + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{x} - \frac{\arccos(ax)^4}{2x^2} + 6a^2\arccos(ax)^2\log\left(1+e^{2i\arccos(ax)}\right) - 6ia^2\arccos(ax)\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) + 3a^2\,\text{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)$$

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

$$\int\frac{\arccos(ax)^4}{x^3}\,dx = -\frac{\arccos(ax)^4}{2x^2} - a^2\left(-2\arccos(ax)^2\left(-i\arccos(ax)+\frac{\sqrt{1-a^2x^2}\arccos(ax)}{ax} + 3\log\left(1+e^{2i\arccos(ax)}\right)\right) + 6i\arccos(ax)\,\text{PolyLog}\left(2,-e^{2i\arccos(ax)}\right) - 3\,\text{PolyLog}\left(3,-e^{2i\arccos(ax)}\right)\right)$$

```
[In] Integrate[ArcCos[a*x]^4/x^3,x]
```

```
[Out] -1/2*ArcCos[a*x]^4/x^2 - a^2*(-2*ArcCos[a*x]^2*((-I)*ArcCos[a*x] + (Sqrt[1
- a^2*x^2]*ArcCos[a*x])/(a*x) + 3*Log[1 + E^((2*I)*ArcCos[a*x])]) + (6*I)*A
rcCos[a*x]*PolyLog[2, -E^((2*I)*ArcCos[a*x])] - 3*PolyLog[3, -E^((2*I)*ArcC
os[a*x])])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

method	result
derivativedivides	$a^2\left(-\frac{\arccos(ax)^3\left(-4ia^2x^2-4ax\sqrt{-a^2x^2+1}+\arccos(ax)\right)}{2a^2x^2} - 4i\arccos(ax)^3 + 6\arccos(ax)^2\ln\left(1+\right.\right.$
default	$a^2\left(-\frac{\arccos(ax)^3\left(-4ia^2x^2-4ax\sqrt{-a^2x^2+1}+\arccos(ax)\right)}{2a^2x^2} - 4i\arccos(ax)^3 + 6\arccos(ax)^2\ln\left(1+\right.\right.$

```
[In] int(arccos(a*x)^4/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] a^2*(-1/2*arccos(a*x)^3*(-4*I*a^2*x^2-4*a*x*(-a^2*x^2+1)^(1/2)+arccos(a*x))
/a^2/x^2-4*I*arccos(a*x)^3+6*arccos(a*x)^2*ln(1+(I*(-a^2*x^2+1)^(1/2)+a*x)^
2)-6*I*arccos(a*x)*polylog(2,-(I*(-a^2*x^2+1)^(1/2)+a*x)^2)+3*polylog(3,-(I
*(-a^2*x^2+1)^(1/2)+a*x)^2))
```

Fricas [F]

$$\int \frac{\arccos(ax)^4}{x^3}\,dx = \int \frac{\arccos{(ax)}^4}{x^3}\,dx$$

```
[In] integrate(arccos(a*x)^4/x^3,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^4/x^3, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^4}{x^3}\,dx = \int \frac{\operatorname{acos}^4{(ax)}}{x^3}\,dx$$

```
[In] integrate(acos(a*x)**4/x**3,x)
```

```
[Out] Integral(acos(a*x)**4/x**3, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^4}{x^3}\,dx = \int \frac{\arccos{(ax)}^4}{x^3}\,dx$$

```
[In] integrate(arccos(a*x)^4/x^3,x, algorithm="maxima")
```

```
[Out] -1/2*(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 4*a*x^2*integrate(sqrt
(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3/(a^2*
x^4 - x^2), x))/x^2
```

Giac [F]

$$\int \frac{\arccos(ax)^4}{x^3} \, dx = \int \frac{\arccos{(ax)}^4}{x^3} \, dx$$

[In] integrate(arccos(a*x)^4/x^3,x, algorithm="giac")

[Out] integrate(arccos(a*x)^4/x^3, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^4}{x^3} \, dx = \int \frac{\mathrm{acos}(a\,x)^4}{x^3} \, dx$$

[In] int(acos(a*x)^4/x^3,x)

[Out] int(acos(a*x)^4/x^3, x)

3.41 $\int \frac{\arccos(ax)^4}{x^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 304

$$\begin{aligned}\int \frac{\arccos(ax)^4}{x^4}\,dx &= -\frac{2a^2\arccos(ax)^2}{x}+\frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{3x^2}\\ &\quad-\frac{\arccos(ax)^4}{3x^3}-8ia^3\arccos(ax)\arctan\left(e^{i\arccos(ax)}\right)\\ &\quad-\frac{4}{3}ia^3\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right)+4ia^3\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)\\ &\quad+2ia^3\arccos(ax)^2\,\text{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)\\ &\quad-4ia^3\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\\ &\quad-2ia^3\arccos(ax)^2\,\text{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\\ &\quad-4a^3\arccos(ax)\,\text{PolyLog}\left(3,-ie^{i\arccos(ax)}\right)\\ &\quad+4a^3\arccos(ax)\,\text{PolyLog}\left(3,ie^{i\arccos(ax)}\right)\\ &\quad-4ia^3\,\text{PolyLog}\left(4,-ie^{i\arccos(ax)}\right)+4ia^3\,\text{PolyLog}\left(4,ie^{i\arccos(ax)}\right)\end{aligned}$$

```
[Out] -2*a^2*arccos(a*x)^2/x-1/3*arccos(a*x)^4/x^3-8*I*a^3*arccos(a*x)*arctan(a*x
+I*(-a^2*x^2+1)^(1/2))-4/3*I*a^3*arccos(a*x)^3*arctan(a*x+I*(-a^2*x^2+1)^(1
/2))+4*I*a^3*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+2*I*a^3*arccos(a*x)^2
*polylog(2,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))-4*I*a^3*polylog(2,I*(a*x+I*(-a^2*
x^2+1)^(1/2)))-2*I*a^3*arccos(a*x)^2*polylog(2,I*(a*x+I*(-a^2*x^2+1)^(1/2))
)-4*a^3*arccos(a*x)*polylog(3,-I*(a*x+I*(-a^2*x^2+1)^(1/2)))+4*a^3*arccos(a
*x)*polylog(3,I*(a*x+I*(-a^2*x^2+1)^(1/2)))-4*I*a^3*polylog(4,-I*(a*x+I*(-a
^2*x^2+1)^(1/2)))+4*I*a^3*polylog(4,I*(a*x+I*(-a^2*x^2+1)^(1/2)))+2/3*a*arc
cos(a*x)^3*(-a^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {4724, 4790, 4804, 4266, 2611, 6744, 2320, 6724, 2317, 2438}

$$\int \frac{\arccos(ax)^4}{x^4} dx = -\frac{4}{3} i a^3 \arccos(ax)^3 \arctan\left(e^{i\arccos(ax)}\right) - 8 i a^3 \arccos(ax) \arctan\left(e^{i\arccos(ax)}\right) + 2 i a^3 \arccos(ax)^2 \text{PolyLog}\left(2, -i e^{i\arccos(ax)}\right) - 2 i a^3 \arccos(ax)^2 \text{PolyLog}\left(2, i e^{i\arccos(ax)}\right) - 4 a^3 \arccos(ax) \text{PolyLog}\left(3, -i e^{i\arccos(ax)}\right) + 4 a^3 \arccos(ax) \text{PolyLog}\left(3, i e^{i\arccos(ax)}\right) + 4 i a^3 \text{PolyLog}\left(2, -i e^{i\arccos(ax)}\right) - 4 i a^3 \text{PolyLog}\left(2, i e^{i\arccos(ax)}\right) - 4 i a^3 \text{PolyLog}\left(4, -i e^{i\arccos(ax)}\right) + 4 i a^3 \text{PolyLog}\left(4, i e^{i\arccos(ax)}\right) + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{3x^2} - \frac{2a^2\arccos(ax)^2}{x} - \frac{\arccos(ax)^4}{3x^3}$$

```
[In] Int[ArcCos[a*x]^4/x^4,x]
```

```
[Out] (-2*a^2*ArcCos[a*x]^2)/x + (2*a*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^3)/(3*x^2) -
ArcCos[a*x]^4/(3*x^3) - (8*I)*a^3*ArcCos[a*x]*ArcTan[E^(I*ArcCos[a*x])] - (
(4*I)/3)*a^3*ArcCos[a*x]^3*ArcTan[E^(I*ArcCos[a*x])] + (4*I)*a^3*PolyLog[2,
 (-I)*E^(I*ArcCos[a*x])] + (2*I)*a^3*ArcCos[a*x]^2*PolyLog[2, (-I)*E^(I*Arc
Cos[a*x])] - (4*I)*a^3*PolyLog[2, I*E^(I*ArcCos[a*x])] - (2*I)*a^3*ArcCos[a
*x]^2*PolyLog[2, I*E^(I*ArcCos[a*x])] - 4*a^3*ArcCos[a*x]*PolyLog[3, (-I)*E
^(I*ArcCos[a*x])] + 4*a^3*ArcCos[a*x]*PolyLog[3, I*E^(I*ArcCos[a*x])] - (4*
I)*a^3*PolyLog[4, (-I)*E^(I*ArcCos[a*x])] + (4*I)*a^3*PolyLog[4, I*E^(I*Arc
Cos[a*x])]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4790

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^2)^(p + 1)*((a + b
*ArcCos[c*x])^n/(d*f*(m + 1))), x] + (Dist[c^2*((m + 2*p + 3)/(f^2*(m + 1))
), Int[(f*x)^(m + 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] + Dist[b*c
*(n/(f*(m + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)^(m + 1)*(1
- c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; FreeQ[{a, b, c
, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && ILtQ[m, -1]
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\arccos(ax)^4}{3x^3} - \frac{1}{3}(4a)\int \frac{\arccos(ax)^3}{x^3\sqrt{1-a^2x^2}}\,dx \\
&= \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{3x^2} - \frac{\arccos(ax)^4}{3x^3} \\
&\quad + \left(2a^2\right)\int \frac{\arccos(ax)^2}{x^2}\,dx - \frac{1}{3}\left(2a^3\right)\int \frac{\arccos(ax)^3}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{2a^2\arccos(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{3x^2} - \frac{\arccos(ax)^4}{3x^3} \\
&\quad + \frac{1}{3}\left(2a^3\right)\operatorname{Subst}\left(\int x^3\sec(x)\,dx, x, \arccos(ax)\right) - \left(4a^3\right)\int \frac{\arccos(ax)}{x\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{2a^2\arccos(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{3x^2} \\
&\quad - \frac{\arccos(ax)^4}{3x^3} - \frac{4}{3}ia^3\arccos(ax)^3\arctan\left(e^{i\arccos(ax)}\right) \\
&\quad - \left(2a^3\right)\operatorname{Subst}\left(\int x^2\log\left(1-ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&\quad + \left(2a^3\right)\operatorname{Subst}\left(\int x^2\log\left(1+ie^{ix}\right)\,dx, x, \arccos(ax)\right) \\
&\quad + \left(4a^3\right)\operatorname{Subst}(\int x\sec(x)\,dx, x, \arccos(ax))
\end{aligned}$$

$$= -\frac{2a^2 \arccos(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{3x^2} - \frac{\arccos(ax)^4}{3x^3}$$
$$- 8ia^3 \arccos(ax) \arctan\left(e^{i\arccos(ax)}\right) - \frac{4}{3}ia^3 \arccos(ax)^3 \arctan\left(e^{i\arccos(ax)}\right)$$
$$+ 2ia^3 \arccos(ax)^2 \operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right)$$
$$- 2ia^3 \arccos(ax)^2 \operatorname{PolyLog}\left(2, ie^{i\arccos(ax)}\right)$$
$$- \left(4ia^3\right) \operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(2, -ie^{ix}\right) dx, x, \arccos(ax)\right)$$
$$+ \left(4ia^3\right) \operatorname{Subst}\left(\int x \operatorname{PolyLog}\left(2, ie^{ix}\right) dx, x, \arccos(ax)\right)$$
$$- \left(4a^3\right) \operatorname{Subst}\left(\int \log\left(1 - ie^{ix}\right) dx, x, \arccos(ax)\right)$$
$$+ \left(4a^3\right) \operatorname{Subst}\left(\int \log\left(1 + ie^{ix}\right) dx, x, \arccos(ax)\right)$$

$$= -\frac{2a^2 \arccos(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2}\arccos(ax)^3}{3x^2} - \frac{\arccos(ax)^4}{3x^3}$$
$$- 8ia^3 \arccos(ax) \arctan\left(e^{i\arccos(ax)}\right) - \frac{4}{3}ia^3 \arccos(ax)^3 \arctan\left(e^{i\arccos(ax)}\right)$$
$$+ 2ia^3 \arccos(ax)^2 \operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right)$$
$$- 2ia^3 \arccos(ax)^2 \operatorname{PolyLog}\left(2, ie^{i\arccos(ax)}\right)$$
$$- 4a^3 \arccos(ax) \operatorname{PolyLog}\left(3, -ie^{i\arccos(ax)}\right)$$
$$+ 4a^3 \arccos(ax) \operatorname{PolyLog}\left(3, ie^{i\arccos(ax)}\right)$$
$$+ \left(4ia^3\right) \operatorname{Subst}\left(\int \frac{\log(1-ix)}{x} dx, x, e^{i\arccos(ax)}\right)$$
$$- \left(4ia^3\right) \operatorname{Subst}\left(\int \frac{\log(1+ix)}{x} dx, x, e^{i\arccos(ax)}\right)$$
$$+ \left(4a^3\right) \operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3, -ie^{ix}\right) dx, x, \arccos(ax)\right)$$
$$- \left(4a^3\right) \operatorname{Subst}\left(\int \operatorname{PolyLog}\left(3, ie^{ix}\right) dx, x, \arccos(ax)\right)$$

$$= -\frac{2a^2 \arccos(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2} \arccos(ax)^3}{3x^2} - \frac{\arccos(ax)^4}{3x^3} - 8ia^3 \arccos(ax) \arctan\left(e^{i\arccos(ax)}\right) - \frac{4}{3} ia^3 \arccos(ax)^3 \arctan\left(e^{i\arccos(ax)}\right) + 4ia^3 \operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) + 2ia^3 \arccos(ax)^2 \operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) - 4ia^3 \operatorname{PolyLog}\left(2, ie^{i\arccos(ax)}\right) - 2ia^3 \arccos(ax)^2 \operatorname{PolyLog}\left(2, ie^{i\arccos(ax)}\right) - 4a^3 \arccos(ax) \operatorname{PolyLog}\left(3, -ie^{i\arccos(ax)}\right) + 4a^3 \arccos(ax) \operatorname{PolyLog}\left(3, ie^{i\arccos(ax)}\right) - \left(4ia^3\right) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3, -ix)}{x} \, dx, x, e^{i\arccos(ax)}\right) + \left(4ia^3\right) \operatorname{Subst}\left(\int \frac{\operatorname{PolyLog}(3, ix)}{x} \, dx, x, e^{i\arccos(ax)}\right)$$

$$= -\frac{2a^2 \arccos(ax)^2}{x} + \frac{2a\sqrt{1-a^2x^2} \arccos(ax)^3}{3x^2} - \frac{\arccos(ax)^4}{3x^3} - 8ia^3 \arccos(ax) \arctan\left(e^{i\arccos(ax)}\right) - \frac{4}{3} ia^3 \arccos(ax)^3 \arctan\left(e^{i\arccos(ax)}\right) + 4ia^3 \operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) + 2ia^3 \arccos(ax)^2 \operatorname{PolyLog}\left(2, -ie^{i\arccos(ax)}\right) - 4ia^3 \operatorname{PolyLog}\left(2, ie^{i\arccos(ax)}\right) - 2ia^3 \arccos(ax)^2 \operatorname{PolyLog}\left(2, ie^{i\arccos(ax)}\right) - 4a^3 \arccos(ax) \operatorname{PolyLog}\left(3, -ie^{i\arccos(ax)}\right) + 4a^3 \arccos(ax) \operatorname{PolyLog}\left(3, ie^{i\arccos(ax)}\right) - 4ia^3 \operatorname{PolyLog}\left(4, -ie^{i\arccos(ax)}\right) + 4ia^3 \operatorname{PolyLog}\left(4, ie^{i\arccos(ax)}\right)$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 1475 vs. 2(304) = 608.

Time = 12.06 (sec) , antiderivative size = 1475, normalized size of antiderivative = 4.85

$$\int \frac{\arccos(ax)^4}{x^4}\,dx = a^3\Bigg(-\frac{1}{6}\arccos(ax)^2\left(12+\arccos(ax)^2\right)$$
$$+4\Big(\arccos(ax)\left(\log\left(1-ie^{i\arccos(ax)}\right)-\log\left(1+ie^{i\arccos(ax)}\right)\right)+i\left(\operatorname{PolyLog}\left(2,-ie^{i\arccos(ax)}\right)-\operatorname{PolyLog}\left(2,ie^{i\arccos(ax)}\right)\right)\Big)$$
$$+\frac{2}{3}\Bigg(\frac{1}{8}\pi^3\log\left(\cot\left(\frac{1}{2}\left(\frac{\pi}{2}-\arccos(ax)\right)\right)\right)$$
$$+\frac{3}{4}\pi^2\Big(\Big(\frac{\pi}{2}-\arccos(ax)\Big)\Big(\log\Big(1-e^{i(\frac{\pi}{2}-\arccos(ax))}\Big)-\log\Big(1+e^{i(\frac{\pi}{2}-\arccos(ax))}\Big)\Big)+i\Big(\operatorname{PolyLog}\Big(2,-e^{i(\frac{\pi}{2}-\mathrm{arcc}}$$
$$-\frac{3}{2}\pi\Big(\Big(\frac{\pi}{2}-\arccos(ax)\Big)^2\Big(\log\Big(1-e^{i(\frac{\pi}{2}-\arccos(ax))}\Big)-\log\Big(1+e^{i(\frac{\pi}{2}-\arccos(ax))}\Big)\Big)+2i\Big(\frac{\pi}{2}-\arccos(ax)\Big)\Big(\mathrm{Pol}$$
$$+8\Bigg(\frac{1}{64}i\Big(\frac{\pi}{2}-\arccos(ax)\Big)^4+\frac{1}{4}i\Big(\frac{\pi}{2}+\frac{1}{2}\Big(-\frac{\pi}{2}+\arccos(ax)\Big)\Big)^4-\frac{1}{8}\Big(\frac{\pi}{2}-\arccos(ax)\Big)^3\log\Big(1+e^{i(\frac{\pi}{2}-\arccos(ax}$$
$$-\frac{-4\arccos(ax)^3+\arccos(ax)^4}{12\left(\cos\left(\frac{1}{2}\arccos(ax)\right)-\sin\left(\frac{1}{2}\arccos(ax)\right)\right)^2}$$
$$-\frac{\arccos(ax)^4\sin\left(\frac{1}{2}\arccos(ax)\right)}{6\left(\cos\left(\frac{1}{2}\arccos(ax)\right)-\sin\left(\frac{1}{2}\arccos(ax)\right)\right)^3}$$
$$+\frac{\arccos(ax)^4\sin\left(\frac{1}{2}\arccos(ax)\right)}{6\left(\cos\left(\frac{1}{2}\arccos(ax)\right)+\sin\left(\frac{1}{2}\arccos(ax)\right)\right)^3}$$
$$-\frac{4\arccos(ax)^3+\arccos(ax)^4}{12\left(\cos\left(\frac{1}{2}\arccos(ax)\right)+\sin\left(\frac{1}{2}\arccos(ax)\right)\right)^2}$$
$$-\frac{-12\arccos(ax)^2\sin\left(\frac{1}{2}\arccos(ax)\right)-\arccos(ax)^4\sin\left(\frac{1}{2}\arccos(ax)\right)}{6\left(\cos\left(\frac{1}{2}\arccos(ax)\right)+\sin\left(\frac{1}{2}\arccos(ax)\right)\right)}$$
$$-\frac{12\arccos(ax)^2\sin\left(\frac{1}{2}\arccos(ax)\right)+\arccos(ax)^4\sin\left(\frac{1}{2}\arccos(ax)\right)}{6\left(\cos\left(\frac{1}{2}\arccos(ax)\right)-\sin\left(\frac{1}{2}\arccos(ax)\right)\right)}\Bigg)$$

```
[In] Integrate[ArcCos[a*x]^4/x^4,x]
```

```
[Out] a^3*(-1/6*(ArcCos[a*x]^2*(12 + ArcCos[a*x]^2)) + 4*(ArcCos[a*x]*(Log[1 - I*
E^(I*ArcCos[a*x])] - Log[1 + I*E^(I*ArcCos[a*x])]) + I*(PolyLog[2, (-I)*E^(
I*ArcCos[a*x])] - PolyLog[2, I*E^(I*ArcCos[a*x])])) + (2*((Pi^3*Log[Cot[(Pi
/2 - ArcCos[a*x])/2]])/8 + (3*Pi^2*((Pi/2 - ArcCos[a*x])*(Log[1 - E^(I*(Pi/
2 - ArcCos[a*x]))] - Log[1 + E^(I*(Pi/2 - ArcCos[a*x]))]) + I*(PolyLog[2, -
E^(I*(Pi/2 - ArcCos[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcCos[a*x]))])))/4 -
```

```
(3*Pi*((Pi/2 - ArcCos[a*x])^2*(Log[1 - E^(I*(Pi/2 - ArcCos[a*x]))] - Log[1
 + E^(I*(Pi/2 - ArcCos[a*x]))]) + (2*I)*(Pi/2 - ArcCos[a*x])*(PolyLog[2, -E
^(I*(Pi/2 - ArcCos[a*x]))] - PolyLog[2, E^(I*(Pi/2 - ArcCos[a*x]))]) + 2*(-
PolyLog[3, -E^(I*(Pi/2 - ArcCos[a*x]))] + PolyLog[3, E^(I*(Pi/2 - ArcCos[a*
x]))])))/2 + 8*((I/64)*(Pi/2 - ArcCos[a*x])^4 + (I/4)*(Pi/2 + (-1/2*Pi + Ar
cCos[a*x])/2)^4 - ((Pi/2 - ArcCos[a*x])^3*Log[1 + E^(I*(Pi/2 - ArcCos[a*x])
)])/8 - (Pi^3*(I*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2) - Log[1 + E^((2*I)*(Pi/
2 + (-1/2*Pi + ArcCos[a*x])/2))]))/8 - (Pi/2 + (-1/2*Pi + ArcCos[a*x])/2)^3
*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2))] + ((3*I)/8)*(Pi/2 -
ArcCos[a*x])^2*PolyLog[2, -E^(I*(Pi/2 - ArcCos[a*x]))] + (3*Pi^2*((I/2)*(Pi
/2 + (-1/2*Pi + ArcCos[a*x])/2)^2 - (Pi/2 + (-1/2*Pi + ArcCos[a*x])/2)*Log[
1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2))] + (I/2)*PolyLog[2, -E^((2
*I)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2))]))/4 + ((3*I)/2)*(Pi/2 + (-1/2*Pi +
 ArcCos[a*x])/2)^2*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2))
] - (3*(Pi/2 - ArcCos[a*x])*PolyLog[3, -E^(I*(Pi/2 - ArcCos[a*x]))])/4 - (3
*Pi*((I/3)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2)^3 - (Pi/2 + (-1/2*Pi + ArcCos
[a*x])/2)^2*Log[1 + E^((2*I)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2))] + I*(Pi/2
 + (-1/2*Pi + ArcCos[a*x])/2)*PolyLog[2, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcCo
s[a*x])/2))] - PolyLog[3, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2))]/2)
)/2 - (3*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2)*PolyLog[3, -E^((2*I)*(Pi/2 + (-
1/2*Pi + ArcCos[a*x])/2))])/2 - ((3*I)/4)*PolyLog[4, -E^(I*(Pi/2 - ArcCos[a
*x]))] - ((3*I)/4)*PolyLog[4, -E^((2*I)*(Pi/2 + (-1/2*Pi + ArcCos[a*x])/2))
])))/3 - (-4*ArcCos[a*x]^3 + ArcCos[a*x]^4)/(12*(Cos[ArcCos[a*x]/2] - Sin[A
rcCos[a*x]/2])^2) - (ArcCos[a*x]^4*Sin[ArcCos[a*x]/2])/(6*(Cos[ArcCos[a*x]/
2] - Sin[ArcCos[a*x]/2])^3) + (ArcCos[a*x]^4*Sin[ArcCos[a*x]/2])/(6*(Cos[Ar
cCos[a*x]/2] + Sin[ArcCos[a*x]/2])^3) - (4*ArcCos[a*x]^3 + ArcCos[a*x]^4)/(
12*(Cos[ArcCos[a*x]/2] + Sin[ArcCos[a*x]/2])^2) - (-12*ArcCos[a*x]^2*Sin[Ar
cCos[a*x]/2] - ArcCos[a*x]^4*Sin[ArcCos[a*x]/2])/(6*(Cos[ArcCos[a*x]/2] + S
in[ArcCos[a*x]/2])) - (12*ArcCos[a*x]^2*Sin[ArcCos[a*x]/2] + ArcCos[a*x]^4*
Sin[ArcCos[a*x]/2])/(6*(Cos[ArcCos[a*x]/2] - Sin[ArcCos[a*x]/2])))
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.38

method	result
derivativedivides	$a^3\left(-\frac{\arccos(ax)^2\left(-2\sqrt{-a^2x^2+1}\,\arccos(ax)ax+\arccos(ax)^2+6a^2x^2\right)}{3a^3x^3}-\frac{2\arccos(ax)^3\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right)}{3}\right.$
default	$a^3\left(-\frac{\arccos(ax)^2\left(-2\sqrt{-a^2x^2+1}\,\arccos(ax)ax+\arccos(ax)^2+6a^2x^2\right)}{3a^3x^3}-\frac{2\arccos(ax)^3\ln\left(1+i\left(i\sqrt{-a^2x^2+1}+ax\right)\right)}{3}\right.$

```
[In] int(arccos(a*x)^4/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] a^3*(-1/3/a^3/x^3*arccos(a*x)^2*(-2*(-a^2*x^2+1)^(1/2)*arccos(a*x)*a*x+arcc
```

```
os(a*x)^2+6*a^2*x^2)-2/3*arccos(a*x)^3*ln(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))+2
*I*polylog(2,-I*(I*(-a^2*x^2+1)^(1/2)+a*x))*arccos(a*x)^2-4*arccos(a*x)*pol
ylog(3,-I*(I*(-a^2*x^2+1)^(1/2)+a*x))-4*I*polylog(4,-I*(I*(-a^2*x^2+1)^(1/2
)+a*x))+2/3*arccos(a*x)^3*ln(1-I*(I*(-a^2*x^2+1)^(1/2)+a*x))-2*I*polylog(2,
I*(I*(-a^2*x^2+1)^(1/2)+a*x))*arccos(a*x)^2+4*arccos(a*x)*polylog(3,I*(I*(-
a^2*x^2+1)^(1/2)+a*x))+4*I*polylog(4,I*(I*(-a^2*x^2+1)^(1/2)+a*x))-4*arccos
(a*x)*ln(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))+4*arccos(a*x)*ln(1-I*(I*(-a^2*x^2+
1)^(1/2)+a*x))+4*I*dilog(1+I*(I*(-a^2*x^2+1)^(1/2)+a*x))-4*I*dilog(1-I*(I*(
-a^2*x^2+1)^(1/2)+a*x)))
```

Fricas [F]

$$\int \frac{\arccos(ax)^4}{x^4} \, dx = \int \frac{\arccos(ax)^4}{x^4} \, dx$$

```
[In] integrate(arccos(a*x)^4/x^4,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^4/x^4, x)
```

Sympy [F]

$$\int \frac{\arccos(ax)^4}{x^4} \, dx = \int \frac{\operatorname{acos}^4(ax)}{x^4} \, dx$$

```
[In] integrate(acos(a*x)**4/x**4,x)
```

```
[Out] Integral(acos(a*x)**4/x**4, x)
```

Maxima [F]

$$\int \frac{\arccos(ax)^4}{x^4} \, dx = \int \frac{\arccos(ax)^4}{x^4} \, dx$$

```
[In] integrate(arccos(a*x)^4/x^4,x, algorithm="maxima")
```

```
[Out] 1/3*(12*a*x^3*integrate(1/3*sqrt(a*x + 1)*sqrt(-a*x + 1)*arctan2(sqrt(a*x +
 1)*sqrt(-a*x + 1), a*x)^3/(a^2*x^5 - x^3), x) - arctan2(sqrt(a*x + 1)*sqrt
(-a*x + 1), a*x)^4)/x^3
```

Giac [F]

$$\int \frac{\arccos(ax)^4}{x^4}\,dx = \int \frac{\arccos\left(ax\right)^4}{x^4}\,dx$$

```
[In] integrate(arccos(a*x)^4/x^4,x, algorithm="giac")
[Out] integrate(arccos(a*x)^4/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\arccos(ax)^4}{x^4}\,dx = \int \frac{\mathrm{acos}(a\,x)^4}{x^4}\,dx$$

```
[In] int(acos(a*x)^4/x^4,x)
[Out] int(acos(a*x)^4/x^4, x)
```

3.42 $\int \frac{x^6}{\arccos(ax)} dx$

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{x^6}{\arccos(ax)} dx = -\frac{5\text{Si}(\arccos(ax))}{64a^7} - \frac{9\text{Si}(3\arccos(ax))}{64a^7} - \frac{5\text{Si}(5\arccos(ax))}{64a^7} - \frac{\text{Si}(7\arccos(ax))}{64a^7}$$

```
[Out] -5/64*Si(arccos(a*x))/a^7-9/64*Si(3*arccos(a*x))/a^7-5/64*Si(5*arccos(a*x))
/a^7-1/64*Si(7*arccos(a*x))/a^7
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4732, 4491, 3380}

$$\int \frac{x^6}{\arccos(ax)} dx = -\frac{5\text{Si}(\arccos(ax))}{64a^7} - \frac{9\text{Si}(3\arccos(ax))}{64a^7} - \frac{5\text{Si}(5\arccos(ax))}{64a^7} - \frac{\text{Si}(7\arccos(ax))}{64a^7}$$

```
[In] Int[x^6/ArcCos[a*x],x]
```

```
[Out] (-5*SinIntegral[ArcCos[a*x]])/(64*a^7) - (9*SinIntegral[3*ArcCos[a*x]])/(64
*a^7) - (5*SinIntegral[5*ArcCos[a*x]])/(64*a^7) - SinIntegral[7*ArcCos[a*x]
]/(64*a^7)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cos^6(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a^7}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{5\sin(x)}{64x}+\frac{9\sin(3x)}{64x}+\frac{5\sin(5x)}{64x}+\frac{\sin(7x)}{64x}\right)\,dx, x, \arccos(ax)\right)}{a^7}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(7x)}{x}\,dx, x, \arccos(ax)\right)}{64a^7}-\frac{5\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{64a^7}\\
&\quad -\frac{5\text{Subst}\left(\int \frac{\sin(5x)}{x}\,dx, x, \arccos(ax)\right)}{64a^7}-\frac{9\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \arccos(ax)\right)}{64a^7}\\
&= -\frac{5\text{Si}(\arccos(ax))}{64a^7}-\frac{9\text{Si}(3\arccos(ax))}{64a^7}-\frac{5\text{Si}(5\arccos(ax))}{64a^7}-\frac{\text{Si}(7\arccos(ax))}{64a^7}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{x^6}{\arccos(ax)}\,dx = -\frac{5\text{Si}(\arccos(ax))+9\text{Si}(3\arccos(ax))+5\text{Si}(5\arccos(ax))+\text{Si}(7\arccos(ax))}{64a^7}$$

```
[In] Integrate[x^6/ArcCos[a*x],x]
```

```
[Out] -1/64*(5*SinIntegral[ArcCos[a*x]] + 9*SinIntegral[3*ArcCos[a*x]] + 5*SinInt
egral[5*ArcCos[a*x]] + SinIntegral[7*ArcCos[a*x]])/a^7
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

method	result	size
derivativedivides	$\frac{-\frac{9\,\mathrm{Si}(3\arccos(ax))}{64}-\frac{5\,\mathrm{Si}(5\arccos(ax))}{64}-\frac{\mathrm{Si}(7\arccos(ax))}{64}-\frac{5\,\mathrm{Si}(\arccos(ax))}{64}}{a^7}$	40
default	$\frac{-\frac{9\,\mathrm{Si}(3\arccos(ax))}{64}-\frac{5\,\mathrm{Si}(5\arccos(ax))}{64}-\frac{\mathrm{Si}(7\arccos(ax))}{64}-\frac{5\,\mathrm{Si}(\arccos(ax))}{64}}{a^7}$	40

```
[In] int(x^6/arccos(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^7*(-9/64*Si(3*arccos(a*x))-5/64*Si(5*arccos(a*x))-1/64*Si(7*arccos(a*x)
)-5/64*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^6}{\arccos(ax)}\,dx = \int \frac{x^6}{\arccos{(ax)}}\,dx$$

```
[In] integrate(x^6/arccos(a*x),x, algorithm="fricas")
[Out] integral(x^6/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{x^6}{\arccos(ax)}\,dx = \int \frac{x^6}{\operatorname{acos}{(ax)}}\,dx$$

```
[In] integrate(x**6/acos(a*x),x)
[Out] Integral(x**6/acos(a*x), x)
```

Maxima [F]

$$\int \frac{x^6}{\arccos(ax)}\,dx = \int \frac{x^6}{\arccos{(ax)}}\,dx$$

```
[In] integrate(x^6/arccos(a*x),x, algorithm="maxima")
[Out] integrate(x^6/arccos(a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.85

$$\int \frac{x^6}{\arccos(ax)}\,dx = -\frac{\mathrm{Si}\left(7\arccos\left(ax\right)\right)}{64\,a^7} - \frac{5\,\mathrm{Si}\left(5\arccos\left(ax\right)\right)}{64\,a^7} - \frac{9\,\mathrm{Si}\left(3\arccos\left(ax\right)\right)}{64\,a^7} - \frac{5\,\mathrm{Si}\left(\arccos\left(ax\right)\right)}{64\,a^7}$$

```
[In] integrate(x^6/arccos(a*x),x, algorithm="giac")
[Out] -1/64*sin_integral(7*arccos(a*x))/a^7 - 5/64*sin_integral(5*arccos(a*x))/a^
7 - 9/64*sin_integral(3*arccos(a*x))/a^7 - 5/64*sin_integral(arccos(a*x))/a
^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\arccos(ax)}\,dx = \int \frac{x^6}{\mathrm{acos}\left(a\,x\right)}\,dx$$

```
[In] int(x^6/acos(a*x),x)
[Out] int(x^6/acos(a*x), x)
```

3.43 $\int \frac{x^5}{\arccos(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 43

$$\int \frac{x^5}{\arccos(ax)}\,dx = -\frac{5\text{Si}(2\arccos(ax))}{32a^6} - \frac{\text{Si}(4\arccos(ax))}{8a^6} - \frac{\text{Si}(6\arccos(ax))}{32a^6}$$

```
[Out] -5/32*Si(2*arccos(a*x))/a^6-1/8*Si(4*arccos(a*x))/a^6-1/32*Si(6*arccos(a*x)
)/a^6
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4732, 4491, 3380}

$$\int \frac{x^5}{\arccos(ax)}\,dx = -\frac{5\text{Si}(2\arccos(ax))}{32a^6} - \frac{\text{Si}(4\arccos(ax))}{8a^6} - \frac{\text{Si}(6\arccos(ax))}{32a^6}$$

```
[In] Int[x^5/ArcCos[a*x],x]
```

```
[Out] (-5*SinIntegral[2*ArcCos[a*x]])/(32*a^6) - SinIntegral[4*ArcCos[a*x]]/(8*a^
6) - SinIntegral[6*ArcCos[a*x]]/(32*a^6)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cos^5(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a^6} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{5\sin(2x)}{32x} + \frac{\sin(4x)}{8x} + \frac{\sin(6x)}{32x}\right)\,dx, x, \arccos(ax)\right)}{a^6} \\
&= -\frac{\text{Subst}\left(\int \frac{\sin(6x)}{x}\,dx, x, \arccos(ax)\right)}{32a^6} - \frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx, x, \arccos(ax)\right)}{8a^6} \\
&\quad - \frac{5\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \arccos(ax)\right)}{32a^6} \\
&= -\frac{5\text{Si}(2\arccos(ax))}{32a^6} - \frac{\text{Si}(4\arccos(ax))}{8a^6} - \frac{\text{Si}(6\arccos(ax))}{32a^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{x^5}{\arccos(ax)}\,dx = -\frac{5\text{Si}(2\arccos(ax)) + 4\text{Si}(4\arccos(ax)) + \text{Si}(6\arccos(ax))}{32a^6}$$

```
[In] Integrate[x^5/ArcCos[a*x],x]
[Out] -1/32*(5*SinIntegral[2*ArcCos[a*x]] + 4*SinIntegral[4*ArcCos[a*x]] + SinInt
egral[6*ArcCos[a*x]])/a^6
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

method	result	size
derivativedivides	$\frac{-\frac{5\,\mathrm{Si}(2\arccos(ax))}{32}-\frac{\mathrm{Si}(4\arccos(ax))}{8}-\frac{\mathrm{Si}(6\arccos(ax))}{32}}{a^6}$	33
default	$\frac{-\frac{5\,\mathrm{Si}(2\arccos(ax))}{32}-\frac{\mathrm{Si}(4\arccos(ax))}{8}-\frac{\mathrm{Si}(6\arccos(ax))}{32}}{a^6}$	33

```
[In] int(x^5/arccos(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^6*(-5/32*Si(2*arccos(a*x))-1/8*Si(4*arccos(a*x))-1/32*Si(6*arccos(a*x))
)
```

Fricas [F]

$$\int \frac{x^5}{\arccos(ax)}\,dx = \int \frac{x^5}{\arccos{(ax)}}\,dx$$

```
[In] integrate(x^5/arccos(a*x),x, algorithm="fricas")
[Out] integral(x^5/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{x^5}{\arccos(ax)}\,dx = \int \frac{x^5}{\operatorname{acos}{(ax)}}\,dx$$

```
[In] integrate(x**5/acos(a*x),x)
[Out] Integral(x**5/acos(a*x), x)
```

Maxima [F]

$$\int \frac{x^5}{\arccos(ax)}\,dx = \int \frac{x^5}{\arccos{(ax)}}\,dx$$

```
[In] integrate(x^5/arccos(a*x),x, algorithm="maxima")
[Out] integrate(x^5/arccos(a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{x^5}{\arccos(ax)}\,dx = -\frac{\text{Si}\,(6\,\arccos\,(ax))}{32\,a^6} - \frac{\text{Si}\,(4\,\arccos\,(ax))}{8\,a^6} - \frac{5\,\text{Si}\,(2\,\arccos\,(ax))}{32\,a^6}$$

```
[In] integrate(x^5/arccos(a*x),x, algorithm="giac")
```

```
[Out] -1/32*sin_integral(6*arccos(a*x))/a^6 - 1/8*sin_integral(4*arccos(a*x))/a^6
 - 5/32*sin_integral(2*arccos(a*x))/a^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\arccos(ax)}\,dx = \int \frac{x^5}{\mathrm{acos}\,(a\,x)}\,dx$$

```
[In] int(x^5/acos(a*x),x)
```

```
[Out] int(x^5/acos(a*x), x)
```

3.44 $\int \frac{x^4}{\arccos(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int \frac{x^4}{\arccos(ax)}\,dx = -\frac{\text{Si}(\arccos(ax))}{8a^5} - \frac{3\text{Si}(3\arccos(ax))}{16a^5} - \frac{\text{Si}(5\arccos(ax))}{16a^5}$$

```
[Out] -1/8*Si(arccos(a*x))/a^5-3/16*Si(3*arccos(a*x))/a^5-1/16*Si(5*arccos(a*x))/
a^5
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4732, 4491, 3380}

$$\int \frac{x^4}{\arccos(ax)}\,dx = -\frac{\text{Si}(\arccos(ax))}{8a^5} - \frac{3\text{Si}(3\arccos(ax))}{16a^5} - \frac{\text{Si}(5\arccos(ax))}{16a^5}$$

```
[In] Int[x^4/ArcCos[a*x],x]
[Out] -1/8*SinIntegral[ArcCos[a*x]]/a^5 - (3*SinIntegral[3*ArcCos[a*x]])/(16*a^5)
 - SinIntegral[5*ArcCos[a*x]]/(16*a^5)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cos^4(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a^5}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{8x}+\frac{3\sin(3x)}{16x}+\frac{\sin(5x)}{16x}\right)dx, x, \arccos(ax)\right)}{a^5}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(5x)}{x}\,dx, x, \arccos(ax)\right)}{16a^5}-\frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{8a^5}\\
&\quad -\frac{3\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \arccos(ax)\right)}{16a^5}\\
&= -\frac{\text{Si}(\arccos(ax))}{8a^5}-\frac{3\text{Si}(3\arccos(ax))}{16a^5}-\frac{\text{Si}(5\arccos(ax))}{16a^5}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$
\int \frac{x^4}{\arccos(ax)}\,dx = -\frac{2\text{Si}(\arccos(ax))+3\text{Si}(3\arccos(ax))+\text{Si}(5\arccos(ax))}{16a^5}
$$

```
[In] Integrate[x^4/ArcCos[a*x],x]
[Out] -1/16*(2*SinIntegral[ArcCos[a*x]] + 3*SinIntegral[3*ArcCos[a*x]] + SinInteg
ral[5*ArcCos[a*x]])/a^5
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{-\frac{3\,\mathrm{Si}(3\arccos(ax))}{16}-\frac{\mathrm{Si}(5\arccos(ax))}{16}-\frac{\mathrm{Si}(\arccos(ax))}{8}}{a^5}$	31
default	$\frac{-\frac{3\,\mathrm{Si}(3\arccos(ax))}{16}-\frac{\mathrm{Si}(5\arccos(ax))}{16}-\frac{\mathrm{Si}(\arccos(ax))}{8}}{a^5}$	31

```
[In] int(x^4/arccos(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^5*(-3/16*Si(3*arccos(a*x))-1/16*Si(5*arccos(a*x))-1/8*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^4}{\arccos(ax)}\,dx = \int \frac{x^4}{\arccos{(ax)}}\,dx$$

```
[In] integrate(x^4/arccos(a*x),x, algorithm="fricas")
[Out] integral(x^4/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)}\,dx = \int \frac{x^4}{\operatorname{acos}{(ax)}}\,dx$$

```
[In] integrate(x**4/acos(a*x),x)
[Out] Integral(x**4/acos(a*x), x)
```

Maxima [F]

$$\int \frac{x^4}{\arccos(ax)}\,dx = \int \frac{x^4}{\arccos{(ax)}}\,dx$$

```
[In] integrate(x^4/arccos(a*x),x, algorithm="maxima")
[Out] integrate(x^4/arccos(a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.85

$$\int \frac{x^4}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}(5\arccos(ax))}{16\,a^5} - \frac{3\operatorname{Si}(3\arccos(ax))}{16\,a^5} - \frac{\operatorname{Si}(\arccos(ax))}{8\,a^5}$$

```
[In] integrate(x^4/arccos(a*x),x, algorithm="giac")
[Out] -1/16*sin_integral(5*arccos(a*x))/a^5 - 3/16*sin_integral(3*arccos(a*x))/a^
5 - 1/8*sin_integral(arccos(a*x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)}\,dx = \int \frac{x^4}{\operatorname{acos}(a\,x)}\,dx$$

```
[In] int(x^4/acos(a*x),x)
[Out] int(x^4/acos(a*x), x)
```

3.45 $\int \frac{x^3}{\arccos(ax)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 29

$$\int \frac{x^3}{\arccos(ax)} \, dx = -\frac{\text{Si}(2\arccos(ax))}{4a^4} - \frac{\text{Si}(4\arccos(ax))}{8a^4}$$

```
[Out] -1/4*Si(2*arccos(a*x))/a^4-1/8*Si(4*arccos(a*x))/a^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4732, 4491, 3380}

$$\int \frac{x^3}{\arccos(ax)} \, dx = -\frac{\text{Si}(2\arccos(ax))}{4a^4} - \frac{\text{Si}(4\arccos(ax))}{8a^4}$$

```
[In] Int[x^3/ArcCos[a*x],x]
[Out] -1/4*SinIntegral[2*ArcCos[a*x]]/a^4 - SinIntegral[4*ArcCos[a*x]]/(8*a^4)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a^4} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4x}+\frac{\sin(4x)}{8x}\right)\,dx, x, \arccos(ax)\right)}{a^4} \\
&= -\frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx, x, \arccos(ax)\right)}{8a^4}-\frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \arccos(ax)\right)}{4a^4} \\
&= -\frac{\text{Si}(2\arccos(ax))}{4a^4}-\frac{\text{Si}(4\arccos(ax))}{8a^4}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

$$\int \frac{x^3}{\arccos(ax)}\,dx = -\frac{2\text{Si}(2\arccos(ax))+\text{Si}(4\arccos(ax))}{8a^4}$$

```
[In] Integrate[x^3/ArcCos[a*x],x]
[Out] -1/8*(2*SinIntegral[2*ArcCos[a*x]] + SinIntegral[4*ArcCos[a*x]])/a^4
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.83

method	result	size
derivativedivides	$\frac{-\frac{\text{Si}(2\arccos(ax))}{4}-\frac{\text{Si}(4\arccos(ax))}{8}}{a^4}$	24
default	$\frac{-\frac{\text{Si}(2\arccos(ax))}{4}-\frac{\text{Si}(4\arccos(ax))}{8}}{a^4}$	24

```
[In] int(x^3/arccos(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^4*(-1/4*Si(2*arccos(a*x))-1/8*Si(4*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\arccos(ax)}\,dx = \int \frac{x^3}{\arccos{(ax)}}\,dx$$

```
[In] integrate(x^3/arccos(a*x),x, algorithm="fricas")
[Out] integral(x^3/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)}\,dx = \int \frac{x^3}{\operatorname{acos}{(ax)}}\,dx$$

```
[In] integrate(x**3/acos(a*x),x)
[Out] Integral(x**3/acos(a*x), x)
```

Maxima [F]

$$\int \frac{x^3}{\arccos(ax)}\,dx = \int \frac{x^3}{\arccos{(ax)}}\,dx$$

```
[In] integrate(x^3/arccos(a*x),x, algorithm="maxima")
[Out] integrate(x^3/arccos(a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.86

$$\int \frac{x^3}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}{(4\arccos{(ax)})}}{8\,a^4} - \frac{\operatorname{Si}{(2\arccos{(ax)})}}{4\,a^4}$$

```
[In] integrate(x^3/arccos(a*x),x, algorithm="giac")
[Out] -1/8*sin_integral(4*arccos(a*x))/a^4 - 1/4*sin_integral(2*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)}\,dx = \int \frac{x^3}{\operatorname{acos}(a\,x)}\,dx$$

```
[In] int(x^3/acos(a*x),x)
```

```
[Out] int(x^3/acos(a*x), x)
```

3.46 $\int \frac{x^2}{\arccos(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 27

$$\int \frac{x^2}{\arccos(ax)}\,dx=-\frac{\operatorname{Si}(\arccos(ax))}{4a^3}-\frac{\operatorname{Si}(3\arccos(ax))}{4a^3}$$

```
[Out] -1/4*Si(arccos(a*x))/a^3-1/4*Si(3*arccos(a*x))/a^3
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4732, 4491, 3380}

$$\int \frac{x^2}{\arccos(ax)}\,dx=-\frac{\operatorname{Si}(\arccos(ax))}{4a^3}-\frac{\operatorname{Si}(3\arccos(ax))}{4a^3}$$

```
[In] Int[x^2/ArcCos[a*x],x]
[Out] -1/4*SinIntegral[ArcCos[a*x]]/a^3 - SinIntegral[3*ArcCos[a*x]]/(4*a^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{4x}+\frac{\sin(3x)}{4x}\right)\,dx, x, \arccos(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{4a^3}-\frac{\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \arccos(ax)\right)}{4a^3}\\
&= -\frac{\text{Si}(\arccos(ax))}{4a^3}-\frac{\text{Si}(3\arccos(ax))}{4a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.74

$$\int \frac{x^2}{\arccos(ax)}\,dx = -\frac{\text{Si}(\arccos(ax))+\text{Si}(3\arccos(ax))}{4a^3}$$

```
[In] Integrate[x^2/ArcCos[a*x],x]
[Out] -1/4*(SinIntegral[ArcCos[a*x]] + SinIntegral[3*ArcCos[a*x]])/a^3
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{-\frac{\text{Si}(3\arccos(ax))}{4}-\frac{\text{Si}(\arccos(ax))}{4}}{a^3}$	22
default	$\frac{-\frac{\text{Si}(3\arccos(ax))}{4}-\frac{\text{Si}(\arccos(ax))}{4}}{a^3}$	22

```
[In] int(x^2/arccos(a*x),x,method=_RETURNVERBOSE)
[Out] 1/a^3*(-1/4*Si(3*arccos(a*x))-1/4*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\arccos(ax)}\,dx = \int \frac{x^2}{\arccos\left(ax\right)}\,dx$$

[In] integrate(x^2/arccos(a*x),x, algorithm="fricas")

[Out] integral(x^2/arccos(a*x), x)

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)}\,dx = \int \frac{x^2}{\operatorname{acos}\left(ax\right)}\,dx$$

[In] integrate(x**2/acos(a*x),x)

[Out] Integral(x**2/acos(a*x), x)

Maxima [F]

$$\int \frac{x^2}{\arccos(ax)}\,dx = \int \frac{x^2}{\arccos\left(ax\right)}\,dx$$

[In] integrate(x^2/arccos(a*x),x, algorithm="maxima")

[Out] integrate(x^2/arccos(a*x), x)

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.85

$$\int \frac{x^2}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}\left(3\arccos\left(ax\right)\right)}{4\,a^3} - \frac{\operatorname{Si}\left(\arccos\left(ax\right)\right)}{4\,a^3}$$

[In] integrate(x^2/arccos(a*x),x, algorithm="giac")

[Out] -1/4*sin_integral(3*arccos(a*x))/a^3 - 1/4*sin_integral(arccos(a*x))/a^3

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)}\,dx = \int \frac{x^2}{\mathrm{acos}\left(a\,x\right)}\,dx$$

```
[In] int(x^2/acos(a*x),x)
[Out] int(x^2/acos(a*x), x)
```

3.47 $\int \frac{x}{\arccos(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 14

$$\int \frac{x}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}(2\arccos(ax))}{2a^2}$$

```
[Out] -1/2*Si(2*arccos(a*x))/a^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4732, 4491, 12, 3380}

$$\int \frac{x}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}(2\arccos(ax))}{2a^2}$$

```
[In] Int[x/ArcCos[a*x],x]
```

```
[Out] -1/2*SinIntegral[2*ArcCos[a*x]]/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx, x, \arccos(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \arccos(ax)\right)}{2a^2}\\
&= -\frac{\text{Si}(2\arccos(ax))}{2a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arccos(ax)}\,dx = -\frac{\text{Si}(2\arccos(ax))}{2a^2}$$

```
[In] Integrate[x/ArcCos[a*x],x]
[Out] -1/2*SinIntegral[2*ArcCos[a*x]]/a^2
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\text{Si}(2\arccos(ax))}{2a^2}$	13
default	$-\frac{\text{Si}(2\arccos(ax))}{2a^2}$	13

```
[In] int(x/arccos(a*x),x,method=_RETURNVERBOSE)
[Out] -1/2*Si(2*arccos(a*x))/a^2
```

Fricas [F]

$$\int \frac{x}{\arccos(ax)}\,dx = \int \frac{x}{\arccos{(ax)}}\,dx$$

```
[In] integrate(x/arccos(a*x),x, algorithm="fricas")
[Out] integral(x/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)}\,dx = \int \frac{x}{\operatorname{acos}{(ax)}}\,dx$$

```
[In] integrate(x/acos(a*x),x)
[Out] Integral(x/acos(a*x), x)
```

Maxima [F]

$$\int \frac{x}{\arccos(ax)}\,dx = \int \frac{x}{\arccos{(ax)}}\,dx$$

```
[In] integrate(x/arccos(a*x),x, algorithm="maxima")
[Out] integrate(x/arccos(a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{x}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}{(2\arccos{(ax)})}}{2\,a^2}$$

```
[In] integrate(x/arccos(a*x),x, algorithm="giac")
[Out] -1/2*sin_integral(2*arccos(a*x))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)}\,dx = \int \frac{x}{\mathrm{acos}\left(a\,x\right)}\,dx$$

```
[In] int(x/acos(a*x),x)
[Out] int(x/acos(a*x), x)
```

3.48 $\int \frac{1}{\arccos(ax)}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \frac{1}{\arccos(ax)}\,dx = -\frac{\text{Si}(\arccos(ax))}{a}$$

```
[Out] -Si(arccos(a*x))/a
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4720, 3380}

$$\int \frac{1}{\arccos(ax)}\,dx = -\frac{\text{Si}(\arccos(ax))}{a}$$

```
[In] Int[ArcCos[a*x]^(-1),x]
```

```
[Out] -(SinIntegral[ArcCos[a*x]]/a)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a}$$
$$= -\frac{\text{Si}(\arccos(ax))}{a}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arccos(ax)}\,dx = -\frac{\text{Si}(\arccos(ax))}{a}$$

```
[In] Integrate[ArcCos[a*x]^(-1),x]
[Out] -(SinIntegral[ArcCos[a*x]]/a)
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$-\frac{\text{Si}(\arccos(ax))}{a}$	11
default	$-\frac{\text{Si}(\arccos(ax))}{a}$	11

```
[In] int(1/arccos(a*x),x,method=_RETURNVERBOSE)
[Out] -Si(arccos(a*x))/a
```

Fricas [F]

$$\int \frac{1}{\arccos(ax)}\,dx = \int \frac{1}{\arccos(ax)}\,dx$$

```
[In] integrate(1/arccos(a*x),x, algorithm="fricas")
[Out] integral(1/arccos(a*x), x)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)}\,dx = \int \frac{1}{\operatorname{acos}(ax)}\,dx$$

```
[In] integrate(1/acos(a*x),x)
[Out] Integral(1/acos(a*x), x)
```

Maxima [F]

$$\int \frac{1}{\arccos(ax)}\,dx = \int \frac{1}{\arccos(ax)}\,dx$$

```
[In] integrate(1/arccos(a*x),x, algorithm="maxima")
[Out] integrate(1/arccos(a*x), x)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arccos(ax)}\,dx = -\frac{\operatorname{Si}(\arccos(ax))}{a}$$

```
[In] integrate(1/arccos(a*x),x, algorithm="giac")
[Out] -sin_integral(arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)}\,dx = \int \frac{1}{\operatorname{acos}(a\,x)}\,dx$$

```
[In] int(1/acos(a*x),x)
[Out] int(1/acos(a*x), x)
```

3.49 $\int \frac{1}{x \arccos(ax)} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arccos(ax)} dx = \text{Int}\left(\frac{1}{x \arccos(ax)}, x\right)$$

```
[Out] Unintegrable(1/x/arccos(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x \arccos(ax)} dx = \int \frac{1}{x \arccos(ax)} dx$$

```
[In] Int[1/(x*ArcCos[a*x]),x]
```

```
[Out] Defer[Int][1/(x*ArcCos[a*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \arccos(ax)} dx$$

Mathematica [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)} dx = \int \frac{1}{x \arccos(ax)} dx$$

```
[In] Integrate[1/(x*ArcCos[a*x]),x]
[Out] Integrate[1/(x*ArcCos[a*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos \left(a x\right)}dx$$

```
[In] int(1/x/arccos(a*x),x)
[Out] int(1/x/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)} dx = \int \frac{1}{x \arccos \left(a x\right)} dx$$

```
[In] integrate(1/x/arccos(a*x),x, algorithm="fricas")
[Out] integral(1/(x*arccos(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{1}{x \arccos(ax)} dx = \int \frac{1}{x \operatorname{acos}\left(a x\right)} dx$$

```
[In] integrate(1/x/acos(a*x),x)
[Out] Integral(1/(x*acos(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)} \, dx = \int \frac{1}{x \arccos{(ax)}} \, dx$$

```
[In] integrate(1/x/arccos(a*x),x, algorithm="maxima")
[Out] integrate(1/(x*arccos(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)} \, dx = \int \frac{1}{x \arccos{(ax)}} \, dx$$

```
[In] integrate(1/x/arccos(a*x),x, algorithm="giac")
[Out] integrate(1/(x*arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)} \, dx = \int \frac{1}{x \operatorname{acos}\left(a\,x\right)} \, dx$$

```
[In] int(1/(x*acos(a*x)),x)
[Out] int(1/(x*acos(a*x)), x)
```

3.50 $\int \frac{1}{x^2 \arccos(ax)} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2 \arccos(ax)} \, dx = \operatorname{Int}\left(\frac{1}{x^2 \arccos(ax)}, x\right)$$

```
[Out] Unintegrable(1/x^2/arccos(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \arccos(ax)} \, dx = \int \frac{1}{x^2 \arccos(ax)} \, dx$$

```
[In] Int[1/(x^2*ArcCos[a*x]),x]
```

```
[Out] Defer[Int][1/(x^2*ArcCos[a*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 \arccos(ax)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)}\,dx = \int \frac{1}{x^2 \arccos(ax)}\,dx$$

```
[In] Integrate[1/(x^2*ArcCos[a*x]),x]
[Out] Integrate[1/(x^2*ArcCos[a*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.75 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \arccos\left(ax\right)}dx$$

```
[In] int(1/x^2/arccos(a*x),x)
[Out] int(1/x^2/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)}\,dx = \int \frac{1}{x^2 \arccos\left(ax\right)}\,dx$$

```
[In] integrate(1/x^2/arccos(a*x),x, algorithm="fricas")
[Out] integral(1/(x^2*arccos(a*x)), x)
```

Sympy [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \arccos(ax)}\,dx = \int \frac{1}{x^2 \operatorname{acos}\left(ax\right)}\,dx$$

```
[In] integrate(1/x**2/acos(a*x),x)
[Out] Integral(1/(x**2*acos(a*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)} \, dx = \int \frac{1}{x^2 \arccos{(ax)}} \, dx$$

```
[In] integrate(1/x^2/arccos(a*x),x, algorithm="maxima")
[Out] integrate(1/(x^2*arccos(a*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)} \, dx = \int \frac{1}{x^2 \arccos{(ax)}} \, dx$$

```
[In] integrate(1/x^2/arccos(a*x),x, algorithm="giac")
[Out] integrate(1/(x^2*arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)} \, dx = \int \frac{1}{x^2 \,\mathrm{acos}\left(a\,x\right)} \, dx$$

```
[In] int(1/(x^2*acos(a*x)),x)
[Out] int(1/(x^2*acos(a*x)), x)
```

3.51 $\int \frac{x^6}{\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \frac{x^6\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{5\,\text{CosIntegral}(\arccos(ax))}{64a^7} - \frac{27\,\text{CosIntegral}(3\arccos(ax))}{64a^7} - \frac{25\,\text{CosIntegral}(5\arccos(ax))}{64a^7} - \frac{7\,\text{CosIntegral}(7\arccos(ax))}{64a^7}$$

```
[Out] -5/64*Ci(arccos(a*x))/a^7-27/64*Ci(3*arccos(a*x))/a^7-25/64*Ci(5*arccos(a*x
))/a^7-7/64*Ci(7*arccos(a*x))/a^7+x^6*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4728, 3383}

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = -\frac{5\,\text{CosIntegral}(\arccos(ax))}{64a^7} - \frac{27\,\text{CosIntegral}(3\arccos(ax))}{64a^7} - \frac{25\,\text{CosIntegral}(5\arccos(ax))}{64a^7} - \frac{7\,\text{CosIntegral}(7\arccos(ax))}{64a^7} + \frac{x^6\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

```
[In] Int[x^6/ArcCos[a*x]^2,x]
```

```
[Out] (x^6*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - (5*CosIntegral[ArcCos[a*x]])/(64*
a^7) - (27*CosIntegral[3*ArcCos[a*x]])/(64*a^7) - (25*CosIntegral[5*ArcCos[
a*x]])/(64*a^7) - (7*CosIntegral[7*ArcCos[a*x]])/(64*a^7)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^6\sqrt{1-a^2x^2}}{a\arccos(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{5\cos(x)}{64x}-\frac{27\cos(3x)}{64x}-\frac{25\cos(5x)}{64x}-\frac{7\cos(7x)}{64x}\right)\,dx,x,\arccos(ax)\right)}{a^7}\\
&= \frac{x^6\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{5\text{Subst}\left(\int\frac{\cos(x)}{x}\,dx,x,\arccos(ax)\right)}{64a^7} - \frac{7\text{Subst}\left(\int\frac{\cos(7x)}{x}\,dx,x,\arccos(ax)\right)}{64a^7}\\
&\quad - \frac{25\text{Subst}\left(\int\frac{\cos(5x)}{x}\,dx,x,\arccos(ax)\right)}{64a^7} - \frac{27\text{Subst}\left(\int\frac{\cos(3x)}{x}\,dx,x,\arccos(ax)\right)}{64a^7}\\
&= \frac{x^6\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{5\,\text{CosIntegral}(\arccos(ax))}{64a^7} - \frac{27\,\text{CosIntegral}(3\arccos(ax))}{64a^7}\\
&\quad - \frac{25\,\text{CosIntegral}(5\arccos(ax))}{64a^7} - \frac{7\,\text{CosIntegral}(7\arccos(ax))}{64a^7}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int\frac{x^6}{\arccos(ax)^2}\,dx = -\frac{-64a^6x^6\sqrt{1-a^2x^2}+5\arccos(ax)\,\text{CosIntegral}(\arccos(ax))+27\arccos(ax)\,\text{CosIntegral}(3\arccos(ax))}{64a^7\arccos(ax)}$$

```
[In] Integrate[x^6/ArcCos[a*x]^2,x]
[Out] -1/64*(-64*a^6*x^6*Sqrt[1 - a^2*x^2] + 5*ArcCos[a*x]*CosIntegral[ArcCos[a*x
]] + 27*ArcCos[a*x]*CosIntegral[3*ArcCos[a*x]] + 25*ArcCos[a*x]*CosIntegral
[5*ArcCos[a*x]] + 7*ArcCos[a*x]*CosIntegral[7*ArcCos[a*x]])/(a^7*ArcCos[a*x
])
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.28

method	result
derivativedivides	$\frac{\frac{9\sin(3\arccos(ax))}{64\arccos(ax)}-\frac{27\,\mathrm{Ci}(3\arccos(ax))}{64}+\frac{5\sin(5\arccos(ax))}{64\arccos(ax)}-\frac{25\,\mathrm{Ci}(5\arccos(ax))}{64}+\frac{\sin(7\arccos(ax))}{64\arccos(ax)}-\frac{7\,\mathrm{Ci}(7\arccos(ax))}{64}+\frac{5\sqrt{-a^2x}}{64\arccos}}{a^7}$
default	$\frac{\frac{9\sin(3\arccos(ax))}{64\arccos(ax)}-\frac{27\,\mathrm{Ci}(3\arccos(ax))}{64}+\frac{5\sin(5\arccos(ax))}{64\arccos(ax)}-\frac{25\,\mathrm{Ci}(5\arccos(ax))}{64}+\frac{\sin(7\arccos(ax))}{64\arccos(ax)}-\frac{7\,\mathrm{Ci}(7\arccos(ax))}{64}+\frac{5\sqrt{-a^2x}}{64\arccos}}{a^7}$

```
[In] int(x^6/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^7*(9/64/arccos(a*x)*sin(3*arccos(a*x))-27/64*Ci(3*arccos(a*x))+5/64/arc
cos(a*x)*sin(5*arccos(a*x))-25/64*Ci(5*arccos(a*x))+1/64*sin(7*arccos(a*x))
/arccos(a*x)-7/64*Ci(7*arccos(a*x))+5/64*(-a^2*x^2+1)^(1/2)/arccos(a*x)-5/6
4*Ci(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \int \frac{x^6}{\arccos\left(ax\right)^2}\,dx$$

```
[In] integrate(x^6/arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral(x^6/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \int \frac{x^6}{\operatorname{acos}^2\left(ax\right)}\,dx$$

```
[In] integrate(x**6/acos(a*x)**2,x)
```

```
[Out] Integral(x**6/acos(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \int \frac{x^6}{\arccos\left(ax\right)^2}\,dx$$

```
[In] integrate(x^6/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] (sqrt(a*x + 1)*sqrt(-a*x + 1)*x^6 - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
 a*x)*integrate((7*a^2*x^7 - 6*x^5)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2
- a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x))/(a*arctan2(sqrt(a*x +
 1)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}x^6}{a\arccos\left(ax\right)} - \frac{7\,\mathrm{Ci}\left(7\,\arccos\left(ax\right)\right)}{64\,a^7} - \frac{25\,\mathrm{Ci}\left(5\,\arccos\left(ax\right)\right)}{64\,a^7} - \frac{27\,\mathrm{Ci}\left(3\,\arccos\left(ax\right)\right)}{64\,a^7} - \frac{5\,\mathrm{Ci}\left(\arccos\left(ax\right)\right)}{64\,a^7}$$

```
[In] integrate(x^6/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x^6/(a*arccos(a*x)) - 7/64*cos_integral(7*arccos(a*x))/a
^7 - 25/64*cos_integral(5*arccos(a*x))/a^7 - 27/64*cos_integral(3*arccos(a*
x))/a^7 - 5/64*cos_integral(arccos(a*x))/a^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\arccos(ax)^2}\,dx = \int \frac{x^6}{\mathrm{acos}\left(a\,x\right)^2}\,dx$$

```
[In] int(x^6/acos(a*x)^2,x)
```

```
[Out] int(x^6/acos(a*x)^2, x)
```

3.52 $\int \frac{x^5}{\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = \frac{x^5\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{5\,\text{CosIntegral}(2\arccos(ax))}{16a^6} - \frac{\text{CosIntegral}(4\arccos(ax))}{2a^6} - \frac{3\,\text{CosIntegral}(6\arccos(ax))}{16a^6}$$

```
[Out] -5/16*Ci(2*arccos(a*x))/a^6-1/2*Ci(4*arccos(a*x))/a^6-3/16*Ci(6*arccos(a*x)
)/a^6+x^5*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4728, 3383}

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = -\frac{5\,\text{CosIntegral}(2\arccos(ax))}{16a^6} - \frac{\text{CosIntegral}(4\arccos(ax))}{2a^6} - \frac{3\,\text{CosIntegral}(6\arccos(ax))}{16a^6} + \frac{x^5\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

```
[In] Int[x^5/ArcCos[a*x]^2,x]
```

```
[Out] (x^5*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - (5*CosIntegral[2*ArcCos[a*x]])/(1
6*a^6) - CosIntegral[4*ArcCos[a*x]]/(2*a^6) - (3*CosIntegral[6*ArcCos[a*x]]
)/(16*a^6)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^5\sqrt{1-a^2x^2}}{a\arccos(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{5\cos(2x)}{16x}-\frac{\cos(4x)}{2x}-\frac{3\cos(6x)}{16x}\right)dx, x, \arccos(ax)\right)}{a^6} \\
&= \frac{x^5\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{3\text{Subst}\left(\int\frac{\cos(6x)}{x}dx, x, \arccos(ax)\right)}{16a^6} \\
&\quad - \frac{5\text{Subst}\left(\int\frac{\cos(2x)}{x}dx, x, \arccos(ax)\right)}{16a^6} - \frac{\text{Subst}\left(\int\frac{\cos(4x)}{x}dx, x, \arccos(ax)\right)}{2a^6} \\
&= \frac{x^5\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{5\,\text{CosIntegral}(2\arccos(ax))}{16a^6} \\
&\quad - \frac{\text{CosIntegral}(4\arccos(ax))}{2a^6} - \frac{3\,\text{CosIntegral}(6\arccos(ax))}{16a^6}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90

$$\int\frac{x^5}{\arccos(ax)^2}dx = -\frac{-\frac{16a^5x^5\sqrt{1-a^2x^2}}{\arccos(ax)} + 5\,\text{CosIntegral}(2\arccos(ax)) + 8\,\text{CosIntegral}(4\arccos(ax)) + 3\,\text{CosIntegral}(6\arccos(a$$

$$16a^6$$

```
[In] Integrate[x^5/ArcCos[a*x]^2,x]
[Out] -1/16*((-16*a^5*x^5*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + 5*CosIntegral[2*ArcCos
[a*x]] + 8*CosIntegral[4*ArcCos[a*x]] + 3*CosIntegral[6*ArcCos[a*x]])/a^6
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

method	result	size
derivativedivides	$\frac{\frac{5\sin(2\arccos(ax))}{32\arccos(ax)}-\frac{5\operatorname{Ci}(2\arccos(ax))}{16}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}-\frac{\operatorname{Ci}(4\arccos(ax))}{2}+\frac{\sin(6\arccos(ax))}{32\arccos(ax)}-\frac{3\operatorname{Ci}(6\arccos(ax))}{16}}{a^6}$	78
default	$\frac{\frac{5\sin(2\arccos(ax))}{32\arccos(ax)}-\frac{5\operatorname{Ci}(2\arccos(ax))}{16}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}-\frac{\operatorname{Ci}(4\arccos(ax))}{2}+\frac{\sin(6\arccos(ax))}{32\arccos(ax)}-\frac{3\operatorname{Ci}(6\arccos(ax))}{16}}{a^6}$	78

```
[In] int(x^5/arccos(a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/a^6*(5/32/arccos(a*x)*sin(2*arccos(a*x))-5/16*Ci(2*arccos(a*x))+1/8/arcco
s(a*x)*sin(4*arccos(a*x))-1/2*Ci(4*arccos(a*x))+1/32/arccos(a*x)*sin(6*arcc
os(a*x))-3/16*Ci(6*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^5}{\arccos(ax)^2}\,dx=\int \frac{x^5}{\arccos{(ax)}^2}\,dx$$

```
[In] integrate(x^5/arccos(a*x)^2,x, algorithm="fricas")
[Out] integral(x^5/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^5}{\arccos(ax)^2}\,dx=\int \frac{x^5}{\operatorname{acos}^2{(ax)}}\,dx$$

```
[In] integrate(x**5/acos(a*x)**2,x)
[Out] Integral(x**5/acos(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^5}{\arccos(ax)^2}\,dx=\int \frac{x^5}{\arccos{(ax)}^2}\,dx$$

```
[In] integrate(x^5/arccos(a*x)^2,x, algorithm="maxima")
[Out] (sqrt(a*x + 1)*sqrt(-a*x + 1)*x^5 - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
 a*x)*integrate((6*a^2*x^6 - 5*x^4)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2
- a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x))/(a*arctan2(sqrt(a*x +
 1)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}x^5}{a\arccos(ax)} - \frac{3\ \mathrm{Ci}\left(6\arccos(ax)\right)}{16\,a^6} - \frac{\mathrm{Ci}\left(4\arccos(ax)\right)}{2\,a^6} - \frac{5\ \mathrm{Ci}\left(2\arccos(ax)\right)}{16\,a^6}$$

```
[In] integrate(x^5/arccos(a*x)^2,x, algorithm="giac")
[Out] sqrt(-a^2*x^2 + 1)*x^5/(a*arccos(a*x)) - 3/16*cos_integral(6*arccos(a*x))/a
^6 - 1/2*cos_integral(4*arccos(a*x))/a^6 - 5/16*cos_integral(2*arccos(a*x))
/a^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\arccos(ax)^2}\,dx = \int \frac{x^5}{\mathrm{acos}\left(a\,x\right)^2}\,dx$$

```
[In] int(x^5/acos(a*x)^2,x)
[Out] int(x^5/acos(a*x)^2, x)
```

3.53 $\int \frac{x^4}{\arccos(ax)^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int \frac{x^4}{\arccos(ax)^2} dx = \frac{x^4\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(\arccos(ax))}{8a^5} - \frac{9\,\text{CosIntegral}(3\arccos(ax))}{16a^5} - \frac{5\,\text{CosIntegral}(5\arccos(ax))}{16a^5}$$

```
[Out] -1/8*Ci(arccos(a*x))/a^5-9/16*Ci(3*arccos(a*x))/a^5-5/16*Ci(5*arccos(a*x))/
a^5+x^4*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4728, 3383}

$$\int \frac{x^4}{\arccos(ax)^2} dx = -\frac{\text{CosIntegral}(\arccos(ax))}{8a^5} - \frac{9\,\text{CosIntegral}(3\arccos(ax))}{16a^5} - \frac{5\,\text{CosIntegral}(5\arccos(ax))}{16a^5} + \frac{x^4\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

```
[In] Int[x^4/ArcCos[a*x]^2,x]
```

```
[Out] (x^4*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/(8*a^5)
- (9*CosIntegral[3*ArcCos[a*x]])/(16*a^5) - (5*CosIntegral[5*ArcCos[a*x]])/
(16*a^5)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^4\sqrt{1-a^2x^2}}{a\arccos(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{\cos(x)}{8x}-\frac{9\cos(3x)}{16x}-\frac{5\cos(5x)}{16x}\right)dx, x, \arccos(ax)\right)}{a^5} \\
&= \frac{x^4\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{Subst}\left(\int\frac{\cos(x)}{x}dx, x, \arccos(ax)\right)}{8a^5} \\
&\quad - \frac{5\text{Subst}\left(\int\frac{\cos(5x)}{x}dx, x, \arccos(ax)\right)}{16a^5} - \frac{9\text{Subst}\left(\int\frac{\cos(3x)}{x}dx, x, \arccos(ax)\right)}{16a^5} \\
&= \frac{x^4\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(\arccos(ax))}{8a^5} \\
&\quad - \frac{9\,\text{CosIntegral}(3\arccos(ax))}{16a^5} - \frac{5\,\text{CosIntegral}(5\arccos(ax))}{16a^5}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.90

$$\int\frac{x^4}{\arccos(ax)^2}dx = -\frac{-\frac{16a^4x^4\sqrt{1-a^2x^2}}{\arccos(ax)} + 2\,\text{CosIntegral}(\arccos(ax)) + 9\,\text{CosIntegral}(3\arccos(ax)) + 5\,\text{CosIntegral}(5\arccos(ax}{16a^5}$$

```
[In] Integrate[x^4/ArcCos[a*x]^2,x]

[Out] -1/16*((-16*a^4*x^4*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + 2*CosIntegral[ArcCos[a
*x]] + 9*CosIntegral[3*ArcCos[a*x]] + 5*CosIntegral[5*ArcCos[a*x]])/a^5
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{\frac{3\sin(3\arccos(ax))}{16\arccos(ax)}-\frac{9\operatorname{Ci}(3\arccos(ax))}{16}+\frac{\sin(5\arccos(ax))}{16\arccos(ax)}-\frac{5\operatorname{Ci}(5\arccos(ax))}{16}+\frac{\sqrt{-a^2x^2+1}}{8\arccos(ax)}-\frac{\operatorname{Ci}(\arccos(ax))}{8}}{a^5}$	81
default	$\frac{\frac{3\sin(3\arccos(ax))}{16\arccos(ax)}-\frac{9\operatorname{Ci}(3\arccos(ax))}{16}+\frac{\sin(5\arccos(ax))}{16\arccos(ax)}-\frac{5\operatorname{Ci}(5\arccos(ax))}{16}+\frac{\sqrt{-a^2x^2+1}}{8\arccos(ax)}-\frac{\operatorname{Ci}(\arccos(ax))}{8}}{a^5}$	81

```
[In] int(x^4/arccos(a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/a^5*(3/16/arccos(a*x)*sin(3*arccos(a*x))-9/16*Ci(3*arccos(a*x))+1/16/arcc
os(a*x)*sin(5*arccos(a*x))-5/16*Ci(5*arccos(a*x))+1/8*(-a^2*x^2+1)^(1/2)/ar
ccos(a*x)-1/8*Ci(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^4}{\arccos(ax)^2}\,dx = \int \frac{x^4}{\arccos{(ax)}^2}\,dx$$

```
[In] integrate(x^4/arccos(a*x)^2,x, algorithm="fricas")
[Out] integral(x^4/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^2}\,dx = \int \frac{x^4}{\operatorname{acos}^2{(ax)}}\,dx$$

```
[In] integrate(x**4/acos(a*x)**2,x)
[Out] Integral(x**4/acos(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^4}{\arccos(ax)^2}\,dx = \int \frac{x^4}{\arccos{(ax)}^2}\,dx$$

```
[In] integrate(x^4/arccos(a*x)^2,x, algorithm="maxima")
[Out] (sqrt(a*x + 1)*sqrt(-a*x + 1)*x^4 - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
 a*x)*integrate((5*a^2*x^5 - 4*x^3)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2
- a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x))/(a*arctan2(sqrt(a*x +
 1)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}x^4}{a\arccos(ax)} - \frac{5\ \text{Ci}\,(5\ \arccos(ax))}{16\,a^5} - \frac{9\ \text{Ci}\,(3\ \arccos(ax))}{16\,a^5} - \frac{\text{Ci}\,(\arccos(ax))}{8\,a^5}$$

```
[In] integrate(x^4/arccos(a*x)^2,x, algorithm="giac")
[Out] sqrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)) - 5/16*cos_integral(5*arccos(a*x))/a
^5 - 9/16*cos_integral(3*arccos(a*x))/a^5 - 1/8*cos_integral(arccos(a*x))/a
^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^2}\,dx = \int \frac{x^4}{\text{acos}\,(a\,x)^2}\,dx$$

```
[In] int(x^4/acos(a*x)^2,x)
[Out] int(x^4/acos(a*x)^2, x)
```

3.54 $\int \frac{x^3}{\arccos(ax)^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 56

$$\int \frac{x^3}{\arccos(ax)^2} \, dx = \frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(2\arccos(ax))}{2a^4} - \frac{\text{CosIntegral}(4\arccos(ax))}{2a^4}$$

```
[Out] -1/2*Ci(2*arccos(a*x))/a^4-1/2*Ci(4*arccos(a*x))/a^4+x^3*(-a^2*x^2+1)^(1/2)
/a/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4728, 3383}

$$\int \frac{x^3}{\arccos(ax)^2} \, dx = -\frac{\text{CosIntegral}(2\arccos(ax))}{2a^4} - \frac{\text{CosIntegral}(4\arccos(ax))}{2a^4} + \frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

```
[In] Int[x^3/ArcCos[a*x]^2,x]
```

```
[Out] (x^3*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[2*ArcCos[a*x]]/(2*a^4
) - CosIntegral[4*ArcCos[a*x]]/(2*a^4)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)} + \frac{\text{Subst}\left(\int\left(-\frac{\cos(2x)}{2x}-\frac{\cos(4x)}{2x}\right)dx, x, \arccos(ax)\right)}{a^4}\\ &= \frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{Subst}\left(\int\frac{\cos(2x)}{x}dx, x, \arccos(ax)\right)}{2a^4} - \frac{\text{Subst}\left(\int\frac{\cos(4x)}{x}dx, x, \arccos(ax)\right)}{2a^4}\\ &= \frac{x^3\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(2\arccos(ax))}{2a^4} - \frac{\text{CosIntegral}(4\arccos(ax))}{2a^4}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\arccos(ax)^2}\,dx = -\frac{-\frac{2a^3x^3\sqrt{1-a^2x^2}}{\arccos(ax)} + \text{CosIntegral}(2\arccos(ax)) + \text{CosIntegral}(4\arccos(ax))}{2a^4}$$

```
[In] Integrate[x^3/ArcCos[a*x]^2,x]
[Out] -1/2*((-2*a^3*x^3*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + CosIntegral[2*ArcCos[a*x
]] + CosIntegral[4*ArcCos[a*x]])/a^4
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{4\arccos(ax)}-\frac{\text{Ci}(2\arccos(ax))}{2}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}-\frac{\text{Ci}(4\arccos(ax))}{2}}{a^4}$	54
default	$\frac{\frac{\sin(2\arccos(ax))}{4\arccos(ax)}-\frac{\text{Ci}(2\arccos(ax))}{2}+\frac{\sin(4\arccos(ax))}{8\arccos(ax)}-\frac{\text{Ci}(4\arccos(ax))}{2}}{a^4}$	54

```
[In] int(x^3/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/4/arccos(a*x)*sin(2*arccos(a*x))-1/2*Ci(2*arccos(a*x))+1/8/arccos(
a*x)*sin(4*arccos(a*x))-1/2*Ci(4*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\arccos(ax)^2}\,dx = \int \frac{x^3}{\arccos\left(ax\right)^2}\,dx$$

```
[In] integrate(x^3/arccos(a*x)^2,x, algorithm="fricas")
[Out] integral(x^3/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^2}\,dx = \int \frac{x^3}{\operatorname{acos}^2\left(ax\right)}\,dx$$

```
[In] integrate(x**3/acos(a*x)**2,x)
[Out] Integral(x**3/acos(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^3}{\arccos(ax)^2}\,dx = \int \frac{x^3}{\arccos\left(ax\right)^2}\,dx$$

```
[In] integrate(x^3/arccos(a*x)^2,x, algorithm="maxima")
[Out] (sqrt(a*x + 1)*sqrt(-a*x + 1)*x^3 - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
 a*x)*integrate((4*a^2*x^4 - 3*x^2)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2
- a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x))/(a*arctan2(sqrt(a*x +
 1)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{x^3}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}x^3}{a\arccos{(ax)}} - \frac{\mathrm{Ci}\left(4\arccos{(ax)}\right)}{2\,a^4} - \frac{\mathrm{Ci}\left(2\arccos{(ax)}\right)}{2\,a^4}$$

```
[In] integrate(x^3/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)) - 1/2*cos_integral(4*arccos(a*x))/a^
4 - 1/2*cos_integral(2*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^2}\,dx = \int \frac{x^3}{{\mathrm{acos}\left(a\,x\right)}^2}\,dx$$

```
[In] int(x^3/acos(a*x)^2,x)
```

```
[Out] int(x^3/acos(a*x)^2, x)
```

3.55 $\int \frac{x^2}{\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{x^2}{\arccos(ax)^2}\,dx=\frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)}-\frac{\text{CosIntegral}(\arccos(ax))}{4a^3}-\frac{3\,\text{CosIntegral}(3\arccos(ax))}{4a^3}$$

```
[Out] -1/4*Ci(arccos(a*x))/a^3-3/4*Ci(3*arccos(a*x))/a^3+x^2*(-a^2*x^2+1)^(1/2)/a
/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {4728, 3383}

$$\int \frac{x^2}{\arccos(ax)^2}\,dx=-\frac{\text{CosIntegral}(\arccos(ax))}{4a^3}-\frac{3\,\text{CosIntegral}(3\arccos(ax))}{4a^3}+\frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)}$$

```
[In] Int[x^2/ArcCos[a*x]^2,x]
```

```
[Out] (x^2*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/(4*a^3)
- (3*CosIntegral[3*ArcCos[a*x]])/(4*a^3)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)} + \frac{\text{Subst}\left(\int \left(-\frac{\cos(x)}{4x} - \frac{3\cos(3x)}{4x}\right) dx, x, \arccos(ax)\right)}{a^3} \\ &= \frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{x} dx, x, \arccos(ax)\right)}{4a^3} - \frac{3\text{Subst}\left(\int \frac{\cos(3x)}{x} dx, x, \arccos(ax)\right)}{4a^3} \\ &= \frac{x^2\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(\arccos(ax))}{4a^3} - \frac{3\,\text{CosIntegral}(3\arccos(ax))}{4a^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = -\frac{-\frac{4a^2x^2\sqrt{1-a^2x^2}}{\arccos(ax)} + \text{CosIntegral}(\arccos(ax)) + 3\,\text{CosIntegral}(3\arccos(ax))}{4a^3}$$

```
[In] Integrate[x^2/ArcCos[a*x]^2,x]
[Out] -1/4*((-4*a^2*x^2*Sqrt[1 - a^2*x^2])/ArcCos[a*x] + CosIntegral[ArcCos[a*x]]
 + 3*CosIntegral[3*ArcCos[a*x]])/a^3
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\frac{\sin(3\arccos(ax))}{4\arccos(ax)} - \frac{3\,\operatorname{Ci}(3\arccos(ax))}{4} + \frac{\sqrt{-a^2x^2+1}}{4\arccos(ax)} - \frac{\operatorname{Ci}(\arccos(ax))}{4}}{a^3}$	57
default	$\frac{\frac{\sin(3\arccos(ax))}{4\arccos(ax)} - \frac{3\,\operatorname{Ci}(3\arccos(ax))}{4} + \frac{\sqrt{-a^2x^2+1}}{4\arccos(ax)} - \frac{\operatorname{Ci}(\arccos(ax))}{4}}{a^3}$	57

```
[In] int(x^2/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/4/arccos(a*x)*sin(3*arccos(a*x))-3/4*Ci(3*arccos(a*x))+1/4*(-a^2*x
^2+1)^(1/2)/arccos(a*x)-1/4*Ci(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \int \frac{x^2}{\arccos\left(ax\right)^2}\,dx$$

```
[In] integrate(x^2/arccos(a*x)^2,x, algorithm="fricas")
[Out] integral(x^2/arccos(a*x)^2, x)
```

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \int \frac{x^2}{\operatorname{acos}^2\left(ax\right)}\,dx$$

```
[In] integrate(x**2/acos(a*x)**2,x)
[Out] Integral(x**2/acos(a*x)**2, x)
```

Maxima [F]

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \int \frac{x^2}{\arccos\left(ax\right)^2}\,dx$$

```
[In] integrate(x^2/arccos(a*x)^2,x, algorithm="maxima")
[Out] (sqrt(a*x + 1)*sqrt(-a*x + 1)*x^2 - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
 a*x)*integrate((3*a^2*x^3 - 2*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2 -
a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x))/(a*arctan2(sqrt(a*x + 1
)*sqrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.89

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2+1}x^2}{a\arccos(ax)} - \frac{3\,\mathrm{Ci}\left(3\,\arccos(ax)\right)}{4\,a^3} - \frac{\mathrm{Ci}\left(\arccos(ax)\right)}{4\,a^3}$$

```
[In] integrate(x^2/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] sqrt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)) - 3/4*cos_integral(3*arccos(a*x))/a^
3 - 1/4*cos_integral(arccos(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^2}\,dx = \int \frac{x^2}{\mathrm{acos}\left(a\,x\right)^2}\,dx$$

```
[In] int(x^2/acos(a*x)^2,x)
```

```
[Out] int(x^2/acos(a*x)^2, x)
```

3.56 $\int \frac{x}{\arccos(ax)^2} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 38

$$\int \frac{x}{\arccos(ax)^2} \, dx = \frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(2\arccos(ax))}{a^2}$$

```
[Out] -Ci(2*arccos(a*x))/a^2+x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4728, 3383}

$$\int \frac{x}{\arccos(ax)^2} \, dx = \frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(2\arccos(ax))}{a^2}$$

```
[In] Int[x/ArcCos[a*x]^2,x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(a*ArcCos[a*x]) - CosIntegral[2*ArcCos[a*x]]/a^2
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
```

```
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \arccos(ax)\right)}{a^2}\\ &= \frac{x\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(2\arccos(ax))}{a^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.97

$$\int \frac{x}{\arccos(ax)^2}\,dx = \frac{\frac{ax\sqrt{1-a^2x^2}}{\arccos(ax)} - \text{CosIntegral}(2\arccos(ax))}{a^2}$$

```
[In] Integrate[x/ArcCos[a*x]^2,x]
[Out] ((a*x*Sqrt[1 - a^2*x^2])/ArcCos[a*x] - CosIntegral[2*ArcCos[a*x]])/a^2
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{2\arccos(ax)} - \text{Ci}(2\arccos(ax))}{a^2}$	30
default	$\frac{\frac{\sin(2\arccos(ax))}{2\arccos(ax)} - \text{Ci}(2\arccos(ax))}{a^2}$	30

```
[In] int(x/arccos(a*x)^2,x,method=_RETURNVERBOSE)
[Out] 1/a^2*(1/2/arccos(a*x)*sin(2*arccos(a*x))-Ci(2*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x}{\arccos(ax)^2}\,dx = \int \frac{x}{\arccos\left(ax\right)^2}\,dx$$

[In] integrate(x/arccos(a*x)^2,x, algorithm="fricas")

[Out] integral(x/arccos(a*x)^2, x)

Sympy [F]

$$\int \frac{x}{\arccos(ax)^2}\,dx = \int \frac{x}{\operatorname{acos}^2\left(ax\right)}\,dx$$

[In] integrate(x/acos(a*x)**2,x)

[Out] Integral(x/acos(a*x)**2, x)

Maxima [F]

$$\int \frac{x}{\arccos(ax)^2}\,dx = \int \frac{x}{\arccos\left(ax\right)^2}\,dx$$

[In] integrate(x/arccos(a*x)^2,x, algorithm="maxima")

```
[Out] -(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate((2*a^2*x^2 - 1)*sq
rt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2 - a)*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x)), x) - sqrt(a*x + 1)*sqrt(-a*x + 1)*x)/(a*arctan2(sqrt(a*x + 1)*s
qrt(-a*x + 1), a*x))
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.95

$$\int \frac{x}{\arccos(ax)^2}\,dx = \frac{\sqrt{-a^2x^2 + 1}x}{a\arccos\left(ax\right)} - \frac{\operatorname{Ci}\left(2\arccos\left(ax\right)\right)}{a^2}$$

[In] integrate(x/arccos(a*x)^2,x, algorithm="giac")

[Out] sqrt(-a^2*x^2 + 1)*x/(a*arccos(a*x)) - cos_integral(2*arccos(a*x))/a^2

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^2}\, dx = \int \frac{x}{\mathrm{acos}\left(a\,x\right)^2}\, dx$$

```
[In] int(x/acos(a*x)^2,x)
[Out] int(x/acos(a*x)^2, x)
```

3.57 $\int \frac{1}{\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 6, antiderivative size = 35

$$\int \frac{1}{\arccos(ax)^2}\,dx = \frac{\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(\arccos(ax))}{a}$$

```
[Out] -Ci(arccos(a*x))/a+(-a^2*x^2+1)^(1/2)/a/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4718, 4810, 3383}

$$\int \frac{1}{\arccos(ax)^2}\,dx = \frac{\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(\arccos(ax))}{a}$$

```
[In] Int[ArcCos[a*x]^(-2),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/a
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
```

```
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-a^2x^2}}{a\arccos(ax)} + a\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx \\
&= \frac{\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \arccos(ax)\right)}{a} \\
&= \frac{\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(\arccos(ax))}{a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00

$$\int \frac{1}{\arccos(ax)^2}\,dx = \frac{\sqrt{1-a^2x^2}}{a\arccos(ax)} - \frac{\text{CosIntegral}(\arccos(ax))}{a}$$

```
[In] Integrate[ArcCos[a*x]^(-2),x]
[Out] Sqrt[1 - a^2*x^2]/(a*ArcCos[a*x]) - CosIntegral[ArcCos[a*x]]/a
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{\frac{\sqrt{-a^2x^2+1}}{\arccos(ax)}-\text{Ci}(\arccos(ax))}{a}$	32
default	$\frac{\frac{\sqrt{-a^2x^2+1}}{\arccos(ax)}-\text{Ci}(\arccos(ax))}{a}$	32

```
[In] int(1/arccos(a*x)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*((-a^2*x^2+1)^(1/2)/arccos(a*x)-Ci(arccos(a*x)))
```

Fricas [F]

$$\int \frac{1}{\arccos(ax)^2} \, dx = \int \frac{1}{\arccos{(ax)}^2} \, dx$$

```
[In] integrate(1/arccos(a*x)^2,x, algorithm="fricas")
[Out] integral(arccos(a*x)^(-2), x)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^2} \, dx = \int \frac{1}{\operatorname{acos}^2{(ax)}} \, dx$$

```
[In] integrate(1/acos(a*x)**2,x)
[Out] Integral(acos(a*x)**(-2), x)
```

Maxima [F]

$$\int \frac{1}{\arccos(ax)^2} \, dx = \int \frac{1}{\arccos{(ax)}^2} \, dx$$

```
[In] integrate(1/arccos(a*x)^2,x, algorithm="maxima")
[Out] -(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(sqrt(a*x + 1)*sq
rt(-a*x + 1)*x/((a^2*x^2 - 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)),
x) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
 a*x))
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.94

$$\int \frac{1}{\arccos(ax)^2} \, dx = -\frac{\operatorname{Ci}{(\arccos{(ax)})}}{a} + \frac{\sqrt{-a^2x^2+1}}{a\arccos{(ax)}}$$

```
[In] integrate(1/arccos(a*x)^2,x, algorithm="giac")
[Out] -cos_integral(arccos(a*x))/a + sqrt(-a^2*x^2 + 1)/(a*arccos(a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^2} \, dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)^2} \,dx$$

```
[In] int(1/acos(a*x)^2,x)
[Out] int(1/acos(a*x)^2, x)
```

3.58 $\int \frac{1}{x \arccos(ax)^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arccos(ax)^2} dx = \text{Int}\left(\frac{1}{x \arccos(ax)^2}, x\right)$$

```
[Out] Unintegrable(1/x/arccos(a*x)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x \arccos(ax)^2} dx = \int \frac{1}{x \arccos(ax)^2} dx$$

```
[In] Int[1/(x*ArcCos[a*x]^2),x]
```

```
[Out] Defer[Int][1/(x*ArcCos[a*x]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \arccos(ax)^2} dx$$

Mathematica [N/A]

Not integrable

Time = 1.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^2} \, dx = \int \frac{1}{x \arccos(ax)^2} \, dx$$

```
[In] Integrate[1/(x*ArcCos[a*x]^2),x]
[Out] Integrate[1/(x*ArcCos[a*x]^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 4.81 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos\left(ax\right)^{2}}dx$$

```
[In] int(1/x/arccos(a*x)^2,x)
[Out] int(1/x/arccos(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^2} \, dx = \int \frac{1}{x \arccos\left(ax\right)^{2}} \, dx$$

```
[In] integrate(1/x/arccos(a*x)^2,x, algorithm="fricas")
[Out] integral(1/(x*arccos(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^2} \, dx = \int \frac{1}{x \operatorname{acos}^{2}{\left(ax \right)}} \, dx$$

```
[In] integrate(1/x/acos(a*x)**2,x)
[Out] Integral(1/(x*acos(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.54 (sec) , antiderivative size = 127, normalized size of antiderivative = 12.70

$$\int \frac{1}{x\arccos(ax)^2}\,dx = \int \frac{1}{x\arccos{(ax)}^2}\,dx$$

```
[In] integrate(1/x/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] -(a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate(sqrt(a*x + 1)*sq
rt(-a*x + 1)/((a^3*x^4 - a*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))
, x) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x))
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arccos(ax)^2}\,dx = \int \frac{1}{x\arccos{(ax)}^2}\,dx$$

```
[In] integrate(1/x/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arccos(ax)^2}\,dx = \int \frac{1}{x\,\mathrm{acos}{(a\,x)}^2}\,dx$$

```
[In] int(1/(x*acos(a*x)^2),x)
```

```
[Out] int(1/(x*acos(a*x)^2), x)
```

3.59 $\int \frac{1}{x^2 \arccos(ax)^2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \text{Int}\left(\frac{1}{x^2 \arccos(ax)^2}, x\right)$$

```
[Out] Unintegrable(1/x^2/arccos(a*x)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \int \frac{1}{x^2 \arccos(ax)^2} \, dx$$

```
[In] Int[1/(x^2*ArcCos[a*x]^2),x]
[Out] Defer[Int][1/(x^2*ArcCos[a*x]^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 \arccos(ax)^2} \, dx$$

Mathematica [N/A]

Not integrable

Time = 14.83 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \int \frac{1}{x^2 \arccos(ax)^2} \, dx$$

```
[In] Integrate[1/(x^2*ArcCos[a*x]^2),x]
[Out] Integrate[1/(x^2*ArcCos[a*x]^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.76 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \arccos\left(ax\right)^2} dx$$

```
[In] int(1/x^2/arccos(a*x)^2,x)
[Out] int(1/x^2/arccos(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \int \frac{1}{x^2 \arccos\left(ax\right)^2} \, dx$$

```
[In] integrate(1/x^2/arccos(a*x)^2,x, algorithm="fricas")
[Out] integral(1/(x^2*arccos(a*x)^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^2} \, dx = \int \frac{1}{x^2 \operatorname{acos}^2\left(ax\right)} \, dx$$

```
[In] integrate(1/x**2/acos(a*x)**2,x)
[Out] Integral(1/(x**2*acos(a*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 136, normalized size of antiderivative = 13.60

$$\int \frac{1}{x^2 \arccos(ax)^2} dx = \int \frac{1}{x^2 \arccos{(ax)}^2} dx$$

```
[In] integrate(1/x^2/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] (a*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)*integrate((a^2*x^2 - 2)*s
qrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^5 - a*x^3)*arctan2(sqrt(a*x + 1)*sqrt(-
a*x + 1), a*x)), x) + sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*x^2*arctan2(sqrt(a*x
 + 1)*sqrt(-a*x + 1), a*x))
```

Giac [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^2} dx = \int \frac{1}{x^2 \arccos{(ax)}^2} dx$$

```
[In] integrate(1/x^2/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccos(a*x)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^2} dx = \int \frac{1}{x^2 \operatorname{acos}{(a\,x)}^2} dx$$

```
[In] int(1/(x^2*acos(a*x)^2),x)
```

```
[Out] int(1/(x^2*acos(a*x)^2), x)
```

3.60 $\int \frac{x^4}{\arccos(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 98

$$\int \frac{x^4}{\arccos(ax)^3}\,dx = \frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{2x^3}{a^2\arccos(ax)} + \frac{5x^5}{2\arccos(ax)} + \frac{\text{Si}(\arccos(ax))}{16a^5} + \frac{27\text{Si}(3\arccos(ax))}{32a^5} + \frac{25\text{Si}(5\arccos(ax))}{32a^5}$$

```
[Out] -2*x^3/a^2/arccos(a*x)+5/2*x^5/arccos(a*x)+1/16*Si(arccos(a*x))/a^5+27/32*S
i(3*arccos(a*x))/a^5+25/32*Si(5*arccos(a*x))/a^5+1/2*x^4*(-a^2*x^2+1)^(1/2)
/a/arccos(a*x)^2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4730, 4808, 4732, 4491, 3380}

$$\int \frac{x^4}{\arccos(ax)^3}\,dx = \frac{\text{Si}(\arccos(ax))}{16a^5} + \frac{27\text{Si}(3\arccos(ax))}{32a^5} + \frac{25\text{Si}(5\arccos(ax))}{32a^5} - \frac{2x^3}{a^2\arccos(ax)} + \frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} + \frac{5x^5}{2\arccos(ax)}$$

```
[In] Int[x^4/ArcCos[a*x]^3,x]
```

```
[Out] (x^4*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - (2*x^3)/(a^2*ArcCos[a*x]) + (
5*x^5)/(2*ArcCos[a*x]) + SinIntegral[ArcCos[a*x]]/(16*a^5) + (27*SinIntegra
l[3*ArcCos[a*x]])/(32*a^5) + (25*SinIntegral[5*ArcCos[a*x]])/(32*a^5)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{2\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^2}\,dx}{a} + \frac{1}{2}(5a)\int \frac{x^5}{\sqrt{1-a^2x^2}\arccos(ax)^2}\,dx$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{2x^3}{a^2\arccos(ax)} + \frac{5x^5}{2\arccos(ax)} - \frac{25}{2}\int \frac{x^4}{\arccos(ax)}\,dx + \frac{6\int \frac{x^2}{\arccos(ax)}\,dx}{a^2}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{2x^3}{a^2\arccos(ax)} + \frac{5x^5}{2\arccos(ax)} - \frac{6\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a^5} + \frac{25\text{Subst}\left(\int \frac{\cos^4(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{2a^5}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{2x^3}{a^2\arccos(ax)} + \frac{5x^5}{2\arccos(ax)} - \frac{6\text{Subst}\left(\int \left(\frac{\sin(x)}{4x} + \frac{\sin(3x)}{4x}\right)\,dx, x, \arccos(ax)\right)}{a^5} + \frac{25\text{Subst}\left(\int \left(\frac{\sin(x)}{8x} + \frac{3\sin(3x)}{16x} + \frac{\sin(5x)}{16x}\right)\,dx, x, \arccos(ax)\right)}{2a^5}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{2x^3}{a^2\arccos(ax)} + \frac{5x^5}{2\arccos(ax)} + \frac{25\text{Subst}\left(\int \frac{\sin(5x)}{x}\,dx, x, \arccos(ax)\right)}{32a^5} - \frac{3\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{2a^5} - \frac{3\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \arccos(ax)\right)}{2a^5} + \frac{25\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{16a^5} + \frac{75\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \arccos(ax)\right)}{32a^5}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{2x^3}{a^2\arccos(ax)} + \frac{5x^5}{2\arccos(ax)} + \frac{\text{Si}(\arccos(ax))}{16a^5} + \frac{27\text{Si}(3\arccos(ax))}{32a^5} + \frac{25\text{Si}(5\arccos(ax))}{32a^5}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.05

$$\int \frac{x^4}{\arccos(ax)^3}\,dx = \frac{16a^4x^4\sqrt{1-a^2x^2} - 64a^3x^3\arccos(ax) + 80a^5x^5\arccos(ax) + 2\arccos(ax)^2\text{Si}(\arccos(ax)) + 27\arccos(ax)}{32a^5\arccos(ax)^2}$$

```
[In] Integrate[x^4/ArcCos[a*x]^3,x]
```

```
[Out] (16*a^4*x^4*Sqrt[1 - a^2*x^2] - 64*a^3*x^3*ArcCos[a*x] + 80*a^5*x^5*ArcCos[
a*x] + 2*ArcCos[a*x]^2*SinIntegral[ArcCos[a*x]] + 27*ArcCos[a*x]^2*SinInteg
ral[3*ArcCos[a*x]] + 25*ArcCos[a*x]^2*SinIntegral[5*ArcCos[a*x]])/(32*a^5*A
rcCos[a*x]^2)
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{3\sin(3\arccos(ax))}{32\arccos(ax)^2}+\frac{9\cos(3\arccos(ax))}{32\arccos(ax)}+\frac{27\ \mathrm{Si}(3\arccos(ax))}{32}+\frac{\sin(5\arccos(ax))}{32\arccos(ax)^2}+\frac{5\cos(5\arccos(ax))}{32\arccos(ax)}+\frac{25\ \mathrm{Si}(5\arccos(ax))}{32}+\frac{\sqrt{-a^2}}{16\arccos}}{a^5}$
default	$\frac{\frac{3\sin(3\arccos(ax))}{32\arccos(ax)^2}+\frac{9\cos(3\arccos(ax))}{32\arccos(ax)}+\frac{27\ \mathrm{Si}(3\arccos(ax))}{32}+\frac{\sin(5\arccos(ax))}{32\arccos(ax)^2}+\frac{5\cos(5\arccos(ax))}{32\arccos(ax)}+\frac{25\ \mathrm{Si}(5\arccos(ax))}{32}+\frac{\sqrt{-a^2}}{16\arccos}}{a^5}$

```
[In] int(x^4/arccos(a*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/a^5*(3/32/arccos(a*x)^2*sin(3*arccos(a*x))+9/32/arccos(a*x)*cos(3*arccos(
a*x))+27/32*Si(3*arccos(a*x))+1/32/arccos(a*x)^2*sin(5*arccos(a*x))+5/32/ar
ccos(a*x)*cos(5*arccos(a*x))+25/32*Si(5*arccos(a*x))+1/16*(-a^2*x^2+1)^(1/2
)/arccos(a*x)^2+1/16/arccos(a*x)*a*x+1/16*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^4}{\arccos(ax)^3}\,dx = \int \frac{x^4}{\arccos\left(ax\right)^3}\,dx$$

```
[In] integrate(x^4/arccos(a*x)^3,x, algorithm="fricas")
[Out] integral(x^4/arccos(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^3}\,dx = \int \frac{x^4}{\operatorname{acos}^3\left(ax\right)}\,dx$$

```
[In] integrate(x**4/acos(a*x)**3,x)
[Out] Integral(x**4/acos(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^4}{\arccos(ax)^3} \, dx = \int \frac{x^4}{\arccos{(ax)}^3} \, dx$$

```
[In] integrate(x^4/arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x^4 - arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x)^2*integrate((25*a^2*x^4 - 12*x^2)/arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x), x) + (5*a^2*x^5 - 4*x^3)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
 a*x))/(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{x^4}{\arccos(ax)^3} \, dx = \frac{5 \, x^5}{2 \, \arccos{(ax)}} + \frac{\sqrt{-a^2 x^2 + 1} x^4}{2 \, a \arccos{(ax)}^2} - \frac{2 \, x^3}{a^2 \arccos{(ax)}} + \frac{25 \, \mathrm{Si}\left(5 \, \arccos{(ax)}\right)}{32 \, a^5} + \frac{27 \, \mathrm{Si}\left(3 \, \arccos{(ax)}\right)}{32 \, a^5} + \frac{\mathrm{Si}\left(\arccos{(ax)}\right)}{16 \, a^5}$$

```
[In] integrate(x^4/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 5/2*x^5/arccos(a*x) + 1/2*sqrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)^2) - 2*x^3/
(a^2*arccos(a*x)) + 25/32*sin_integral(5*arccos(a*x))/a^5 + 27/32*sin_integ
ral(3*arccos(a*x))/a^5 + 1/16*sin_integral(arccos(a*x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^3} \, dx = \int \frac{x^4}{{\mathrm{acos}\left(a\,x\right)}^3} \,d x$$

```
[In] int(x^4/acos(a*x)^3,x)
```

```
[Out] int(x^4/acos(a*x)^3, x)
```

3.61 $\int \frac{x^3}{\arccos(ax)^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 83

$$\int \frac{x^3}{\arccos(ax)^3}\,dx=\frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}-\frac{3x^2}{2a^2\arccos(ax)}+\frac{2x^4}{\arccos(ax)}+\frac{\mathrm{Si}(2\arccos(ax))}{2a^4}+\frac{\mathrm{Si}(4\arccos(ax))}{a^4}$$

```
[Out] -3/2*x^2/a^2/arccos(a*x)+2*x^4/arccos(a*x)+1/2*Si(2*arccos(a*x))/a^4+Si(4*a
rccos(a*x))/a^4+1/2*x^3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^2
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4730, 4808, 4732, 4491, 3380, 12}

$$\int \frac{x^3}{\arccos(ax)^3}\,dx=\frac{\mathrm{Si}(2\arccos(ax))}{2a^4}+\frac{\mathrm{Si}(4\arccos(ax))}{a^4}-\frac{3x^2}{2a^2\arccos(ax)}+\frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}+\frac{2x^4}{\arccos(ax)}$$

```
[In] Int[x^3/ArcCos[a*x]^3,x]
```

```
[Out] (x^3*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - (3*x^2)/(2*a^2*ArcCos[a*x]) +
 (2*x^4)/ArcCos[a*x] + SinIntegral[2*ArcCos[a*x]]/(2*a^4) + SinIntegral[4*A
rcCos[a*x]]/a^4
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{3\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^2}\,dx}{2a} + (2a)\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)^2}\,dx \\
&= \frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{3x^2}{2a^2\arccos(ax)} + \frac{2x^4}{\arccos(ax)} - 8\int \frac{x^3}{\arccos(ax)}\,dx + \frac{3\int \frac{x}{\arccos(ax)}\,dx}{a^2}
\end{aligned}
$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{3x^2}{2a^2\arccos(ax)} + \frac{2x^4}{\arccos(ax)} - \frac{3\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a^4} + \frac{8\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a^4}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{3x^2}{2a^2\arccos(ax)} + \frac{2x^4}{\arccos(ax)} - \frac{3\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx, x, \arccos(ax)\right)}{a^4} + \frac{8\text{Subst}\left(\int \left(\frac{\sin(2x)}{4x} + \frac{\sin(4x)}{8x}\right)\,dx, x, \arccos(ax)\right)}{a^4}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{3x^2}{2a^2\arccos(ax)} + \frac{2x^4}{\arccos(ax)} + \frac{\text{Subst}\left(\int \frac{\sin(4x)}{x}\,dx, x, \arccos(ax)\right)}{a^4} - \frac{3\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \arccos(ax)\right)}{2a^4} + \frac{2\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \arccos(ax)\right)}{a^4}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{3x^2}{2a^2\arccos(ax)} + \frac{2x^4}{\arccos(ax)} + \frac{\text{Si}(2\arccos(ax))}{2a^4} + \frac{\text{Si}(4\arccos(ax))}{a^4}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\arccos(ax)^3}\,dx = \frac{\frac{a^2x^2\left(ax\sqrt{1-a^2x^2}+(-3+4a^2x^2)\arccos(ax)\right)}{\arccos(ax)^2} + \text{Si}(2\arccos(ax)) + 2\text{Si}(4\arccos(ax))}{2a^4}$$

```
[In] Integrate[x^3/ArcCos[a*x]^3,x]
[Out] ((a^2*x^2*(a*x*Sqrt[1 - a^2*x^2] + (-3 + 4*a^2*x^2)*ArcCos[a*x]))/ArcCos[a*
x]^2 + SinIntegral[2*ArcCos[a*x]] + 2*SinIntegral[4*ArcCos[a*x]])/(2*a^4)
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{8\arccos(ax)^2}+\frac{\cos(2\arccos(ax))}{4\arccos(ax)}+\frac{\text{Si}(2\arccos(ax))}{2}+\frac{\sin(4\arccos(ax))}{16\arccos(ax)^2}+\frac{\cos(4\arccos(ax))}{4\arccos(ax)}+\text{Si}(4\arccos(ax))}{a^4}$	82
default	$\frac{\frac{\sin(2\arccos(ax))}{8\arccos(ax)^2}+\frac{\cos(2\arccos(ax))}{4\arccos(ax)}+\frac{\text{Si}(2\arccos(ax))}{2}+\frac{\sin(4\arccos(ax))}{16\arccos(ax)^2}+\frac{\cos(4\arccos(ax))}{4\arccos(ax)}+\text{Si}(4\arccos(ax))}{a^4}$	82

```
[In] int(x^3/arccos(a*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/a^4*(1/8/arccos(a*x)^2*sin(2*arccos(a*x))+1/4/arccos(a*x)*cos(2*arccos(a*
x))+1/2*Si(2*arccos(a*x))+1/16/arccos(a*x)^2*sin(4*arccos(a*x))+1/4/arccos(
a*x)*cos(4*arccos(a*x))+Si(4*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\arccos(ax)^3}\,dx = \int \frac{x^3}{\arccos\left(ax\right)^3}\,dx$$

```
[In] integrate(x^3/arccos(a*x)^3,x, algorithm="fricas")
[Out] integral(x^3/arccos(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^3}\,dx = \int \frac{x^3}{\operatorname{acos}^3\left(ax\right)}\,dx$$

```
[In] integrate(x**3/acos(a*x)**3,x)
[Out] Integral(x**3/acos(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^3}{\arccos(ax)^3}\,dx = \int \frac{x^3}{\arccos\left(ax\right)^3}\,dx$$

```
[In] integrate(x^3/arccos(a*x)^3,x, algorithm="maxima")
[Out] 1/2*(sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x^3 - 2*arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x)^2*integrate((8*a^2*x^3 - 3*x)/arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x), x) + (4*a^2*x^4 - 3*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a
*x))/(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{x^3}{\arccos(ax)^3}\,dx = \frac{2\,x^4}{\arccos{(ax)}} + \frac{\sqrt{-a^2x^2+1}x^3}{2\,a\arccos{(ax)}^2} - \frac{3\,x^2}{2\,a^2\arccos{(ax)}} + \frac{\mathrm{Si}\,(4\,\arccos{(ax)})}{a^4} + \frac{\mathrm{Si}\,(2\,\arccos{(ax)})}{2\,a^4}$$

```
[In] integrate(x^3/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 2*x^4/arccos(a*x) + 1/2*sqrt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)^2) - 3/2*x^2/
(a^2*arccos(a*x)) + sin_integral(4*arccos(a*x))/a^4 + 1/2*sin_integral(2*ar
ccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^3}\,dx = \int \frac{x^3}{{\mathrm{acos}\left(a\,x\right)}^3}\,dx$$

```
[In] int(x^3/acos(a*x)^3,x)
```

```
[Out] int(x^3/acos(a*x)^3, x)
```

3.62 $\int \frac{x^2}{\arccos(ax)^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int \frac{x^2}{\arccos(ax)^3} dx = \frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{x}{a^2\arccos(ax)} + \frac{3x^3}{2\arccos(ax)} + \frac{\text{Si}(\arccos(ax))}{8a^3} + \frac{9\text{Si}(3\arccos(ax))}{8a^3}$$

```
[Out] -x/a^2/arccos(a*x)+3/2*x^3/arccos(a*x)+1/8*Si(arccos(a*x))/a^3+9/8*Si(3*arc
cos(a*x))/a^3+1/2*x^2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^2
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4730, 4808, 4732, 4491, 3380, 4720}

$$\int \frac{x^2}{\arccos(ax)^3} dx = \frac{\text{Si}(\arccos(ax))}{8a^3} + \frac{9\text{Si}(3\arccos(ax))}{8a^3} + \frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{x}{a^2\arccos(ax)} + \frac{3x^3}{2\arccos(ax)}$$

```
[In] Int[x^2/ArcCos[a*x]^3,x]
```

```
[Out] (x^2*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - x/(a^2*ArcCos[a*x]) + (3*x^3)
/(2*ArcCos[a*x]) + SinIntegral[ArcCos[a*x]]/(8*a^3) + (9*SinIntegral[3*ArcC
os[a*x]])/(8*a^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rule steps

$$\text{integral} = \frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^2}\,dx}{a} + \frac{1}{2}(3a)\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^2}\,dx$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{x}{a^2\arccos(ax)} + \frac{3x^3}{2\arccos(ax)} - \frac{9}{2}\int \frac{x^2}{\arccos(ax)}\,dx + \frac{\int \frac{1}{\arccos(ax)}\,dx}{a^2}$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{x}{a^2\arccos(ax)} + \frac{3x^3}{2\arccos(ax)} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a^3} + \frac{9\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{2a^3}$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{x}{a^2\arccos(ax)} + \frac{3x^3}{2\arccos(ax)} - \frac{\text{Si}(\arccos(ax))}{a^3} + \frac{9\text{Subst}\left(\int \left(\frac{\sin(x)}{4x} + \frac{\sin(3x)}{4x}\right)\,dx, x, \arccos(ax)\right)}{2a^3}$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{x}{a^2\arccos(ax)} + \frac{3x^3}{2\arccos(ax)} - \frac{\text{Si}(\arccos(ax))}{a^3} + \frac{9\text{Subst}\left(\int \frac{\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{8a^3} + \frac{9\text{Subst}\left(\int \frac{\sin(3x)}{x}\,dx, x, \arccos(ax)\right)}{8a^3}$$

$$= \frac{x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{x}{a^2\arccos(ax)} + \frac{3x^3}{2\arccos(ax)} + \frac{\text{Si}(\arccos(ax))}{8a^3} + \frac{9\text{Si}(3\arccos(ax))}{8a^3}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \frac{\frac{4ax\left(ax\sqrt{1-a^2x^2}+(-2+3a^2x^2)\arccos(ax)\right)}{\arccos(ax)^2} + \text{Si}(\arccos(ax)) + 9\text{Si}(3\arccos(ax))}{8a^3}$$

```
[In] Integrate[x^2/ArcCos[a*x]^3,x]
[Out] ((4*a*x*(a*x*Sqrt[1 - a^2*x^2] + (-2 + 3*a^2*x^2)*ArcCos[a*x]))/ArcCos[a*x]
^2 + SinIntegral[ArcCos[a*x]] + 9*SinIntegral[3*ArcCos[a*x]])/(8*a^3)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\sin(3\arccos(ax))}{8\arccos(ax)^2}+\frac{3\cos(3\arccos(ax))}{8\arccos(ax)}+\frac{9\,\text{Si}(3\arccos(ax))}{8}+\frac{\sqrt{-a^2x^2+1}}{8\arccos(ax)^2}+\frac{ax}{8\arccos(ax)}+\frac{\text{Si}(\arccos(ax))}{8}}{a^3}$	82
default	$\frac{\frac{\sin(3\arccos(ax))}{8\arccos(ax)^2}+\frac{3\cos(3\arccos(ax))}{8\arccos(ax)}+\frac{9\,\text{Si}(3\arccos(ax))}{8}+\frac{\sqrt{-a^2x^2+1}}{8\arccos(ax)^2}+\frac{ax}{8\arccos(ax)}+\frac{\text{Si}(\arccos(ax))}{8}}{a^3}$	82

```
[In] int(x^2/arccos(a*x)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^3*(1/8/arccos(a*x)^2*sin(3*arccos(a*x))+3/8/arccos(a*x)*cos(3*arccos(a*
x))+9/8*Si(3*arccos(a*x))+1/8*(-a^2*x^2+1)^(1/2)/arccos(a*x)^2+1/8/arccos(a
*x)*a*x+1/8*Si(arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \int \frac{x^2}{\arccos\left(ax\right)^3}\,dx$$

```
[In] integrate(x^2/arccos(a*x)^3,x, algorithm="fricas")
```

```
[Out] integral(x^2/arccos(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \int \frac{x^2}{\operatorname{acos}^3\left(ax\right)}\,dx$$

```
[In] integrate(x**2/acos(a*x)**3,x)
```

```
[Out] Integral(x**2/acos(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \int \frac{x^2}{\arccos\left(ax\right)^3}\,dx$$

```
[In] integrate(x^2/arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x^2 - arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x)^2*integrate((9*a^2*x^2 - 2)/arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
 a*x), x) + (3*a^2*x^3 - 2*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(
a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.88

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \frac{3\,x^3}{2\,\arccos\left(ax\right)} + \frac{\sqrt{-a^2x^2+1}x^2}{2\,a\arccos\left(ax\right)^2} - \frac{x}{a^2\arccos\left(ax\right)} + \frac{9\,\mathrm{Si}\left(3\,\arccos\left(ax\right)\right)}{8\,a^3} + \frac{\mathrm{Si}\left(\arccos\left(ax\right)\right)}{8\,a^3}$$

```
[In] integrate(x^2/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 3/2*x^3/arccos(a*x) + 1/2*sqrt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)^2) - x/(a^2
*arccos(a*x)) + 9/8*sin_integral(3*arccos(a*x))/a^3 + 1/8*sin_integral(arcc
os(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^3}\,dx = \int \frac{x^2}{{\mathrm{acos}\left(a\,x\right)}^3}\,dx$$

```
[In] int(x^2/acos(a*x)^3,x)
```

```
[Out] int(x^2/acos(a*x)^3, x)
```

3.63 $\int \frac{x}{\arccos(ax)^3} dx$

Optimal result

Integrand size = 8, antiderivative size = 63

$$\int \frac{x}{\arccos(ax)^3} dx = \frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{1}{2a^2\arccos(ax)} + \frac{x^2}{\arccos(ax)} + \frac{\text{Si}(2\arccos(ax))}{a^2}$$

```
[Out] -1/2/a^2/arccos(a*x)+x^2/arccos(a*x)+Si(2*arccos(a*x))/a^2+1/2*x*(-a^2*x^2+
1)^(1/2)/a/arccos(a*x)^2
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.875, Rules used = {4730, 4808, 4732, 4491, 12, 3380, 4738}

$$\int \frac{x}{\arccos(ax)^3} dx = \frac{\text{Si}(2\arccos(ax))}{a^2} + \frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{1}{2a^2\arccos(ax)} + \frac{x^2}{\arccos(ax)}$$

```
[In] Int[x/ArcCos[a*x]^3,x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) - 1/(2*a^2*ArcCos[a*x]) + x^2/Arc
Cos[a*x] + SinIntegral[2*ArcCos[a*x]]/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^2}\,dx}{2a} + a\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^2}\,dx$$

$$= \frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{1}{2a^2\arccos(ax)} + \frac{x^2}{\arccos(ax)} - 2\int \frac{x}{\arccos(ax)}\,dx$$

$$= \frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{1}{2a^2\arccos(ax)} + \frac{x^2}{\arccos(ax)} + \frac{2\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{x}\,dx, x, \arccos(ax)\right)}{a^2}$$

$$= \frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{1}{2a^2\arccos(ax)} + \frac{x^2}{\arccos(ax)} + \frac{2\text{Subst}\left(\int \frac{\sin(2x)}{2x}\,dx, x, \arccos(ax)\right)}{a^2}$$

$$= \frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{1}{2a^2\arccos(ax)} + \frac{x^2}{\arccos(ax)} + \frac{\text{Subst}\left(\int \frac{\sin(2x)}{x}\,dx, x, \arccos(ax)\right)}{a^2}$$

$$= \frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} - \frac{1}{2a^2\arccos(ax)} + \frac{x^2}{\arccos(ax)} + \frac{\text{Si}(2\arccos(ax))}{a^2}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \frac{x}{\arccos(ax)^3}\,dx = \frac{x\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} + \frac{-1+2a^2x^2}{2a^2\arccos(ax)} + \frac{\text{Si}(2\arccos(ax))}{a^2}$$

```
[In] Integrate[x/ArcCos[a*x]^3,x]
[Out] (x*Sqrt[1 - a^2*x^2])/(2*a*ArcCos[a*x]^2) + (-1 + 2*a^2*x^2)/(2*a^2*ArcCos[
a*x]) + SinIntegral[2*ArcCos[a*x]]/a^2
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.68

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{4\arccos(ax)^2}+\frac{\cos(2\arccos(ax))}{2\arccos(ax)}+\text{Si}(2\arccos(ax))}{a^2}$	43
default	$\frac{\frac{\sin(2\arccos(ax))}{4\arccos(ax)^2}+\frac{\cos(2\arccos(ax))}{2\arccos(ax)}+\text{Si}(2\arccos(ax))}{a^2}$	43

```
[In] int(x/arccos(a*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/a^2*(1/4/arccos(a*x)^2*sin(2*arccos(a*x))+1/2/arccos(a*x)*cos(2*arccos(a*
x))+Si(2*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x}{\arccos(ax)^3}\,dx = \int \frac{x}{\arccos\left(ax\right)^3}\,dx$$

```
[In] integrate(x/arccos(a*x)^3,x, algorithm="fricas")
[Out] integral(x/arccos(a*x)^3, x)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)^3}\,dx = \int \frac{x}{\operatorname{acos}^3\left(ax\right)}\,dx$$

```
[In] integrate(x/acos(a*x)**3,x)
[Out] Integral(x/acos(a*x)**3, x)
```

Maxima [F]

$$\int \frac{x}{\arccos(ax)^3}\,dx = \int \frac{x}{\arccos\left(ax\right)^3}\,dx$$

```
[In] integrate(x/arccos(a*x)^3,x, algorithm="maxima")
[Out] -1/2*(4*a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate(x/arctan
2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x), x) - sqrt(a*x + 1)*sqrt(-a*x + 1)*a*x
 - (2*a^2*x^2 - 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^2*arctan2
(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [A] (verification not implemented)

none

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.90

$$\int \frac{x}{\arccos(ax)^3}\,dx = \frac{x^2}{\arccos\left(ax\right)} + \frac{\mathrm{Si}\left(2\arccos\left(ax\right)\right)}{a^2} + \frac{\sqrt{-a^2x^2+1}x}{2\,a\arccos\left(ax\right)^2} - \frac{1}{2\,a^2\arccos\left(ax\right)}$$

```
[In] integrate(x/arccos(a*x)^3,x, algorithm="giac")
[Out] x^2/arccos(a*x) + sin_integral(2*arccos(a*x))/a^2 + 1/2*sqrt(-a^2*x^2 + 1)*
x/(a*arccos(a*x)^2) - 1/2/(a^2*arccos(a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^3}\,dx = \int \frac{x}{\operatorname{acos}\left(a\,x\right)^3}\,dx$$

```
[In] int(x/acos(a*x)^3,x)
[Out] int(x/acos(a*x)^3, x)
```

3.64 $\int \frac{1}{\arccos(ax)^3} \, dx$

Optimal result

Integrand size = 6, antiderivative size = 51

$$\int \frac{1}{\arccos(ax)^3} \, dx = \frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} + \frac{x}{2\arccos(ax)} + \frac{\operatorname{Si}(\arccos(ax))}{2a}$$

```
[Out] 1/2*x/arccos(a*x)+1/2*Si(arccos(a*x))/a+1/2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x
)^2
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4718, 4808, 4720, 3380}

$$\int \frac{1}{\arccos(ax)^3} \, dx = \frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2} + \frac{\operatorname{Si}(\arccos(ax))}{2a} + \frac{x}{2\arccos(ax)}$$

```
[In] Int[ArcCos[a*x]^(-3),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(2*a*ArcCos[a*x]^2) + x/(2*ArcCos[a*x]) + SinIntegral[Arc
Cos[a*x]]/(2*a)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
```

```
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}+\frac{1}{2}a\int\frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^2}\,dx\\ &= \frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}+\frac{x}{2\arccos(ax)}-\frac{1}{2}\int\frac{1}{\arccos(ax)}\,dx\\ &= \frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}+\frac{x}{2\arccos(ax)}+\frac{\text{Subst}\left(\int\frac{\sin(x)}{x}\,dx,x,\arccos(ax)\right)}{2a}\\ &= \frac{\sqrt{1-a^2x^2}}{2a\arccos(ax)^2}+\frac{x}{2\arccos(ax)}+\frac{\text{Si}(\arccos(ax))}{2a}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int\frac{1}{\arccos(ax)^3}\,dx=\frac{\sqrt{1-a^2x^2}+ax\arccos(ax)+\arccos(ax)^2\text{Si}(\arccos(ax))}{2a\arccos(ax)^2}$$

```
[In] Integrate[ArcCos[a*x]^(-3),x]
[Out] (Sqrt[1 - a^2*x^2] + a*x*ArcCos[a*x] + ArcCos[a*x]^2*SinIntegral[ArcCos[a*x
]])/(2*a*ArcCos[a*x]^2)
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$\frac{\frac{\sqrt{-a^2x^2+1}}{2\arccos(ax)^2}+\frac{ax}{2\arccos(ax)}+\frac{\mathrm{Si}(\arccos(ax))}{2}}{a}$	43
default	$\frac{\frac{\sqrt{-a^2x^2+1}}{2\arccos(ax)^2}+\frac{ax}{2\arccos(ax)}+\frac{\mathrm{Si}(\arccos(ax))}{2}}{a}$	43

```
[In] int(1/arccos(a*x)^3,x,method=_RETURNVERBOSE)
[Out] 1/a*(1/2*(-a^2*x^2+1)^(1/2)/arccos(a*x)^2+1/2/arccos(a*x)*a*x+1/2*Si(arccos
(a*x)))
```

Fricas [F]

$$\int \frac{1}{\arccos(ax)^3}\,dx = \int \frac{1}{\arccos\left(ax\right)^3}\,dx$$

```
[In] integrate(1/arccos(a*x)^3,x, algorithm="fricas")
[Out] integral(arccos(a*x)^(-3), x)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^3}\,dx = \int \frac{1}{\operatorname{acos}^3\left(ax\right)}\,dx$$

```
[In] integrate(1/acos(a*x)**3,x)
[Out] Integral(acos(a*x)**(-3), x)
```

Maxima [F]

$$\int \frac{1}{\arccos(ax)^3}\,dx = \int \frac{1}{\arccos\left(ax\right)^3}\,dx$$

```
[In] integrate(1/arccos(a*x)^3,x, algorithm="maxima")
[Out] -1/2*(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate(1/arctan2(sq
rt(a*x + 1)*sqrt(-a*x + 1), a*x), x) - a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x
+ 1), a*x) - sqrt(a*x + 1)*sqrt(-a*x + 1))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a
*x + 1), a*x)^2)
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.84

$$\int \frac{1}{\arccos(ax)^3}\,dx = \frac{x}{2\,\arccos\left(ax\right)} + \frac{\mathrm{Si}\left(\arccos\left(ax\right)\right)}{2\,a} + \frac{\sqrt{-a^2x^2+1}}{2\,a\arccos\left(ax\right)^2}$$

```
[In] integrate(1/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] 1/2*x/arccos(a*x) + 1/2*sin_integral(arccos(a*x))/a + 1/2*sqrt(-a^2*x^2 + 1
)/(a*arccos(a*x)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^3}\,dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)^3}\,dx$$

```
[In] int(1/acos(a*x)^3,x)
```

```
[Out] int(1/acos(a*x)^3, x)
```

3.65 $\int \frac{1}{x \arccos(ax)^3} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arccos(ax)^3} dx = \text{Int}\left(\frac{1}{x \arccos(ax)^3}, x\right)$$

```
[Out] Unintegrable(1/x/arccos(a*x)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x \arccos(ax)^3} dx = \int \frac{1}{x \arccos(ax)^3} dx$$

```
[In] Int[1/(x*ArcCos[a*x]^3),x]
```

```
[Out] Defer[Int][1/(x*ArcCos[a*x]^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \arccos(ax)^3} dx$$

Mathematica [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^3} \, dx = \int \frac{1}{x \arccos(ax)^3} \, dx$$

```
[In] Integrate[1/(x*ArcCos[a*x]^3),x]
[Out] Integrate[1/(x*ArcCos[a*x]^3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^3} dx$$

```
[In] int(1/x/arccos(a*x)^3,x)
[Out] int(1/x/arccos(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^3} \, dx = \int \frac{1}{x \arccos(ax)^3} \, dx$$

```
[In] integrate(1/x/arccos(a*x)^3,x, algorithm="fricas")
[Out] integral(1/(x*arccos(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^3} \, dx = \int \frac{1}{x \operatorname{acos}^3(ax)} \, dx$$

```
[In] integrate(1/x/acos(a*x)**3,x)
[Out] Integral(1/(x*acos(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 124, normalized size of antiderivative = 12.40

$$\int \frac{1}{x \arccos(ax)^3} \, dx = \int \frac{1}{x \arccos{(ax)}^3} \, dx$$

```
[In] integrate(1/x/arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*x^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate(1/(x^3*ar
ctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + sqrt(a*x + 1)*sqrt(-a*x + 1
)*a*x + arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^2*x^2*arctan2(sqrt(a
*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^3} \, dx = \int \frac{1}{x \arccos{(ax)}^3} \, dx$$

```
[In] integrate(1/x/arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^3} \, dx = \int \frac{1}{x \operatorname{acos}{(a\,x)}^3} \, dx$$

```
[In] int(1/(x*acos(a*x)^3),x)
```

```
[Out] int(1/(x*acos(a*x)^3), x)
```

3.66 $\int \frac{1}{x^2 \arccos(ax)^3} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \operatorname{Int}\left(\frac{1}{x^2 \arccos(ax)^3}, x\right)$$

```
[Out] Unintegrable(1/x^2/arccos(a*x)^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \arccos(ax)^3} \, dx$$

```
[In] Int[1/(x^2*ArcCos[a*x]^3),x]
```

```
[Out] Defer[Int][1/(x^2*ArcCos[a*x]^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 \arccos(ax)^3} \, dx$$

Mathematica [N/A]

Not integrable

Time = 7.67 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \arccos(ax)^3} \, dx$$

```
[In] Integrate[1/(x^2*ArcCos[a*x]^3),x]
[Out] Integrate[1/(x^2*ArcCos[a*x]^3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.67 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \arccos{(ax)}^3} dx$$

```
[In] int(1/x^2/arccos(a*x)^3,x)
[Out] int(1/x^2/arccos(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^3} \, dx$$

```
[In] integrate(1/x^2/arccos(a*x)^3,x, algorithm="fricas")
[Out] integral(1/(x^2*arccos(a*x)^3), x)
```

Sympy [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \operatorname{acos}^3{(ax)}} \, dx$$

```
[In] integrate(1/x**2/acos(a*x)**3,x)
[Out] Integral(1/(x**2*acos(a*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 143, normalized size of antiderivative = 14.30

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^3} \, dx$$

```
[In] integrate(1/x^2/arccos(a*x)^3,x, algorithm="maxima")
[Out] -1/2*(x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2*integrate((a^2*x^2 -
 6)/(x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) - sqrt(a*x + 1)*sq
rt(-a*x + 1)*a*x + (a^2*x^2 - 2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)
)/(a^2*x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)
```

Giac [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^3} \, dx$$

```
[In] integrate(1/x^2/arccos(a*x)^3,x, algorithm="giac")
[Out] integrate(1/(x^2*arccos(a*x)^3), x)
```

Mupad [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^3} \, dx = \int \frac{1}{x^2 \,\mathrm{acos}\left(a\,x\right)^3} \, dx$$

```
[In] int(1/(x^2*acos(a*x)^3),x)
[Out] int(1/(x^2*acos(a*x)^3), x)
```

3.67 $\int \frac{x^4}{\arccos(ax)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 158

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{2x^3}{3a^2\arccos(ax)^2} + \frac{5x^5}{6\arccos(ax)^2} + \frac{2x^2\sqrt{1-a^2x^2}}{a^3\arccos(ax)} - \frac{25x^4\sqrt{1-a^2x^2}}{6a\arccos(ax)} + \frac{\text{CosIntegral}(\arccos(ax))}{48a^5} + \frac{27\,\text{CosIntegral}(3\arccos(ax))}{32a^5} + \frac{125\,\text{CosIntegral}(5\arccos(ax))}{96a^5}$$

```
[Out] -2/3*x^3/a^2/arccos(a*x)^2+5/6*x^5/arccos(a*x)^2+1/48*Ci(arccos(a*x))/a^5+2
7/32*Ci(3*arccos(a*x))/a^5+125/96*Ci(5*arccos(a*x))/a^5+1/3*x^4*(-a^2*x^2+1
)^(1/2)/a/arccos(a*x)^3+2*x^2*(-a^2*x^2+1)^(1/2)/a^3/arccos(a*x)-25/6*x^4*(
-a^2*x^2+1)^(1/2)/a/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4730, 4808, 4728, 3383}

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \frac{\text{CosIntegral}(\arccos(ax))}{48a^5} + \frac{27\,\text{CosIntegral}(3\arccos(ax))}{32a^5} + \frac{125\,\text{CosIntegral}(5\arccos(ax))}{96a^5} - \frac{2x^3}{3a^2\arccos(ax)^2} - \frac{25x^4\sqrt{1-a^2x^2}}{6a\arccos(ax)} + \frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} + \frac{2x^2\sqrt{1-a^2x^2}}{a^3\arccos(ax)} + \frac{5x^5}{6\arccos(ax)^2}$$

```
[In] Int[x^4/ArcCos[a*x]^4,x]
```

```
[Out] (x^4*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - (2*x^3)/(3*a^2*ArcCos[a*x]^2)
 + (5*x^5)/(6*ArcCos[a*x]^2) + (2*x^2*Sqrt[1 - a^2*x^2])/(a^3*ArcCos[a*x])
- (25*x^4*Sqrt[1 - a^2*x^2])/(6*a*ArcCos[a*x]) + CosIntegral[ArcCos[a*x]]/(
48*a^5) + (27*CosIntegral[3*ArcCos[a*x]])/(32*a^5) + (125*CosIntegral[5*Arc
Cos[a*x]])/(96*a^5)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{4\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx}{3a} + \frac{1}{3}(5a)\int \frac{x^5}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{2x^3}{3a^2\arccos(ax)^2} + \frac{5x^5}{6\arccos(ax)^2} - \frac{25}{6}\int \frac{x^4}{\arccos(ax)^2}\,dx + \frac{2\int \frac{x^2}{\arccos(ax)^2}\,dx}{a^2}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{2x^3}{3a^2\arccos(ax)^2} + \frac{5x^5}{6\arccos(ax)^2} + \frac{2x^2\sqrt{1-a^2x^2}}{a^3\arccos(ax)} - \frac{25x^4\sqrt{1-a^2x^2}}{6a\arccos(ax)} + \frac{2\text{Subst}\left(\int\left(-\frac{\cos(x)}{4x}-\frac{3\cos(3x)}{4x}\right)dx,x,\arccos(ax)\right)}{a^5} - \frac{25\text{Subst}\left(\int\left(-\frac{\cos(x)}{8x}-\frac{9\cos(3x)}{16x}-\frac{5\cos(5x)}{16x}\right)dx,x,\arccos(ax)\right)}{6a^5}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{2x^3}{3a^2\arccos(ax)^2} + \frac{5x^5}{6\arccos(ax)^2} + \frac{2x^2\sqrt{1-a^2x^2}}{a^3\arccos(ax)} - \frac{25x^4\sqrt{1-a^2x^2}}{6a\arccos(ax)} - \frac{\text{Subst}\left(\int\frac{\cos(x)}{x}dx,x,\arccos(ax)\right)}{2a^5} + \frac{25\text{Subst}\left(\int\frac{\cos(x)}{x}dx,x,\arccos(ax)\right)}{48a^5} + \frac{125\text{Subst}\left(\int\frac{\cos(5x)}{x}dx,x,\arccos(ax)\right)}{96a^5} - \frac{3\text{Subst}\left(\int\frac{\cos(3x)}{x}dx,x,\arccos(ax)\right)}{2a^5} + \frac{75\text{Subst}\left(\int\frac{\cos(3x)}{x}dx,x,\arccos(ax)\right)}{32a^5}$$

$$= \frac{x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{2x^3}{3a^2\arccos(ax)^2} + \frac{5x^5}{6\arccos(ax)^2} + \frac{2x^2\sqrt{1-a^2x^2}}{a^3\arccos(ax)} - \frac{25x^4\sqrt{1-a^2x^2}}{6a\arccos(ax)} + \frac{\text{CosIntegral}(\arccos(ax))}{48a^5} + \frac{27\,\text{CosIntegral}(3\arccos(ax))}{32a^5} + \frac{125\,\text{CosIntegral}(5\arccos(ax))}{96a^5}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.01

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \frac{32a^4x^4\sqrt{1-a^2x^2} - 64a^3x^3\arccos(ax) + 80a^5x^5\arccos(ax) + 192a^2x^2\sqrt{1-a^2x^2}\arccos(ax)^2 - 400a^4x^4\sqrt{1}}{}$$

```
[In] Integrate[x^4/ArcCos[a*x]^4,x]
```

```
[Out] (32*a^4*x^4*Sqrt[1 - a^2*x^2] - 64*a^3*x^3*ArcCos[a*x] + 80*a^5*x^5*ArcCos[
a*x] + 192*a^2*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2 - 400*a^4*x^4*Sqrt[1 - a
^2*x^2]*ArcCos[a*x]^2 + 2*ArcCos[a*x]^3*CosIntegral[ArcCos[a*x]] + 81*ArcCo
s[a*x]^3*CosIntegral[3*ArcCos[a*x]] + 125*ArcCos[a*x]^3*CosIntegral[5*ArcCo
s[a*x]])/(96*a^5*ArcCos[a*x]^3)
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{\frac{\sqrt{-a^2x^2+1}}{24\arccos(ax)^3}+\frac{ax}{48\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{48\arccos(ax)}+\frac{\mathrm{Ci}(\arccos(ax))}{48}+\frac{\sin(3\arccos(ax))}{16\arccos(ax)^3}+\frac{3\cos(3\arccos(ax))}{32\arccos(ax)^2}-\frac{9\sin(3\arccos(ax))}{32\arccos(ax)}+\frac{27}{}}{a^5}$
default	$\frac{\frac{\sqrt{-a^2x^2+1}}{24\arccos(ax)^3}+\frac{ax}{48\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{48\arccos(ax)}+\frac{\mathrm{Ci}(\arccos(ax))}{48}+\frac{\sin(3\arccos(ax))}{16\arccos(ax)^3}+\frac{3\cos(3\arccos(ax))}{32\arccos(ax)^2}-\frac{9\sin(3\arccos(ax))}{32\arccos(ax)}+\frac{27}{}}{a^5}$

```
[In] int(x^4/arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^5*(1/24*(-a^2*x^2+1)^(1/2)/arccos(a*x)^3+1/48/arccos(a*x)^2*a*x-1/48*(-
a^2*x^2+1)^(1/2)/arccos(a*x)+1/48*Ci(arccos(a*x))+1/16/arccos(a*x)^3*sin(3*
arccos(a*x))+3/32/arccos(a*x)^2*cos(3*arccos(a*x))-9/32/arccos(a*x)*sin(3*a
rccos(a*x))+27/32*Ci(3*arccos(a*x))+1/48/arccos(a*x)^3*sin(5*arccos(a*x))+5
/96/arccos(a*x)^2*cos(5*arccos(a*x))-25/96/arccos(a*x)*sin(5*arccos(a*x))+1
25/96*Ci(5*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \int \frac{x^4}{\arccos\left(ax\right)^4}\,dx$$

```
[In] integrate(x^4/arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^4/arccos(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \int \frac{x^4}{\operatorname{acos}^4\left(ax\right)}\,dx$$

```
[In] integrate(x**4/acos(a*x)**4,x)
```

```
[Out] Integral(x**4/acos(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \int \frac{x^4}{\arccos\left(ax\right)^4}\,dx$$

```
[In] integrate(x^4/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6*(125*
a^4*x^5 - 136*a^2*x^3 + 24*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^2 - a^3)
*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + (2*a^2*x^4 - (25*a^2*x^4
 - 12*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*sqrt
(-a*x + 1) + (5*a^3*x^5 - 4*a*x^3)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*
x))/(a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.87

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \frac{5\,x^5}{6\,\arccos\left(ax\right)^2} - \frac{25\,\sqrt{-a^2x^2+1}x^4}{6\,a\arccos\left(ax\right)} + \frac{\sqrt{-a^2x^2+1}x^4}{3\,a\arccos\left(ax\right)^3} - \frac{2\,x^3}{3\,a^2\arccos\left(ax\right)^2} + \frac{2\,\sqrt{-a^2x^2+1}x^2}{a^3\arccos\left(ax\right)} + \frac{125\,\mathrm{Ci}\left(5\,\arccos\left(ax\right)\right)}{96\,a^5} + \frac{27\,\mathrm{Ci}\left(3\,\arccos\left(ax\right)\right)}{32\,a^5} + \frac{\mathrm{Ci}\left(\arccos\left(ax\right)\right)}{48\,a^5}$$

```
[In] integrate(x^4/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 5/6*x^5/arccos(a*x)^2 - 25/6*sqrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)) + 1/3*s
qrt(-a^2*x^2 + 1)*x^4/(a*arccos(a*x)^3) - 2/3*x^3/(a^2*arccos(a*x)^2) + 2*s
qrt(-a^2*x^2 + 1)*x^2/(a^3*arccos(a*x)) + 125/96*cos_integral(5*arccos(a*x)
)/a^5 + 27/32*cos_integral(3*arccos(a*x))/a^5 + 1/48*cos_integral(arccos(a*
x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^4}\,dx = \int \frac{x^4}{{\mathrm{acos}\left(a\,x\right)}^4}\,dx$$

```
[In] int(x^4/acos(a*x)^4,x)
```

```
[Out] int(x^4/acos(a*x)^4, x)
```

3.68 $\int \frac{x^3}{\arccos(ax)^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 143

$$\int \frac{x^3}{\arccos(ax)^4} dx = \frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x^2}{2a^2\arccos(ax)^2} + \frac{2x^4}{3\arccos(ax)^2} + \frac{x\sqrt{1-a^2x^2}}{a^3\arccos(ax)} - \frac{8x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)} + \frac{\text{CosIntegral}(2\arccos(ax))}{3a^4} + \frac{4\,\text{CosIntegral}(4\arccos(ax))}{3a^4}$$

```
[Out] -1/2*x^2/a^2/arccos(a*x)^2+2/3*x^4/arccos(a*x)^2+1/3*Ci(2*arccos(a*x))/a^4+
4/3*Ci(4*arccos(a*x))/a^4+1/3*x^3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^3+x*(-a^
2*x^2+1)^(1/2)/a^3/arccos(a*x)-8/3*x^3*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4730, 4808, 4728, 3383}

$$\int \frac{x^3}{\arccos(ax)^4} dx = \frac{\text{CosIntegral}(2\arccos(ax))}{3a^4} + \frac{4\,\text{CosIntegral}(4\arccos(ax))}{3a^4} - \frac{x^2}{2a^2\arccos(ax)^2} - \frac{8x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)} + \frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} + \frac{x\sqrt{1-a^2x^2}}{a^3\arccos(ax)} + \frac{2x^4}{3\arccos(ax)^2}$$

```
[In] Int[x^3/ArcCos[a*x]^4,x]
```

```
[Out] (x^3*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - x^2/(2*a^2*ArcCos[a*x]^2) + (
2*x^4)/(3*ArcCos[a*x]^2) + (x*Sqrt[1 - a^2*x^2])/(a^3*ArcCos[a*x]) - (8*x^3
*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]) + CosIntegral[2*ArcCos[a*x]]/(3*a^4)
+ (4*CosIntegral[4*ArcCos[a*x]])/(3*a^4)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx}{a} + \frac{1}{3}(4a)\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx \\ &= \frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x^2}{2a^2\arccos(ax)^2} + \frac{2x^4}{3\arccos(ax)^2} - \frac{8}{3}\int \frac{x^3}{\arccos(ax)^2}\,dx + \frac{\int \frac{x}{\arccos(ax)^2}\,dx}{a^2}\end{aligned}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x^2}{2a^2\arccos(ax)^2} + \frac{2x^4}{3\arccos(ax)^2} + \frac{x\sqrt{1-a^2x^2}}{a^3\arccos(ax)} - \frac{8x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \arccos(ax)\right)}{a^4} - \frac{8\text{Subst}\left(\int \left(-\frac{\cos(2x)}{2x} - \frac{\cos(4x)}{2x}\right)\,dx, x, \arccos(ax)\right)}{3a^4}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x^2}{2a^2\arccos(ax)^2} + \frac{2x^4}{3\arccos(ax)^2} + \frac{x\sqrt{1-a^2x^2}}{a^3\arccos(ax)} - \frac{8x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)} - \frac{\text{CosIntegral}(2\arccos(ax))}{a^4} + \frac{4\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \arccos(ax)\right)}{3a^4} + \frac{4\text{Subst}\left(\int \frac{\cos(4x)}{x}\,dx, x, \arccos(ax)\right)}{3a^4}$$

$$= \frac{x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x^2}{2a^2\arccos(ax)^2} + \frac{2x^4}{3\arccos(ax)^2} + \frac{x\sqrt{1-a^2x^2}}{a^3\arccos(ax)} - \frac{8x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)} + \frac{\text{CosIntegral}(2\arccos(ax))}{3a^4} + \frac{4\,\text{CosIntegral}(4\arccos(ax))}{3a^4}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.75

$$\int \frac{x^3}{\arccos(ax)^4}\,dx = \frac{\frac{ax\left(2a^2x^2\sqrt{1-a^2x^2}+ax(-3+4a^2x^2)\arccos(ax)-2\sqrt{1-a^2x^2}(-3+8a^2x^2)\arccos(ax)^2\right)}{\arccos(ax)^3} + 2\,\text{CosIntegral}(2\arccos(ax)) + 8\,\text{CosIn}}{6a^4}$$

```
[In] Integrate[x^3/ArcCos[a*x]^4,x]
```

```
[Out] ((a*x*(2*a^2*x^2*Sqrt[1 - a^2*x^2] + a*x*(-3 + 4*a^2*x^2)*ArcCos[a*x] - 2*S
qrt[1 - a^2*x^2]*(-3 + 8*a^2*x^2)*ArcCos[a*x]^2))/ArcCos[a*x]^3 + 2*CosInte
gral[2*ArcCos[a*x]] + 8*CosIntegral[4*ArcCos[a*x]])/(6*a^4)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{12\arccos(ax)^3}+\frac{\cos(2\arccos(ax))}{12\arccos(ax)^2}-\frac{\sin(2\arccos(ax))}{6\arccos(ax)}+\frac{\text{Ci}(2\arccos(ax))}{3}+\frac{\sin(4\arccos(ax))}{24\arccos(ax)^3}+\frac{\cos(4\arccos(ax))}{12\arccos(ax)^2}-\frac{\sin(4\arccos(ax))}{3\arccos(ax)}}{a^4}$
default	$\frac{\frac{\sin(2\arccos(ax))}{12\arccos(ax)^3}+\frac{\cos(2\arccos(ax))}{12\arccos(ax)^2}-\frac{\sin(2\arccos(ax))}{6\arccos(ax)}+\frac{\text{Ci}(2\arccos(ax))}{3}+\frac{\sin(4\arccos(ax))}{24\arccos(ax)^3}+\frac{\cos(4\arccos(ax))}{12\arccos(ax)^2}-\frac{\sin(4\arccos(ax))}{3\arccos(ax)}}{a^4}$

```
[In] int(x^3/arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^4*(1/12/arccos(a*x)^3*sin(2*arccos(a*x))+1/12/arccos(a*x)^2*cos(2*arcco
s(a*x))-1/6/arccos(a*x)*sin(2*arccos(a*x))+1/3*Ci(2*arccos(a*x))+1/24/arcco
s(a*x)^3*sin(4*arccos(a*x))+1/12/arccos(a*x)^2*cos(4*arccos(a*x))-1/3/arcco
s(a*x)*sin(4*arccos(a*x))+4/3*Ci(4*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x^3}{\arccos(ax)^4}\,dx = \int \frac{x^3}{\arccos\left(ax\right)^4}\,dx$$

```
[In] integrate(x^3/arccos(a*x)^4,x, algorithm="fricas")
```

```
[Out] integral(x^3/arccos(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^4}\,dx = \int \frac{x^3}{\operatorname{acos}^4\left(ax\right)}\,dx$$

```
[In] integrate(x**3/acos(a*x)**4,x)
```

```
[Out] Integral(x**3/acos(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x^3}{\arccos(ax)^4}\,dx = \int \frac{x^3}{\arccos\left(ax\right)^4}\,dx$$

```
[In] integrate(x^3/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/3*(32*a
^4*x^4 - 30*a^2*x^2 + 3)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^2 - a^3)*arct
an2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + 2*(a^2*x^3 - (8*a^2*x^3 - 3*x
)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*sqrt(-a*x + 1
) + (4*a^3*x^4 - 3*a*x^2)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^3*
arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.87

$$\int \frac{x^3}{\arccos(ax)^4}\,dx = \frac{2\,x^4}{3\,\arccos\left(ax\right)^2} - \frac{8\,\sqrt{-a^2x^2+1}x^3}{3\,a\arccos\left(ax\right)} + \frac{\sqrt{-a^2x^2+1}x^3}{3\,a\arccos\left(ax\right)^3} - \frac{x^2}{2\,a^2\arccos\left(ax\right)^2} + \frac{\sqrt{-a^2x^2+1}x}{a^3\arccos\left(ax\right)} + \frac{4\,\mathrm{Ci}\left(4\,\arccos\left(ax\right)\right)}{3\,a^4} + \frac{\mathrm{Ci}\left(2\,\arccos\left(ax\right)\right)}{3\,a^4}$$

```
[In] integrate(x^3/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 2/3*x^4/arccos(a*x)^2 - 8/3*sqrt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)) + 1/3*sq
rt(-a^2*x^2 + 1)*x^3/(a*arccos(a*x)^3) - 1/2*x^2/(a^2*arccos(a*x)^2) + sqrt
(-a^2*x^2 + 1)*x/(a^3*arccos(a*x)) + 4/3*cos_integral(4*arccos(a*x))/a^4 +
1/3*cos_integral(2*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^4}\,dx = \int \frac{x^3}{\mathrm{acos}\left(a\,x\right)^4}\,dx$$

```
[In] int(x^3/acos(a*x)^4,x)
```

```
[Out] int(x^3/acos(a*x)^4, x)
```

3.69 $\int \frac{x^2}{\arccos(ax)^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 141

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x}{3a^2\arccos(ax)^2} + \frac{x^3}{2\arccos(ax)^2} + \frac{\sqrt{1-a^2x^2}}{3a^3\arccos(ax)} - \frac{3x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)} + \frac{\text{CosIntegral}(\arccos(ax))}{24a^3} + \frac{9\,\text{CosIntegral}(3\arccos(ax))}{8a^3}$$

```
[Out] -1/3*x/a^2/arccos(a*x)^2+1/2*x^3/arccos(a*x)^2+1/24*Ci(arccos(a*x))/a^3+9/8
*Ci(3*arccos(a*x))/a^3+1/3*x^2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^3+1/3*(-a^2
*x^2+1)^(1/2)/a^3/arccos(a*x)-3/2*x^2*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4730, 4808, 4728, 3383, 4718, 4810}

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \frac{\text{CosIntegral}(\arccos(ax))}{24a^3} + \frac{9\,\text{CosIntegral}(3\arccos(ax))}{8a^3} - \frac{3x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)} + \frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x}{3a^2\arccos(ax)^2} + \frac{\sqrt{1-a^2x^2}}{3a^3\arccos(ax)} + \frac{x^3}{2\arccos(ax)^2}$$

```
[In] Int[x^2/ArcCos[a*x]^4,x]
```

```
[Out] (x^2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - x/(3*a^2*ArcCos[a*x]^2) + x^3
/(2*ArcCos[a*x]^2) + Sqrt[1 - a^2*x^2]/(3*a^3*ArcCos[a*x]) - (3*x^2*Sqrt[1
```

```
- a^2*x^2])/(2*a*ArcCos[a*x]) + CosIntegral[ArcCos[a*x]]/(24*a^3) + (9*CosI
ntegral[3*ArcCos[a*x]])/(8*a^3)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
```

```
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{2\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx}{3a} + a\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx \\
&= \frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x}{3a^2\arccos(ax)^2} + \frac{x^3}{2\arccos(ax)^2} - \frac{3}{2}\int \frac{x^2}{\arccos(ax)^2}\,dx + \frac{\int \frac{1}{\arccos(ax)^2}\,dx}{3a^2} \\
&= \frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x}{3a^2\arccos(ax)^2} + \frac{x^3}{2\arccos(ax)^2} + \frac{\sqrt{1-a^2x^2}}{3a^3\arccos(ax)} - \frac{3x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)} \\
&\quad - \frac{3\text{Subst}\left(\int \left(-\frac{\cos(x)}{4x} - \frac{3\cos(3x)}{4x}\right)\,dx, x, \arccos(ax)\right)}{2a^3} + \frac{\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx}{3a} \\
&= \frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x}{3a^2\arccos(ax)^2} + \frac{x^3}{2\arccos(ax)^2} + \frac{\sqrt{1-a^2x^2}}{3a^3\arccos(ax)} \\
&\quad - \frac{3x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \arccos(ax)\right)}{3a^3} \\
&\quad + \frac{3\text{Subst}\left(\int \frac{\cos(x)}{x}\,dx, x, \arccos(ax)\right)}{8a^3} + \frac{9\text{Subst}\left(\int \frac{\cos(3x)}{x}\,dx, x, \arccos(ax)\right)}{8a^3} \\
&= \frac{x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{x}{3a^2\arccos(ax)^2} + \frac{x^3}{2\arccos(ax)^2} + \frac{\sqrt{1-a^2x^2}}{3a^3\arccos(ax)} \\
&\quad - \frac{3x^2\sqrt{1-a^2x^2}}{2a\arccos(ax)} + \frac{\text{CosIntegral}(\arccos(ax))}{24a^3} + \frac{9\,\text{CosIntegral}(3\arccos(ax))}{8a^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.79

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \frac{\frac{8a^2x^2\sqrt{1-a^2x^2}}{\arccos(ax)^3} + \frac{4ax(-2+3a^2x^2)}{\arccos(ax)^2} - \frac{4\sqrt{1-a^2x^2}(-2+9a^2x^2)}{\arccos(ax)} - 80\,\text{CosIntegral}(\arccos(ax)) + 27(3\,\text{CosIntegral}(\arccos(}{24a^3}$$

```
[In] Integrate[x^2/ArcCos[a*x]^4,x]
```

```
[Out] ((8*a^2*x^2*Sqrt[1 - a^2*x^2])/ArcCos[a*x]^3 + (4*a*x*(-2 + 3*a^2*x^2))/Arc
Cos[a*x]^2 - (4*Sqrt[1 - a^2*x^2]*(-2 + 9*a^2*x^2))/ArcCos[a*x] - 80*CosInt
egral[ArcCos[a*x]] + 27*(3*CosIntegral[ArcCos[a*x]] + CosIntegral[3*ArcCos[
a*x]]))/(24*a^3)
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.83

method	result
derivativedivides	$\frac{\frac{\sqrt{-a^2x^2+1}}{12\arccos(ax)^3}+\frac{ax}{24\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{24\arccos(ax)}+\frac{\mathrm{Ci}(\arccos(ax))}{24}+\frac{\sin(3\arccos(ax))}{12\arccos(ax)^3}+\frac{\cos(3\arccos(ax))}{8\arccos(ax)^2}-\frac{3\sin(3\arccos(ax))}{8\arccos(ax)}+\frac{9\ \mathrm{Ci}(}{}}{a^3}$
default	$\frac{\frac{\sqrt{-a^2x^2+1}}{12\arccos(ax)^3}+\frac{ax}{24\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{24\arccos(ax)}+\frac{\mathrm{Ci}(\arccos(ax))}{24}+\frac{\sin(3\arccos(ax))}{12\arccos(ax)^3}+\frac{\cos(3\arccos(ax))}{8\arccos(ax)^2}-\frac{3\sin(3\arccos(ax))}{8\arccos(ax)}+\frac{9\ \mathrm{Ci}(}{}}{a^3}$

[In] int(x^2/arccos(a*x)^4,x,method=_RETURNVERBOSE)

[Out] 1/a^3*(1/12*(-a^2*x^2+1)^(1/2)/arccos(a*x)^3+1/24/arccos(a*x)^2*a*x-1/24*(-a^2*x^2+1)^(1/2)/arccos(a*x)+1/24*Ci(arccos(a*x))+1/12/arccos(a*x)^3*sin(3*arccos(a*x))+1/8/arccos(a*x)^2*cos(3*arccos(a*x))-3/8/arccos(a*x)*sin(3*arccos(a*x))+9/8*Ci(3*arccos(a*x)))

Fricas [F]

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \int \frac{x^2}{\arccos\left(ax\right)^4}\,dx$$

[In] integrate(x^2/arccos(a*x)^4,x, algorithm="fricas")

[Out] integral(x^2/arccos(a*x)^4, x)

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \int \frac{x^2}{\operatorname{acos}^4\left(ax\right)}\,dx$$

[In] integrate(x**2/acos(a*x)**4,x)

[Out] Integral(x**2/acos(a*x)**4, x)

Maxima [F]

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \int \frac{x^2}{\arccos{(ax)}^4}\,dx$$

```
[In] integrate(x^2/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6*(27*a
^2*x^3 - 20*x)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2 - a)*arctan2(sqrt(a*x
 + 1)*sqrt(-a*x + 1), a*x)), x) + (2*a^2*x^2 - (9*a^2*x^2 - 2)*arctan2(sqrt
(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*sqrt(-a*x + 1) + (3*a^3*x^3
 - 2*a*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^3*arctan2(sqrt(a*x
 + 1)*sqrt(-a*x + 1), a*x)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \frac{x^3}{2\arccos{(ax)}^2} - \frac{3\sqrt{-a^2x^2+1}x^2}{2\,a\arccos{(ax)}} + \frac{\sqrt{-a^2x^2+1}x^2}{3\,a\arccos{(ax)}^3} + \frac{9\,\mathrm{Ci}\left(3\arccos{(ax)}\right)}{8\,a^3} + \frac{\mathrm{Ci}\left(\arccos{(ax)}\right)}{24\,a^3} - \frac{x}{3\,a^2\arccos{(ax)}^2} + \frac{\sqrt{-a^2x^2+1}}{3\,a^3\arccos{(ax)}}$$

```
[In] integrate(x^2/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/2*x^3/arccos(a*x)^2 - 3/2*sqrt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)) + 1/3*sq
rt(-a^2*x^2 + 1)*x^2/(a*arccos(a*x)^3) + 9/8*cos_integral(3*arccos(a*x))/a^
3 + 1/24*cos_integral(arccos(a*x))/a^3 - 1/3*x/(a^2*arccos(a*x)^2) + 1/3*sq
rt(-a^2*x^2 + 1)/(a^3*arccos(a*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^4}\,dx = \int \frac{x^2}{{\mathrm{acos}\left(a\,x\right)}^4}\,dx$$

```
[In] int(x^2/acos(a*x)^4,x)
```

```
[Out] int(x^2/acos(a*x)^4, x)
```

3.70 $\int \frac{x}{\arccos(ax)^4}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 97

$$\int \frac{x}{\arccos(ax)^4}\,dx = \frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{1}{6a^2\arccos(ax)^2} + \frac{x^2}{3\arccos(ax)^2} - \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)} + \frac{2\,\text{CosIntegral}(2\arccos(ax))}{3a^2}$$

```
[Out] -1/6/a^2/arccos(a*x)^2+1/3*x^2/arccos(a*x)^2+2/3*Ci(2*arccos(a*x))/a^2+1/3*
x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^3-2/3*x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4730, 4808, 4728, 3383, 4738}

$$\int \frac{x}{\arccos(ax)^4}\,dx = \frac{2\,\text{CosIntegral}(2\arccos(ax))}{3a^2} - \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)} + \frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{1}{6a^2\arccos(ax)^2} + \frac{x^2}{3\arccos(ax)^2}$$

```
[In] Int[x/ArcCos[a*x]^4,x]
```

```
[Out] (x*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^3) - 1/(6*a^2*ArcCos[a*x]^2) + x^2/(
3*ArcCos[a*x]^2) - (2*x*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]) + (2*CosIntegr
al[2*ArcCos[a*x]])/(3*a^2)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx}{3a} + \frac{1}{3}(2a)\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx \\ &= \frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{1}{6a^2\arccos(ax)^2} + \frac{x^2}{3\arccos(ax)^2} - \frac{2}{3}\int \frac{x}{\arccos(ax)^2}\,dx\end{aligned}$$

$$= \frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{1}{6a^2\arccos(ax)^2} + \frac{x^2}{3\arccos(ax)^2} - \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)} + \frac{2\text{Subst}\left(\int \frac{\cos(2x)}{x}\,dx, x, \arccos(ax)\right)}{3a^2}$$

$$= \frac{x\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} - \frac{1}{6a^2\arccos(ax)^2} + \frac{x^2}{3\arccos(ax)^2} - \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)} + \frac{2\,\text{CosIntegral}(2\arccos(ax))}{3a^2}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.89

$$\int \frac{x}{\arccos(ax)^4}\,dx = \frac{2ax\sqrt{1-a^2x^2} + (-1+2a^2x^2)\arccos(ax) - 4ax\sqrt{1-a^2x^2}\arccos(ax)^2 + 4\arccos(ax)^3\,\text{CosIntegral}(2\,\text{arc}}{6a^2\arccos(ax)^3}$$

```
[In] Integrate[x/ArcCos[a*x]^4,x]
```

```
[Out] (2*a*x*Sqrt[1 - a^2*x^2] + (-1 + 2*a^2*x^2)*ArcCos[a*x] - 4*a*x*Sqrt[1 - a^
2*x^2]*ArcCos[a*x]^2 + 4*ArcCos[a*x]^3*CosIntegral[2*ArcCos[a*x]])/(6*a^2*A
rcCos[a*x]^3)
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(ax))}{6\arccos(ax)^3}+\frac{\cos(2\arccos(ax))}{6\arccos(ax)^2}-\frac{\sin(2\arccos(ax))}{3\arccos(ax)}+\frac{2\ \text{Ci}(2\arccos(ax))}{3}}{a^2}$	60
default	$\frac{\frac{\sin(2\arccos(ax))}{6\arccos(ax)^3}+\frac{\cos(2\arccos(ax))}{6\arccos(ax)^2}-\frac{\sin(2\arccos(ax))}{3\arccos(ax)}+\frac{2\ \text{Ci}(2\arccos(ax))}{3}}{a^2}$	60

```
[In] int(x/arccos(a*x)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*(1/6/arccos(a*x)^3*sin(2*arccos(a*x))+1/6/arccos(a*x)^2*cos(2*arccos(
a*x))-1/3/arccos(a*x)*sin(2*arccos(a*x))+2/3*Ci(2*arccos(a*x)))
```

Fricas [F]

$$\int \frac{x}{\arccos(ax)^4}\,dx = \int \frac{x}{\arccos{(ax)}^4}\,dx$$

```
[In] integrate(x/arccos(a*x)^4,x, algorithm="fricas")
[Out] integral(x/arccos(a*x)^4, x)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)^4}\,dx = \int \frac{x}{\operatorname{acos}^4{(ax)}}\,dx$$

```
[In] integrate(x/acos(a*x)**4,x)
[Out] Integral(x/acos(a*x)**4, x)
```

Maxima [F]

$$\int \frac{x}{\arccos(ax)^4}\,dx = \int \frac{x}{\arccos{(ax)}^4}\,dx$$

```
[In] integrate(x/arccos(a*x)^4,x, algorithm="maxima")
[Out] 1/6*(6*a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(2/3*(2*a^
2*x^2 - 1)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^3*x^2 - a)*arctan2(sqrt(a*x + 1
)*sqrt(-a*x + 1), a*x)), x) - 2*(2*a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1)
, a*x)^2 - a*x)*sqrt(a*x + 1)*sqrt(-a*x + 1) + (2*a^2*x^2 - 1)*arctan2(sqrt
(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
a*x)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

$$\int \frac{x}{\arccos(ax)^4}\,dx = \frac{x^2}{3\arccos{(ax)}^2} - \frac{2\sqrt{-a^2x^2+1}x}{3\,a\arccos{(ax)}} + \frac{2\operatorname{Ci}{(2\arccos{(ax)})}}{3\,a^2} + \frac{\sqrt{-a^2x^2+1}x}{3\,a\arccos{(ax)}^3} - \frac{1}{6\,a^2\arccos{(ax)}^2}$$

```
[In] integrate(x/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/3*x^2/arccos(a*x)^2 - 2/3*sqrt(-a^2*x^2 + 1)*x/(a*arccos(a*x)) + 2/3*cos_
integral(2*arccos(a*x))/a^2 + 1/3*sqrt(-a^2*x^2 + 1)*x/(a*arccos(a*x)^3) -
1/6/(a^2*arccos(a*x)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^4}\,dx = \int \frac{x}{\mathrm{acos}\left(a\,x\right)^4}\,dx$$

```
[In] int(x/acos(a*x)^4,x)
```

```
[Out] int(x/acos(a*x)^4, x)
```

3.71 $\int \frac{1}{\arccos(ax)^4} dx$

Optimal result

Integrand size = 6, antiderivative size = 78

$$\int \frac{1}{\arccos(ax)^4} dx = \frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} + \frac{x}{6\arccos(ax)^2} - \frac{\sqrt{1-a^2x^2}}{6a\arccos(ax)} + \frac{\text{CosIntegral}(\arccos(ax))}{6a}$$

```
[Out] 1/6*x/arccos(a*x)^2+1/6*Ci(arccos(a*x))/a+1/3*(-a^2*x^2+1)^(1/2)/a/arccos(a
*x)^3-1/6*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4718, 4808, 4810, 3383}

$$\int \frac{1}{\arccos(ax)^4} dx = -\frac{\sqrt{1-a^2x^2}}{6a\arccos(ax)} + \frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3} + \frac{\text{CosIntegral}(\arccos(ax))}{6a} + \frac{x}{6\arccos(ax)^2}$$

```
[In] Int[ArcCos[a*x]^(-4),x]
```

```
[Out] Sqrt[1 - a^2*x^2]/(3*a*ArcCos[a*x]^3) + x/(6*ArcCos[a*x]^2) - Sqrt[1 - a^2*
x^2]/(6*a*ArcCos[a*x]) + CosIntegral[ArcCos[a*x]]/(6*a)
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+\frac{1}{3}a\int\frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^3}\,dx\\
&= \frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+\frac{x}{6\arccos(ax)^2}-\frac{1}{6}\int\frac{1}{\arccos(ax)^2}\,dx\\
&= \frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+\frac{x}{6\arccos(ax)^2}-\frac{\sqrt{1-a^2x^2}}{6a\arccos(ax)}-\frac{1}{6}a\int\frac{x}{\sqrt{1-a^2x^2}\arccos(ax)}\,dx\\
&= \frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+\frac{x}{6\arccos(ax)^2}-\frac{\sqrt{1-a^2x^2}}{6a\arccos(ax)}+\frac{\text{Subst}\left(\int\frac{\cos(x)}{x}\,dx,x,\arccos(ax)\right)}{6a}\\
&= \frac{\sqrt{1-a^2x^2}}{3a\arccos(ax)^3}+\frac{x}{6\arccos(ax)^2}-\frac{\sqrt{1-a^2x^2}}{6a\arccos(ax)}+\frac{\text{CosIntegral}(\arccos(ax))}{6a}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.91

$$\int \frac{1}{\arccos(ax)^4} \, dx = \frac{2\sqrt{1-a^2x^2} + ax\arccos(ax) - \sqrt{1-a^2x^2}\arccos(ax)^2 + \arccos(ax)^3 \operatorname{CosIntegral}(\arccos(ax))}{6a\arccos(ax)^3}$$

```
[In] Integrate[ArcCos[a*x]^(-4),x]
[Out] (2*Sqrt[1 - a^2*x^2] + a*x*ArcCos[a*x] - Sqrt[1 - a^2*x^2]*ArcCos[a*x]^2 +
ArcCos[a*x]^3*CosIntegral[ArcCos[a*x]])/(6*a*ArcCos[a*x]^3)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\frac{\sqrt{-a^2x^2+1}}{3\arccos(ax)^3}+\frac{ax}{6\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{6\arccos(ax)}+\frac{\operatorname{Ci}(\arccos(ax))}{6}}{a}$	63
default	$\frac{\frac{\sqrt{-a^2x^2+1}}{3\arccos(ax)^3}+\frac{ax}{6\arccos(ax)^2}-\frac{\sqrt{-a^2x^2+1}}{6\arccos(ax)}+\frac{\operatorname{Ci}(\arccos(ax))}{6}}{a}$	63

```
[In] int(1/arccos(a*x)^4,x,method=_RETURNVERBOSE)
[Out] 1/a*(1/3*(-a^2*x^2+1)^(1/2)/arccos(a*x)^3+1/6/arccos(a*x)^2*a*x-1/6*(-a^2*x
^2+1)^(1/2)/arccos(a*x)+1/6*Ci(arccos(a*x)))
```

Fricas [F]

$$\int \frac{1}{\arccos(ax)^4} \, dx = \int \frac{1}{\arccos\left(ax\right)^4} \, dx$$

```
[In] integrate(1/arccos(a*x)^4,x, algorithm="fricas")
[Out] integral(arccos(a*x)^(-4), x)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^4}\,dx = \int \frac{1}{\operatorname{acos}^4(ax)}\,dx$$

```
[In] integrate(1/acos(a*x)**4,x)
```

```
[Out] Integral(acos(a*x)**(-4), x)
```

Maxima [F]

$$\int \frac{1}{\arccos(ax)^4}\,dx = \int \frac{1}{\arccos(ax)^4}\,dx$$

```
[In] integrate(1/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^2*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6*sqrt(
a*x + 1)*sqrt(-a*x + 1)*x/((a^2*x^2 - 1)*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
1), a*x)), x) + a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x) - sqrt(a*x +
 1)*sqrt(-a*x + 1)*(arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2 - 2))/(a*a
rctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.85

$$\int \frac{1}{\arccos(ax)^4}\,dx = \frac{\operatorname{Ci}(\arccos(ax))}{6a} + \frac{x}{6\arccos(ax)^2} - \frac{\sqrt{-a^2x^2+1}}{6a\arccos(ax)} + \frac{\sqrt{-a^2x^2+1}}{3a\arccos(ax)^3}$$

```
[In] integrate(1/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] 1/6*cos_integral(arccos(a*x))/a + 1/6*x/arccos(a*x)^2 - 1/6*sqrt(-a^2*x^2 +
 1)/(a*arccos(a*x)) + 1/3*sqrt(-a^2*x^2 + 1)/(a*arccos(a*x)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^4}\,dx = \int \frac{1}{\operatorname{acos}\left(a\,x\right)^4}\,dx$$

```
[In] int(1/acos(a*x)^4,x)
[Out] int(1/acos(a*x)^4, x)
```

3.72 $\int \frac{1}{x \arccos(ax)^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x \arccos(ax)^4} dx = \operatorname{Int}\left(\frac{1}{x \arccos(ax)^4}, x\right)$$

```
[Out] Unintegrable(1/x/arccos(a*x)^4,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x \arccos(ax)^4} dx = \int \frac{1}{x \arccos(ax)^4} dx$$

```
[In] Int[1/(x*ArcCos[a*x]^4),x]
```

```
[Out] Defer[Int][1/(x*ArcCos[a*x]^4), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \arccos(ax)^4} dx$$

Mathematica [N/A]

Not integrable

Time = 3.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arccos(ax)^4}\,dx = \int \frac{1}{x\arccos(ax)^4}\,dx$$

```
[In] Integrate[1/(x*ArcCos[a*x]^4),x]
[Out] Integrate[1/(x*ArcCos[a*x]^4), x]
```

Maple [N/A] (verified)

Not integrable

Time = 3.00 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\arccos\left(ax\right)^{4}}dx$$

```
[In] int(1/x/arccos(a*x)^4,x)
[Out] int(1/x/arccos(a*x)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x\arccos(ax)^4}\,dx = \int \frac{1}{x\arccos\left(ax\right)^{4}}\,dx$$

```
[In] integrate(1/x/arccos(a*x)^4,x, algorithm="fricas")
[Out] integral(1/(x*arccos(a*x)^4), x)
```

Sympy [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\arccos(ax)^4}\,dx = \int \frac{1}{x\operatorname{acos}^{4}\left(ax\right)}\,dx$$

```
[In] integrate(1/x/acos(a*x)**4,x)
[Out] Integral(1/(x*acos(a*x)**4), x)
```

Maxima [N/A]

Not integrable

Time = 4.42 (sec) , antiderivative size = 200, normalized size of antiderivative = 20.00

$$\int \frac{1}{x \arccos(ax)^4} \, dx = \int \frac{1}{x \arccos{(ax)}^4} \, dx$$

```
[In] integrate(1/x/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^3*x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/3*(
2*a^2*x^2 - 3)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^6 - a^3*x^4)*arctan2(sq
rt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) + a*x*arctan2(sqrt(a*x + 1)*sqrt(-a*x
 + 1), a*x) + 2*(a^2*x^2 + arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sq
rt(a*x + 1)*sqrt(-a*x + 1))/(a^3*x^3*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1),
a*x)^3)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^4} \, dx = \int \frac{1}{x \arccos{(ax)}^4} \, dx$$

```
[In] integrate(1/x/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)^4), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x \arccos(ax)^4} \, dx = \int \frac{1}{x \operatorname{acos}{(a\,x)}^4} \, dx$$

```
[In] int(1/(x*acos(a*x)^4),x)
```

```
[Out] int(1/(x*acos(a*x)^4), x)
```

3.73 $\int \frac{1}{x^2 \arccos(ax)^4} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{1}{x^2 \arccos(ax)^4} dx = \operatorname{Int}\left(\frac{1}{x^2 \arccos(ax)^4}, x\right)$$

```
[Out] Unintegrable(1/x^2/arccos(a*x)^4,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2 \arccos(ax)^4} dx = \int \frac{1}{x^2 \arccos(ax)^4} dx$$

```
[In] Int[1/(x^2*ArcCos[a*x]^4),x]
```

```
[Out] Defer[Int][1/(x^2*ArcCos[a*x]^4), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2 \arccos(ax)^4} dx$$

Mathematica [N/A]

Not integrable

Time = 17.34 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \arccos(ax)^4} \, dx$$

```
[In] Integrate[1/(x^2*ArcCos[a*x]^4),x]
[Out] Integrate[1/(x^2*ArcCos[a*x]^4), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.62 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2 \arccos{(ax)}^4} dx$$

```
[In] int(1/x^2/arccos(a*x)^4,x)
[Out] int(1/x^2/arccos(a*x)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^4} \, dx$$

```
[In] integrate(1/x^2/arccos(a*x)^4,x, algorithm="fricas")
[Out] integral(1/(x^2*arccos(a*x)^4), x)
```

Sympy [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \operatorname{acos}^4{(ax)}} \, dx$$

```
[In] integrate(1/x**2/acos(a*x)**4,x)
[Out] Integral(1/(x**2*acos(a*x)**4), x)
```

Maxima [N/A]

Not integrable

Time = 5.04 (sec) , antiderivative size = 229, normalized size of antiderivative = 22.90

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^4} \, dx$$

```
[In] integrate(1/x^2/arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] -1/6*(6*a^3*x^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3*integrate(1/6*
(a^4*x^4 - 20*a^2*x^2 + 24)*sqrt(a*x + 1)*sqrt(-a*x + 1)/((a^5*x^7 - a^3*x^
5)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)), x) - (2*a^2*x^2 - (a^2*x^2
- 6)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2)*sqrt(a*x + 1)*sqrt(-a*x
+ 1) + (a^3*x^3 - 2*a*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))/(a^3*x
^4*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3)
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \arccos{(ax)}^4} \, dx$$

```
[In] integrate(1/x^2/arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate(1/(x^2*arccos(a*x)^4), x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{1}{x^2 \arccos(ax)^4} \, dx = \int \frac{1}{x^2 \, \mathrm{acos}{(a\,x)}^4} \, dx$$

```
[In] int(1/(x^2*acos(a*x)^4),x)
```

```
[Out] int(1/(x^2*acos(a*x)^4), x)
```

3.74 $\int x^4 \sqrt{\arccos(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 121

$$\int x^4 \sqrt{\arccos(ax)}\,dx = \frac{1}{5}x^5\sqrt{\arccos(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^5} - \frac{\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{80a^5}$$

```
[Out] -1/800*FresnelC(10^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*10^(1/2)*Pi^(1/2)/a^5-
1/96*FresnelC(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a^5-1/16
*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^5+1/5*x^5*
arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4726, 4810, 3393, 3385, 3433}

$$\int x^4 \sqrt{\arccos(ax)}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^5} - \frac{\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{80a^5} + \frac{1}{5}x^5\sqrt{\arccos(ax)}$$

```
[In] Int[x^4*Sqrt[ArcCos[a*x]],x]
```

```
[Out] (x^5*Sqrt[ArcCos[a*x]])/5 - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]])/(8*a^5) - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(16*a^5)
 - (Sqrt[Pi/10]*FresnelC[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(80*a^5)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{5}x^5\sqrt{\arccos(ax)} + \frac{1}{10}a\int \frac{x^5}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx$$

$$= \frac{1}{5}x^5\sqrt{\arccos(ax)} - \frac{\text{Subst}\left(\int \frac{\cos^5(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{10a^5}$$

$$= \frac{1}{5}x^5\sqrt{\arccos(ax)} - \frac{\text{Subst}\left(\int \left(\frac{5\cos(x)}{8\sqrt{x}} + \frac{5\cos(3x)}{16\sqrt{x}} + \frac{\cos(5x)}{16\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{10a^5}$$

$$= \frac{1}{5}x^5\sqrt{\arccos(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(5x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{160a^5} - \frac{\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{32a^5} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{16a^5}$$

$$= \frac{1}{5}x^5\sqrt{\arccos(ax)} - \frac{\text{Subst}\left(\int \cos(5x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{80a^5} - \frac{\text{Subst}\left(\int \cos(3x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{16a^5} - \frac{\text{Subst}\left(\int \cos(x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{8a^5}$$

$$= \frac{1}{5}x^5\sqrt{\arccos(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^5} - \frac{\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{80a^5}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.60

$$\int x^4\sqrt{\arccos(ax)}\,dx$$

$$= \frac{i\left(150\sqrt{-i\arccos(ax)}\Gamma\left(\frac{3}{2}, -i\arccos(ax)\right) - 150\sqrt{i\arccos(ax)}\Gamma\left(\frac{3}{2}, i\arccos(ax)\right) + 25\sqrt{3}\sqrt{-i\arccos(ax)}\right.}{}$$

```
[In] Integrate[x^4*Sqrt[ArcCos[a*x]],x]
```

```
[Out] ((I/2400)*(150*Sqrt[(-I)*ArcCos[a*x]]*Gamma[3/2, (-I)*ArcCos[a*x]] - 150*Sq
rt[I*ArcCos[a*x]]*Gamma[3/2, I*ArcCos[a*x]] + 25*Sqrt[3]*Sqrt[(-I)*ArcCos[a
*x]]*Gamma[3/2, (-3*I)*ArcCos[a*x]] - 25*Sqrt[3]*Sqrt[I*ArcCos[a*x]]*Gamma[
3/2, (3*I)*ArcCos[a*x]] + 3*Sqrt[5]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[3/2, (-5*I
)*ArcCos[a*x]] - 3*Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[3/2, (5*I)*ArcCos[a*x]
]))/(a^5*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.18

method	result
default	$\frac{-3\sqrt{5}\,\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-25\sqrt{3}\,\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-150\sqrt{2}\,\sqrt{\text{arcco}}}{2400a^{5}\sqrt{\text{arccos}}}$

```
[In] int(x^4*arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2400/a^5/arccos(a*x)^(1/2)*(-3*5^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)
*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))-25*3^(1/2)*2^(1/2)*ar
ccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2
))-150*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(
a*x)^(1/2))+300*arccos(a*x)*a*x+150*arccos(a*x)*cos(3*arccos(a*x))+30*arcco
s(a*x)*cos(5*arccos(a*x)))
```

Fricas [F(-2)]

Exception generated.

$$\int x^4\sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*arccos(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^4\sqrt{\arccos(ax)}\,dx = \int x^4\sqrt{\operatorname{acos}(ax)}\,dx$$

```
[In] integrate(x**4*acos(a*x)**(1/2),x)
[Out] Integral(x**4*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^4 \sqrt{\arccos(ax)}\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.04

$$\begin{aligned}\int x^4 \sqrt{\arccos(ax)}\, dx = &\frac{(i+1)\,\sqrt{10}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{10}\sqrt{\arccos\left(ax\right)}\right)}{3200\,a^5} \\ &- \frac{(i-1)\,\sqrt{10}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{10}\sqrt{\arccos\left(ax\right)}\right)}{3200\,a^5} \\ &+ \frac{(i+1)\,\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{6}\sqrt{\arccos\left(ax\right)}\right)}{384\,a^5} \\ &- \frac{(i-1)\,\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{6}\sqrt{\arccos\left(ax\right)}\right)}{384\,a^5} \\ &+ \frac{(i+1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos\left(ax\right)}\right)}{64\,a^5} \\ &- \frac{(i-1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos\left(ax\right)}\right)}{64\,a^5} \\ &+ \frac{\sqrt{\arccos\left(ax\right)}e^{\left(5i\,\arccos\left(ax\right)\right)}}{160\,a^5} + \frac{\sqrt{\arccos\left(ax\right)}e^{\left(3i\,\arccos\left(ax\right)\right)}}{32\,a^5} \\ &+ \frac{\sqrt{\arccos\left(ax\right)}e^{\left(i\,\arccos\left(ax\right)\right)}}{16\,a^5} + \frac{\sqrt{\arccos\left(ax\right)}e^{\left(-i\,\arccos\left(ax\right)\right)}}{16\,a^5} \\ &+ \frac{\sqrt{\arccos\left(ax\right)}e^{\left(-3i\,\arccos\left(ax\right)\right)}}{32\,a^5} + \frac{\sqrt{\arccos\left(ax\right)}e^{\left(-5i\,\arccos\left(ax\right)\right)}}{160\,a^5}\end{aligned}$$

```
[In] integrate(x^4*arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] (1/3200*I + 1/3200)*sqrt(10)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(10)*sqrt(arcco
s(a*x)))/a^5 - (1/3200*I - 1/3200)*sqrt(10)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqr
t(10)*sqrt(arccos(a*x)))/a^5 + (1/384*I + 1/384)*sqrt(6)*sqrt(pi)*erf((1/2*
```

```
I - 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 - (1/384*I - 1/384)*sqrt(6)*sqrt(pi
)*erf(-(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 + (1/64*I + 1/64)*sqrt(
2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^5 - (1/64*I - 1/
64)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^5 + 1/
160*sqrt(arccos(a*x))*e^(5*I*arccos(a*x))/a^5 + 1/32*sqrt(arccos(a*x))*e^(3
*I*arccos(a*x))/a^5 + 1/16*sqrt(arccos(a*x))*e^(I*arccos(a*x))/a^5 + 1/16*s
qrt(arccos(a*x))*e^(-I*arccos(a*x))/a^5 + 1/32*sqrt(arccos(a*x))*e^(-3*I*ar
ccos(a*x))/a^5 + 1/160*sqrt(arccos(a*x))*e^(-5*I*arccos(a*x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \sqrt{\arccos(ax)}\,dx = \int x^4\,\sqrt{\mathrm{acos}\,(a\,x)}\,dx$$

```
[In] int(x^4*acos(a*x)^(1/2),x)
[Out] int(x^4*acos(a*x)^(1/2), x)
```

3.75 $\int x^3\sqrt{\arccos(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 95

$$\int x^3\sqrt{\arccos(ax)}\,dx = -\frac{3\sqrt{\arccos(ax)}}{32a^4}+\frac{1}{4}x^4\sqrt{\arccos(ax)} -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{64a^4}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{16a^4}$$

```
[Out] -1/128*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-
1/16*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4-3/32*arccos(a*x)^(
1/2)/a^4+1/4*x^4*arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4726, 4810, 3393, 3385, 3433}

$$\int x^3\sqrt{\arccos(ax)}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{64a^4}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{16a^4} -\frac{3\sqrt{\arccos(ax)}}{32a^4}+\frac{1}{4}x^4\sqrt{\arccos(ax)}$$

```
[In] Int[x^3*Sqrt[ArcCos[a*x]],x]
```

```
[Out] (-3*Sqrt[ArcCos[a*x]])/(32*a^4) + (x^4*Sqrt[ArcCos[a*x]])/4 - (Sqrt[Pi/2]*F
resnelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(64*a^4) - (Sqrt[Pi]*FresnelC[(2*S
qrt[ArcCos[a*x]])/Sqrt[Pi]])/(16*a^4)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\sqrt{\arccos(ax)}+\frac{1}{8}a\int \frac{x^4}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx\\
&= \frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\text{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{8a^4}\\
&= \frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}}+\frac{\cos(2x)}{2\sqrt{x}}+\frac{\cos(4x)}{8\sqrt{x}}\right)\,dx,x,\arccos(ax)\right)}{8a^4}\\
&= -\frac{3\sqrt{\arccos(ax)}}{32a^4}+\frac{1}{4}x^4\sqrt{\arccos(ax)}-\frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{64a^4}\\
&\quad -\frac{\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{16a^4}
\end{aligned}$$

$$= -\frac{3\sqrt{\arccos(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\arccos(ax)} - \frac{\text{Subst}\left(\int \cos\left(4x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{32a^4} - \frac{\text{Subst}\left(\int \cos\left(2x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{8a^4}$$

$$= -\frac{3\sqrt{\arccos(ax)}}{32a^4} + \frac{1}{4}x^4\sqrt{\arccos(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{64a^4} - \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{16a^4}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.38

$$\int x^3\sqrt{\arccos(ax)}\,dx = \frac{i\left(4\sqrt{2}\sqrt{-i\arccos(ax)}\Gamma\left(\frac{3}{2},-2i\arccos(ax)\right) - 4\sqrt{2}\sqrt{i\arccos(ax)}\Gamma\left(\frac{3}{2},2i\arccos(ax)\right) + \sqrt{-i\arccos(ax)}\Gamma\right.}{128a^4\sqrt{\arccos(ax)}}$$

```
[In] Integrate[x^3*Sqrt[ArcCos[a*x]],x]
```

```
[Out] ((I/128)*(4*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[3/2, (-2*I)*ArcCos[a*x]] -
 4*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[3/2, (2*I)*ArcCos[a*x]] + Sqrt[(-I)*Ar
cCos[a*x]]*Gamma[3/2, (-4*I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[3/2,
(4*I)*ArcCos[a*x]]))/(a^4*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

method	result
default	$\frac{-\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+16\arccos(ax)\cos(2\arccos(ax))+4\arccos(ax)\cos(4\arccos(ax))-8\sqrt{\arccos(ax)}}{128a^4\sqrt{\arccos(ax)}}$

```
[In] int(x^3*arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/a^4/arccos(a*x)^(1/2)*(-2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2
*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))+16*arccos(a*x)*cos(2*arccos(a*x))+4*ar
ccos(a*x)*cos(4*arccos(a*x))-8*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arccos
(a*x)^(1/2)/Pi^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \sqrt{\arccos(ax)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arccos(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3 \sqrt{\arccos(ax)}\, dx = \int x^3 \sqrt{\operatorname{acos}(ax)}\, dx$$

```
[In] integrate(x**3*acos(a*x)**(1/2),x)
[Out] Integral(x**3*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \sqrt{\arccos(ax)}\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*arccos(a*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.61

$$\int x^3\sqrt{\arccos(ax)}\,dx = \frac{(i+1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left((i-1)\,\sqrt{2}\sqrt{\arccos(ax)}\right)}{512\,a^4} - \frac{(i-1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\,\sqrt{2}\sqrt{\arccos(ax)}\right)}{512\,a^4} + \frac{(i+1)\,\sqrt{\pi}\,\mathrm{erf}\left((i-1)\,\sqrt{\arccos(ax)}\right)}{64\,a^4} - \frac{(i-1)\,\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\,\sqrt{\arccos(ax)}\right)}{64\,a^4} + \frac{\sqrt{\arccos(ax)}e^{(4i\arccos(ax))}}{64\,a^4} + \frac{\sqrt{\arccos(ax)}e^{(2i\arccos(ax))}}{16\,a^4} + \frac{\sqrt{\arccos(ax)}e^{(-2i\arccos(ax))}}{16\,a^4} + \frac{\sqrt{\arccos(ax)}e^{(-4i\arccos(ax))}}{64\,a^4}$$

```
[In] integrate(x^3*arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] (1/512*I + 1/512)*sqrt(2)*sqrt(pi)*erf((I - 1)*sqrt(2)*sqrt(arccos(a*x)))/a
^4 - (1/512*I - 1/512)*sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arccos(a*
x)))/a^4 + (1/64*I + 1/64)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^4 - (1
/64*I - 1/64)*sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^4 + 1/64*sqrt(arcc
os(a*x))*e^(4*I*arccos(a*x))/a^4 + 1/16*sqrt(arccos(a*x))*e^(2*I*arccos(a*x
))/a^4 + 1/16*sqrt(arccos(a*x))*e^(-2*I*arccos(a*x))/a^4 + 1/64*sqrt(arccos
(a*x))*e^(-4*I*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{\arccos(ax)}\,dx = \int x^3\,\sqrt{\mathrm{acos}\,(a\,x)}\,dx$$

```
[In] int(x^3*acos(a*x)^(1/2),x)
```

```
[Out] int(x^3*acos(a*x)^(1/2), x)
```

3.76 $\int x^2 \sqrt{\arccos(ax)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 86

$$\int x^2 \sqrt{\arccos(ax)}\,dx = \frac{1}{3}x^3\sqrt{\arccos(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^3} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{12a^3}$$

```
[Out] -1/72*FresnelC(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a^3-1/8
*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3+1/3*x^3*
arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {4726, 4810, 3393, 3385, 3433}

$$\int x^2 \sqrt{\arccos(ax)}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^3} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{12a^3} + \frac{1}{3}x^3\sqrt{\arccos(ax)}$$

```
[In] Int[x^2*Sqrt[ArcCos[a*x]],x]
```

```
[Out] (x^3*Sqrt[ArcCos[a*x]])/3 - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]])/(4*a^3) - (Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(12*a^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\sqrt{\arccos(ax)} + \frac{1}{6}a\int \frac{x^3}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx \\
&= \frac{1}{3}x^3\sqrt{\arccos(ax)} - \frac{\text{Subst}\left(\int \frac{\cos^3(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{6a^3} \\
&= \frac{1}{3}x^3\sqrt{\arccos(ax)} - \frac{\text{Subst}\left(\int \left(\frac{3\cos(x)}{4\sqrt{x}} + \frac{\cos(3x)}{4\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{6a^3} \\
&= \frac{1}{3}x^3\sqrt{\arccos(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{24a^3} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{8a^3}
\end{aligned}
$$

$$= \frac{1}{3}x^3\sqrt{\arccos(ax)} - \frac{\text{Subst}\left(\int \cos\left(3x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{12a^3} - \frac{\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{4a^3}$$

$$= \frac{1}{3}x^3\sqrt{\arccos(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^3} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{12a^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.49

$$\int x^2\sqrt{\arccos(ax)}\,dx$$
$$= \frac{i\left(9\sqrt{-i\arccos(ax)}\Gamma\left(\frac{3}{2}, -i\arccos(ax)\right) - 9\sqrt{i\arccos(ax)}\Gamma\left(\frac{3}{2}, i\arccos(ax)\right) + \sqrt{3}\left(\sqrt{-i\arccos(ax)}\Gamma\left(\frac{3}{2}, -\right.\right.\right.}{72a^3\sqrt{\arccos(ax)}}$$

```
[In] Integrate[x^2*Sqrt[ArcCos[a*x]],x]
```

```
[Out] ((I/72)*(9*Sqrt[(-I)*ArcCos[a*x]]*Gamma[3/2, (-I)*ArcCos[a*x]] - 9*Sqrt[I*A
rcCos[a*x]]*Gamma[3/2, I*ArcCos[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcCos[a*x]]*Gam
ma[3/2, (-3*I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[3/2, (3*I)*ArcCos[a
*x]])))/(a^3*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

method	result
default	$\frac{-\sqrt{3}\,\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-9\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+18\arccos(ax)ax+6\,\text{arcc}}{72a^3\sqrt{\arccos(ax)}}$

```
[In] int(x^2*arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72/a^3/arccos(a*x)^(1/2)*(-3^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*Fre
snelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))-9*2^(1/2)*arccos(a*x)^(1/
2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))+18*arccos(a*x)*a*x
+6*arccos(a*x)*cos(3*arccos(a*x)))
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \sqrt{\arccos(ax)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*arccos(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2 \sqrt{\arccos(ax)}\, dx = \int x^2 \sqrt{\operatorname{acos}(ax)}\, dx$$

```
[In] integrate(x**2*acos(a*x)**(1/2),x)
[Out] Integral(x**2*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \sqrt{\arccos(ax)}\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*arccos(a*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

$$\int x^2\sqrt{\arccos(ax)}\,dx = \frac{(i+1)\ \sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\ \sqrt{6}\sqrt{\arccos{(ax)}}\right)}{288\,a^3} - \frac{(i-1)\ \sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{6}\sqrt{\arccos{(ax)}}\right)}{288\,a^3} + \frac{(i+1)\ \sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\ \sqrt{2}\sqrt{\arccos{(ax)}}\right)}{32\,a^3} - \frac{(i-1)\ \sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\arccos{(ax)}}\right)}{32\,a^3} + \frac{\sqrt{\arccos{(ax)}}e^{(3i\,\arccos(ax))}}{24\,a^3} + \frac{\sqrt{\arccos{(ax)}}e^{(i\,\arccos(ax))}}{8\,a^3} + \frac{\sqrt{\arccos{(ax)}}e^{(-i\,\arccos(ax))}}{8\,a^3} + \frac{\sqrt{\arccos{(ax)}}e^{(-3i\,\arccos(ax))}}{24\,a^3}$$

```
[In] integrate(x^2*arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] (1/288*I + 1/288)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*sqrt(arccos(a*
x)))/a^3 - (1/288*I - 1/288)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*sq
rt(arccos(a*x)))/a^3 + (1/32*I + 1/32)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*s
qrt(2)*sqrt(arccos(a*x)))/a^3 - (1/32*I - 1/32)*sqrt(2)*sqrt(pi)*erf(-(1/2*
I + 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^3 + 1/24*sqrt(arccos(a*x))*e^(3*I*arc
cos(a*x))/a^3 + 1/8*sqrt(arccos(a*x))*e^(I*arccos(a*x))/a^3 + 1/8*sqrt(arcc
os(a*x))*e^(-I*arccos(a*x))/a^3 + 1/24*sqrt(arccos(a*x))*e^(-3*I*arccos(a*x
))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{\arccos(ax)}\,dx = \int x^2\,\sqrt{\mathrm{acos}\left(a\,x\right)}\,dx$$

```
[In] int(x^2*acos(a*x)^(1/2),x)
```

```
[Out] int(x^2*acos(a*x)^(1/2), x)
```

3.77 $\int x\sqrt{\arccos(ax)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 59

$$\int x\sqrt{\arccos(ax)}\,dx = -\frac{\sqrt{\arccos(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\arccos(ax)} - \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^2}$$

```
[Out] -1/8*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2-1/4*arccos(a*x)^(1
/2)/a^2+1/2*x^2*arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4726, 4810, 3393, 3385, 3433}

$$\int x\sqrt{\arccos(ax)}\,dx = -\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^2} - \frac{\sqrt{\arccos(ax)}}{4a^2} + \frac{1}{2}x^2\sqrt{\arccos(ax)}$$

```
[In] Int[x*Sqrt[ArcCos[a*x]],x]
```

```
[Out] -1/4*Sqrt[ArcCos[a*x]]/a^2 + (x^2*Sqrt[ArcCos[a*x]])/2 - (Sqrt[Pi]*FresnelC
[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(8*a^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2\sqrt{\arccos(ax)}+\frac{1}{4}a\int\frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx\\
&= \frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\text{Subst}\left(\int\frac{\cos^2(x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{4a^2}\\
&= \frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\text{Subst}\left(\int\left(\frac{1}{2\sqrt{x}}+\frac{\cos(2x)}{2\sqrt{x}}\right)\,dx,x,\arccos(ax)\right)}{4a^2}\\
&= -\frac{\sqrt{\arccos(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\text{Subst}\left(\int\frac{\cos(2x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{8a^2}\\
&= -\frac{\sqrt{\arccos(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\text{Subst}\left(\int\cos\left(2x^2\right)\,dx,x,\sqrt{\arccos(ax)}\right)}{4a^2}\\
&= -\frac{\sqrt{\arccos(ax)}}{4a^2}+\frac{1}{2}x^2\sqrt{\arccos(ax)}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.83

$$\int x\sqrt{\arccos(ax)}\,dx = \frac{\frac{1}{4}\sqrt{\arccos(ax)}\cos(2\arccos(ax)) - \frac{1}{8}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^2}$$

```
[In] Integrate[x*Sqrt[ArcCos[a*x]],x]
[Out] ((Sqrt[ArcCos[a*x]]*Cos[2*ArcCos[a*x]])/4 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcC
os[a*x]])/Sqrt[Pi]])/8)/a^2
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.73

method	result	size
default	$\frac{2\cos(2\arccos(ax))\sqrt{\arccos(ax)}\,\sqrt{\pi}-\pi\ \text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^2\sqrt{\pi}}$	43

```
[In] int(x*arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/8/a^2*(2*cos(2*arccos(a*x))*arccos(a*x)^(1/2)*Pi^(1/2)-Pi*FresnelC(2*arcc
os(a*x)^(1/2)/Pi^(1/2)))/Pi^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*arccos(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x\sqrt{\arccos(ax)}\,dx = \int x\sqrt{\operatorname{acos}(ax)}\,dx$$

```
[In] integrate(x*acos(a*x)**(1/2),x)
[Out] Integral(x*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{\arccos(ax)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*arccos(a*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int x\sqrt{\arccos(ax)}\,dx = \frac{(i+1)\sqrt{\pi}\operatorname{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{32\,a^2} - \frac{(i-1)\sqrt{\pi}\operatorname{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{32\,a^2} + \frac{\sqrt{\arccos(ax)}e^{(2i\arccos(ax))}}{8\,a^2} + \frac{\sqrt{\arccos(ax)}e^{(-2i\arccos(ax))}}{8\,a^2}$$

```
[In] integrate(x*arccos(a*x)^(1/2),x, algorithm="giac")
[Out] (1/32*I + 1/32)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^2 - (1/32*I - 1/3
2)*sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^2 + 1/8*sqrt(arccos(a*x))*e^(
2*I*arccos(a*x))/a^2 + 1/8*sqrt(arccos(a*x))*e^(-2*I*arccos(a*x))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\arccos(ax)}\,dx = \int x\,\sqrt{\mathrm{acos}\left(a\,x\right)}\,dx$$

```
[In] int(x*acos(a*x)^(1/2),x)
[Out] int(x*acos(a*x)^(1/2), x)
```

3.78 $\int \sqrt{\arccos(ax)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 44

$$\int \sqrt{\arccos(ax)}\,dx = x\sqrt{\arccos(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

```
[Out] -1/2*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a+x*arcc
os(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4716, 4810, 3385, 3433}

$$\int \sqrt{\arccos(ax)}\,dx = x\sqrt{\arccos(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

```
[In] Int[Sqrt[ArcCos[a*x]],x]
```

```
[Out] x*Sqrt[ArcCos[a*x]] - (Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\sqrt{\arccos(ax)} + \frac{1}{2}a\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx \\
&= x\sqrt{\arccos(ax)} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{2a} \\
&= x\sqrt{\arccos(ax)} - \frac{\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{a} \\
&= x\sqrt{\arccos(ax)} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.57

$$\int \sqrt{\arccos(ax)}\,dx = \frac{i\left(\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{3}{2}, -i\arccos(ax)\right) - \sqrt{i\arccos(ax)}\,\Gamma\left(\frac{3}{2}, i\arccos(ax)\right)\right)}{2a\sqrt{\arccos(ax)}}$$

```
[In] Integrate[Sqrt[ArcCos[a*x]],x]
[Out] ((I/2)*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[3/2, (-I)*ArcCos[a*x]] - Sqrt[I*ArcCos
[a*x]]*Gamma[3/2, I*ArcCos[a*x]]))/(a*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{-\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+2\arccos(ax)ax}{2a\sqrt{\arccos(ax)}}$	49

```
[In] int(arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2/a/arccos(a*x)^(1/2)*(-2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/
2)/Pi^(1/2)*arccos(a*x)^(1/2))+2*arccos(a*x)*a*x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\arccos(ax)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arccos(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{\arccos(ax)}\,dx = \int \sqrt{\operatorname{acos}(ax)}\,dx$$

```
[In] integrate(acos(a*x)**(1/2),x)
[Out] Integral(sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{\arccos(ax)}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.89

$$\int \sqrt{\arccos(ax)}\,dx = \frac{(i+1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos\left(ax\right)}\right)}{8\,a} - \frac{(i-1)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos\left(ax\right)}\right)}{8\,a} + \frac{\sqrt{\arccos\left(ax\right)}e^{\left(i\arccos\left(ax\right)\right)}}{2\,a} + \frac{\sqrt{\arccos\left(ax\right)}e^{\left(-i\arccos\left(ax\right)\right)}}{2\,a}$$

```
[In] integrate(arccos(a*x)^(1/2),x, algorithm="giac")
[Out] (1/8*I + 1/8)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))
/a - (1/8*I - 1/8)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(
a*x)))/a + 1/2*sqrt(arccos(a*x))*e^(I*arccos(a*x))/a + 1/2*sqrt(arccos(a*x)
)*e^(-I*arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\arccos(ax)}\,dx = \int \sqrt{\mathrm{acos}\left(a\,x\right)}\,dx$$

```
[In] int(acos(a*x)^(1/2),x)
[Out] int(acos(a*x)^(1/2), x)
```

3.79 $\int \frac{\sqrt{\arccos(ax)}}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\sqrt{\arccos(ax)}}{x} \, dx = \operatorname{Int}\left(\frac{\sqrt{\arccos(ax)}}{x}, x\right)$$

```
[Out] Unintegrable(arccos(a*x)^(1/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{\arccos(ax)}}{x} \, dx = \int \frac{\sqrt{\arccos(ax)}}{x} \, dx$$

```
[In] Int[Sqrt[ArcCos[a*x]]/x,x]
```

```
[Out] Defer[Int][Sqrt[ArcCos[a*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{\arccos(ax)}}{x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{\arccos(ax)}}{x} \, dx = \int \frac{\sqrt{\arccos(ax)}}{x} \, dx$$

```
[In] Integrate[Sqrt[ArcCos[a*x]]/x,x]
[Out] Integrate[Sqrt[ArcCos[a*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arccos(ax)}}{x} dx$$

```
[In] int(arccos(a*x)^(1/2)/x,x)
[Out] int(arccos(a*x)^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos(ax)}}{x} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arccos(a*x)^(1/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{\arccos(ax)}}{x} \, dx = \int \frac{\sqrt{\operatorname{acos}(ax)}}{x} \, dx$$

```
[In] integrate(acos(a*x)**(1/2)/x,x)
[Out] Integral(sqrt(acos(a*x))/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\arccos(ax)}}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^(1/2)/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arccos(ax)}}{x}\,dx = \int \frac{\sqrt{\arccos{(ax)}}}{x}\,dx$$

```
[In] integrate(arccos(a*x)^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(arccos(a*x))/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\arccos(ax)}}{x}\,dx = \int \frac{\sqrt{\mathrm{acos}\left(a\,x\right)}}{x}\,dx$$

```
[In] int(acos(a*x)^(1/2)/x,x)
[Out] int(acos(a*x)^(1/2)/x, x)
```

3.80 $\int x^4 \arccos(ax)^{3/2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 282

$$\begin{aligned}\int x^4 \arccos(ax)^{3/2} \, dx = &-\frac{4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^5} \\ &-\frac{2x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^3}-\frac{3x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{50a} \\ &+\frac{1}{5}x^5\arccos(ax)^{3/2}+\frac{11\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{400a^5} \\ &+\frac{2\sqrt{2\pi}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{25a^5}+\frac{\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{50a^5} \\ &+\frac{3\sqrt{\frac{3\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{800a^5}+\frac{3\sqrt{\frac{\pi}{10}}\operatorname{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{800a^5}\end{aligned}$$

```
[Out] 1/5*x^5*arccos(a*x)^(3/2)+3/8000*FresnelS(10^(1/2)/Pi^(1/2)*arccos(a*x)^(1/
2))*10^(1/2)*Pi^(1/2)/a^5+1/192*FresnelS(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)
)*6^(1/2)*Pi^(1/2)/a^5+3/32*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^
(1/2)*Pi^(1/2)/a^5-4/25*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a^5-2/25*x^2*(
-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a^3-3/50*x^4*(-a^2*x^2+1)^(1/2)*arccos(
a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4726, 4796, 4768, 4720, 3386, 3432, 4732, 4491}

$$\int x^4 \arccos(ax)^{3/2}\,dx = \frac{2\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{25a^5} + \frac{11\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{400a^5} + \frac{3\sqrt{\frac{3\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{800a^5} + \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{50a^5} + \frac{3\sqrt{\frac{\pi}{10}}\,\text{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{800a^5} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{50a} - \frac{4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^3} + \frac{1}{5}x^5\arccos(ax)^{3/2}$$

```
[In] Int[x^4*ArcCos[a*x]^(3/2),x]
```

```
[Out] (-4*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(25*a^5) - (2*x^2*Sqrt[1 - a^2*x^2
]*Sqrt[ArcCos[a*x]])/(25*a^3) - (3*x^4*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])
/(50*a) + (x^5*ArcCos[a*x]^(3/2))/5 + (11*Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sq
rt[ArcCos[a*x]]])/(400*a^5) + (2*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos
[a*x]]])/(25*a^5) + (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(50
*a^5) + (3*Sqrt[(3*Pi)/2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(800*a^5)
 + (3*Sqrt[Pi/10]*FresnelS[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(800*a^5)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{5}x^5\arccos(ax)^{3/2} + \frac{1}{10}(3a)\int\frac{x^5\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{3x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{50a} + \frac{1}{5}x^5\arccos(ax)^{3/2} \\
&\quad - \frac{3}{100}\int\frac{x^4}{\sqrt{\arccos(ax)}}\,dx + \frac{6\int\frac{x^3\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}\,dx}{25a}
\end{aligned}
$$

$$= -\frac{2x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{50a} + \frac{1}{5}x^5\arccos(ax)^{3/2} + \frac{3\text{Subst}\left(\int \frac{\cos^4(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{100a^5} + \frac{4\int \frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}\,dx}{25a^3} - \frac{\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx}{25a^2}$$

$$= -\frac{4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{50a} + \frac{1}{5}x^5\arccos(ax)^{3/2} + \frac{3\text{Subst}\left(\int \left(\frac{\sin(x)}{8\sqrt{x}} + \frac{3\sin(3x)}{16\sqrt{x}} + \frac{\sin(5x)}{16\sqrt{x}}\right)dx, x, \arccos(ax)\right)}{100a^5} + \frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{25a^5} - \frac{2\int \frac{1}{\sqrt{\arccos(ax)}}\,dx}{25a^4}$$

$$= -\frac{4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{50a} + \frac{1}{5}x^5\arccos(ax)^{3/2} + \frac{3\text{Subst}\left(\int \frac{\sin(5x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{1600a^5} + \frac{3\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{800a^5} + \frac{9\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{1600a^5} + \frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{4\sqrt{x}} + \frac{\sin(3x)}{4\sqrt{x}}\right)dx, x, \arccos(ax)\right)}{25a^5} + \frac{2\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{25a^5}$$

$$= -\frac{4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{50a} + \frac{1}{5}x^5\arccos(ax)^{3/2} + \frac{3\text{Subst}\left(\int \sin\left(5x^2\right)dx, x, \sqrt{\arccos(ax)}\right)}{800a^5} + \frac{3\text{Subst}\left(\int \sin\left(x^2\right)dx, x, \sqrt{\arccos(ax)}\right)}{400a^5} + \frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{100a^5} + \frac{\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{100a^5} + \frac{9\text{Subst}\left(\int \sin\left(3x^2\right)dx, x, \sqrt{\arccos(ax)}\right)}{800a^5} + \frac{4\text{Subst}\left(\int \sin\left(x^2\right)dx, x, \sqrt{\arccos(ax)}\right)}{25a^5}$$

$$= -\frac{4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{50a} + \frac{1}{5}x^5\arccos(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{400a^5} + \frac{2\sqrt{2\pi}\,\mathrm{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{25a^5} + \frac{3\sqrt{\frac{3\pi}{2}}\,\mathrm{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{800a^5} + \frac{3\sqrt{\frac{\pi}{10}}\,\mathrm{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{800a^5} + \frac{\mathrm{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{50a^5} + \frac{\mathrm{Subst}\left(\int \sin\left(3x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{50a^5}$$

$$= -\frac{4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^5} - \frac{2x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{25a^3} - \frac{3x^4\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{50a} + \frac{1}{5}x^5\arccos(ax)^{3/2} + \frac{11\sqrt{\frac{\pi}{2}}\,\mathrm{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{400a^5} + \frac{2\sqrt{2\pi}\,\mathrm{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{25a^5} + \frac{\sqrt{\frac{\pi}{6}}\,\mathrm{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{50a^5} + \frac{3\sqrt{\frac{3\pi}{2}}\,\mathrm{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{800a^5} + \frac{3\sqrt{\frac{\pi}{10}}\,\mathrm{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{800a^5}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.66

$$\int x^4\arccos(ax)^{3/2}\,dx = -\frac{2250\left(\sqrt{-i\arccos(ax)}\Gamma\left(\frac{5}{2},-i\arccos(ax)\right)+\sqrt{i\arccos(ax)}\Gamma\left(\frac{5}{2},i\arccos(ax)\right)\right)+125\sqrt{3}\left(\sqrt{-i\arccos(ax}\right.}{}$$

```
[In] Integrate[x^4*ArcCos[a*x]^(3/2),x]
```

```
[Out] -1/36000*(2250*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-I)*ArcCos[a*x]] + Sqrt[
I*ArcCos[a*x]]*Gamma[5/2, I*ArcCos[a*x]]) + 125*Sqrt[3]*(Sqrt[(-I)*ArcCos[a
*x]]*Gamma[5/2, (-3*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (3*I)*
ArcCos[a*x]]) + 9*Sqrt[5]*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-5*I)*ArcCos[
a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (5*I)*ArcCos[a*x]]))/(a^5*Sqrt[ArcCo
s[a*x]])
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.68

method	result
default	$\frac{9\ \text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{5}\,\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}+125\ \text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{3}\,\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}+3000\arccos(ax)^{2}}{}$

```
[In] int(x^4*arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24000/a^5*(9*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))*5^(1/2)
*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)+125*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*a
rccos(a*x)^(1/2))*3^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)+3000*arccos(a*
x)^2*a*x+2250*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*arccos(a
*x)^(1/2)*Pi^(1/2)+1500*arccos(a*x)^2*cos(3*arccos(a*x))+300*arccos(a*x)^2*
cos(5*arccos(a*x))-4500*arccos(a*x)*(-a^2*x^2+1)^(1/2)-750*arccos(a*x)*sin(
3*arccos(a*x))-90*arccos(a*x)*sin(5*arccos(a*x)))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x^4 \arccos(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4*arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^4 \arccos(ax)^{3/2}\,dx = \int x^4 \operatorname{acos}^{\frac{3}{2}}(ax)\,dx$$

```
[In] integrate(x**4*acos(a*x)**(3/2),x)
```

```
[Out] Integral(x**4*acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^4 \arccos(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*arccos(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.26

$$\begin{aligned}
\int x^4 \arccos(ax)^{3/2}\,dx &= \frac{\arccos(ax)^{\frac{3}{2}} e^{(5i \arccos(ax))}}{160\,a^5} + \frac{\arccos(ax)^{\frac{3}{2}} e^{(3i \arccos(ax))}}{32\,a^5} \\
&+ \frac{\arccos(ax)^{\frac{3}{2}} e^{(i \arccos(ax))}}{16\,a^5} + \frac{\arccos(ax)^{\frac{3}{2}} e^{(-i \arccos(ax))}}{16\,a^5} \\
&+ \frac{\arccos(ax)^{\frac{3}{2}} e^{(-3i \arccos(ax))}}{32\,a^5} + \frac{\arccos(ax)^{\frac{3}{2}} e^{(-5i \arccos(ax))}}{160\,a^5} \\
&+ \frac{(3i-3)\,\sqrt{10}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{10}\sqrt{\arccos(ax)}\right)}{32000\,a^5} \\
&- \frac{(3i+3)\,\sqrt{10}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{10}\sqrt{\arccos(ax)}\right)}{32000\,a^5} \\
&+ \frac{(i-1)\,\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{6}\sqrt{\arccos(ax)}\right)}{768\,a^5} \\
&- \frac{(i+1)\,\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{6}\sqrt{\arccos(ax)}\right)}{768\,a^5} \\
&+ \frac{(3i-3)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos(ax)}\right)}{128\,a^5} \\
&- \frac{(3i+3)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos(ax)}\right)}{128\,a^5} \\
&+ \frac{3i\,\sqrt{\arccos(ax)}e^{(5i \arccos(ax))}}{1600\,a^5} + \frac{i\,\sqrt{\arccos(ax)}e^{(3i \arccos(ax))}}{64\,a^5} \\
&+ \frac{3i\,\sqrt{\arccos(ax)}e^{(i \arccos(ax))}}{32\,a^5} - \frac{3i\,\sqrt{\arccos(ax)}e^{(-i \arccos(ax))}}{32\,a^5} \\
&- \frac{i\,\sqrt{\arccos(ax)}e^{(-3i \arccos(ax))}}{64\,a^5} - \frac{3i\,\sqrt{\arccos(ax)}e^{(-5i \arccos(ax))}}{1600\,a^5}
\end{aligned}$$

```
[In] integrate(x^4*arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/160*arccos(a*x)^(3/2)*e^(5*I*arccos(a*x))/a^5 + 1/32*arccos(a*x)^(3/2)*e^
(3*I*arccos(a*x))/a^5 + 1/16*arccos(a*x)^(3/2)*e^(I*arccos(a*x))/a^5 + 1/16
*arccos(a*x)^(3/2)*e^(-I*arccos(a*x))/a^5 + 1/32*arccos(a*x)^(3/2)*e^(-3*I*
arccos(a*x))/a^5 + 1/160*arccos(a*x)^(3/2)*e^(-5*I*arccos(a*x))/a^5 + (3/32
000*I - 3/32000)*sqrt(10)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(10)*sqrt(arccos(a
*x)))/a^5 - (3/32000*I + 3/32000)*sqrt(10)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt
(10)*sqrt(arccos(a*x)))/a^5 + (1/768*I - 1/768)*sqrt(6)*sqrt(pi)*erf((1/2*I
 - 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 - (1/768*I + 1/768)*sqrt(6)*sqrt(pi)
*erf(-(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 + (3/128*I - 3/128)*sqrt
(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^5 - (3/128*I +
3/128)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^5 +
 3/1600*I*sqrt(arccos(a*x))*e^(5*I*arccos(a*x))/a^5 + 1/64*I*sqrt(arccos(a*
x))*e^(3*I*arccos(a*x))/a^5 + 3/32*I*sqrt(arccos(a*x))*e^(I*arccos(a*x))/a^
5 - 3/32*I*sqrt(arccos(a*x))*e^(-I*arccos(a*x))/a^5 - 1/64*I*sqrt(arccos(a*
x))*e^(-3*I*arccos(a*x))/a^5 - 3/1600*I*sqrt(arccos(a*x))*e^(-5*I*arccos(a*
x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^{3/2} \, dx = \int x^4 \operatorname{acos}(a\,x)^{3/2} \, dx$$

```
[In] int(x^4*acos(a*x)^(3/2),x)
```

```
[Out] int(x^4*acos(a*x)^(3/2), x)
```

3.81 $\int x^3 \arccos(ax)^{3/2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 157

$$\int x^3 \arccos(ax)^{3/2} \, dx = -\frac{9x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{64a^3} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{32a} - \frac{3\arccos(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{512a^4} + \frac{3\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{64a^4}$$

```
[Out] -3/32*arccos(a*x)^(3/2)/a^4+1/4*x^4*arccos(a*x)^(3/2)+3/1024*FresnelS(2*2^(
1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4+3/64*FresnelS(2*arcco
s(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4-9/64*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(
1/2)/a^3-3/32*x^3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4726, 4796, 4738, 4732, 4491, 12, 3386, 3432}

$$\int x^3 \arccos(ax)^{3/2} \, dx = \frac{3\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{512a^4} + \frac{3\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{64a^4} - \frac{3\arccos(ax)^{3/2}}{32a^4} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{32a} - \frac{9x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{64a^3} + \frac{1}{4}x^4\arccos(ax)^{3/2}$$

```
[In] Int[x^3*ArcCos[a*x]^(3/2),x]
```

```
[Out] (-9*x*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(64*a^3) - (3*x^3*Sqrt[1 - a^2*x
^2]*Sqrt[ArcCos[a*x]])/(32*a) - (3*ArcCos[a*x]^(3/2))/(32*a^4) + (x^4*ArcCo
s[a*x]^(3/2))/4 + (3*Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(
512*a^4) + (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(64*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
```

```
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\arccos(ax)^{3/2}+\frac{1}{8}(3a)\int\frac{x^4\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{3x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{32a}+\frac{1}{4}x^4\arccos(ax)^{3/2}\\
&\quad -\frac{3}{64}\int\frac{x^3}{\sqrt{\arccos(ax)}}\,dx+\frac{9\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}\,dx}{32a}\\
&= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{64a^3}-\frac{3x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{32a}+\frac{1}{4}x^4\arccos(ax)^{3/2}\\
&\quad +\frac{3\text{Subst}\left(\int\frac{\cos^3(x)\sin(x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{64a^4}+\frac{9\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}\,dx}{64a^3}-\frac{9\int\frac{x}{\sqrt{\arccos(ax)}}\,dx}{128a^2}\\
&= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{64a^3}-\frac{3x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{32a}-\frac{3\arccos(ax)^{3/2}}{32a^4}\\
&\quad +\frac{1}{4}x^4\arccos(ax)^{3/2}+\frac{3\text{Subst}\left(\int\left(\frac{\sin(2x)}{4\sqrt{x}}+\frac{\sin(4x)}{8\sqrt{x}}\right)dx,x,\arccos(ax)\right)}{64a^4}\\
&\quad +\frac{9\text{Subst}\left(\int\frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{128a^4}\\
&= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{64a^3}-\frac{3x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{32a}\\
&\quad -\frac{3\arccos(ax)^{3/2}}{32a^4}+\frac{1}{4}x^4\arccos(ax)^{3/2}+\frac{3\text{Subst}\left(\int\frac{\sin(4x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{512a^4}\\
&\quad +\frac{3\text{Subst}\left(\int\frac{\sin(2x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{256a^4}+\frac{9\text{Subst}\left(\int\frac{\sin(2x)}{2\sqrt{x}}\,dx,x,\arccos(ax)\right)}{128a^4}
\end{aligned}$$

$$= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{64a^3} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{32a} - \frac{3\arccos(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{3/2} + \frac{3\text{Subst}\left(\int \sin(4x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{256a^4} + \frac{3\text{Subst}\left(\int \sin(2x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{128a^4} + \frac{9\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{256a^4}$$

$$= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{64a^3} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{32a} - \frac{3\arccos(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{512a^4} + \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{256a^4} + \frac{9\text{Subst}\left(\int \sin(2x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{128a^4}$$

$$= -\frac{9x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{64a^3} - \frac{3x^3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{32a} - \frac{3\arccos(ax)^{3/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{512a^4} + \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{64a^4}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.82

$$\int x^3\arccos(ax)^{3/2}\,dx = -\frac{8\sqrt{2}\sqrt{-i\arccos(ax)}\Gamma\left(\frac{5}{2},-2i\arccos(ax)\right) + 8\sqrt{2}\sqrt{i\arccos(ax)}\Gamma\left(\frac{5}{2},2i\arccos(ax)\right) + \sqrt{-i\arccos(ax)}\Gamma\left(\frac{5}{2},\right.}{512a^4\sqrt{\arccos(ax)}}$$

```
[In] Integrate[x^3*ArcCos[a*x]^(3/2),x]
```

```
[Out] -1/512*(8*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-2*I)*ArcCos[a*x]] + 8
*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (2*I)*ArcCos[a*x]] + Sqrt[(-I)*ArcC
os[a*x]]*Gamma[5/2, (-4*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (4
*I)*ArcCos[a*x]])/(a^4*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.75

method	result
default	$\frac{128 \arccos(ax)^{\frac{3}{2}} \sqrt{\pi}\, \cos(2 \arccos(ax))+32 \arccos(ax)^{\frac{3}{2}} \sqrt{\pi}\, \cos(4 \arccos(ax))+3\pi\sqrt{2}\, \mathrm{FresnelS}\left(\frac{2\sqrt{2}\, \sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-96 \sqrt{\arccos(ax)}\, \sqrt{\cdots}}{1024 a^{4} \sqrt{\pi}}$

[In] int(x^3*arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/1024/a^4/Pi^(1/2)*(128*arccos(a*x)^(3/2)*Pi^(1/2)*cos(2*arccos(a*x))+32*a
rccos(a*x)^(3/2)*Pi^(1/2)*cos(4*arccos(a*x))+3*Pi*2^(1/2)*FresnelS(2*2^(1/2
)/Pi^(1/2)*arccos(a*x)^(1/2))-96*arccos(a*x)^(1/2)*Pi^(1/2)*sin(2*arccos(a*
x))-12*arccos(a*x)^(1/2)*Pi^(1/2)*sin(4*arccos(a*x))+48*Pi*FresnelS(2*arcco
s(a*x)^(1/2)/Pi^(1/2)))

Fricas [F(-2)]

Exception generated.

$$\int x^{3} \arccos(ax)^{3/2}\, dx = \text{Exception raised: TypeError}$$

[In] integrate(x^3*arccos(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [F]

$$\int x^{3} \arccos(ax)^{3/2}\, dx = \int x^{3} \operatorname{acos}^{\frac{3}{2}}(ax)\, dx$$

[In] integrate(x**3*acos(a*x)**(3/2),x)

[Out] Integral(x**3*acos(a*x)**(3/2), x)

Maxima [F(-2)]

Exception generated.

$$\int x^3 \arccos(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.43

$$\int x^3 \arccos(ax)^{3/2}\,dx = \frac{\arccos(ax)^{\frac{3}{2}}e^{(4i\arccos(ax))}}{64a^4} + \frac{\arccos(ax)^{\frac{3}{2}}e^{(2i\arccos(ax))}}{16a^4} + \frac{\arccos(ax)^{\frac{3}{2}}e^{(-2i\arccos(ax))}}{16a^4} + \frac{\arccos(ax)^{\frac{3}{2}}e^{(-4i\arccos(ax))}}{64a^4} + \frac{(3i-3)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{4096a^4} - \frac{(3i+3)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{4096a^4} + \frac{(3i-3)\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{256a^4} - \frac{(3i+3)\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{256a^4} + \frac{3i\sqrt{\arccos(ax)}e^{(4i\arccos(ax))}}{512a^4} + \frac{3i\sqrt{\arccos(ax)}e^{(2i\arccos(ax))}}{64a^4} - \frac{3i\sqrt{\arccos(ax)}e^{(-2i\arccos(ax))}}{64a^4} - \frac{3i\sqrt{\arccos(ax)}e^{(-4i\arccos(ax))}}{512a^4}$$

```
[In] integrate(x^3*arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*arccos(a*x)^(3/2)*e^(4*I*arccos(a*x))/a^4 + 1/16*arccos(a*x)^(3/2)*e^(
2*I*arccos(a*x))/a^4 + 1/16*arccos(a*x)^(3/2)*e^(-2*I*arccos(a*x))/a^4 + 1/
64*arccos(a*x)^(3/2)*e^(-4*I*arccos(a*x))/a^4 + (3/4096*I - 3/4096)*sqrt(2)
*sqrt(pi)*erf((I - 1)*sqrt(2)*sqrt(arccos(a*x)))/a^4 - (3/4096*I + 3/4096)*
sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arccos(a*x)))/a^4 + (3/256*I - 3
/256)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^4 - (3/256*I + 3/256)*sqrt(
```

```
pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^4 + 3/512*I*sqrt(arccos(a*x))*e^(4*I*
arccos(a*x))/a^4 + 3/64*I*sqrt(arccos(a*x))*e^(2*I*arccos(a*x))/a^4 - 3/64*
I*sqrt(arccos(a*x))*e^(-2*I*arccos(a*x))/a^4 - 3/512*I*sqrt(arccos(a*x))*e^
(-4*I*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^{3/2}\,dx = \int x^3 \operatorname{acos}(a\,x)^{3/2}\,dx$$

```
[In] int(x^3*acos(a*x)^(3/2),x)
```

```
[Out] int(x^3*acos(a*x)^(3/2), x)
```

3.82 $\int x^2 \arccos(ax)^{3/2} dx$

Optimal result

Integrand size = 12, antiderivative size = 147

$$\int x^2 \arccos(ax)^{3/2} dx = -\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{6a} + \frac{1}{3}x^3 \arccos(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{24a^3}$$

```
[Out] 1/3*x^3*arccos(a*x)^(3/2)+1/144*FresnelS(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)
)*6^(1/2)*Pi^(1/2)/a^3+3/16*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^
(1/2)*Pi^(1/2)/a^3-1/3*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a^3-1/6*x^2*(-a
^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4726, 4796, 4768, 4720, 3386, 3432, 4732, 4491}

$$\int x^2 \arccos(ax)^{3/2} dx = \frac{3\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{6}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{24a^3} - \frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{6a} - \frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^3} + \frac{1}{3}x^3 \arccos(ax)^{3/2}$$

```
[In] Int[x^2*ArcCos[a*x]^(3/2),x]
```

```
[Out] -1/3*(Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/a^3 - (x^2*Sqrt[1 - a^2*x^2]*Sqr
t[ArcCos[a*x]])/(6*a) + (x^3*ArcCos[a*x]^(3/2))/3 + (3*Sqrt[Pi/2]*FresnelS[
Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(8*a^3) + (Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sq
rt[ArcCos[a*x]]])/(24*a^3)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
```

```
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\arccos(ax)^{3/2}+\frac{1}{2}a\int\frac{x^3\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}\,dx\\
&=-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{6a}+\frac{1}{3}x^3\arccos(ax)^{3/2}-\frac{1}{12}\int\frac{x^2}{\sqrt{\arccos(ax)}}\,dx+\frac{\int\frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}\,dx}{3a}\\
&=-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^3}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{6a}+\frac{1}{3}x^3\arccos(ax)^{3/2}\\
&\quad+\frac{\text{Subst}\left(\int\frac{\cos^2(x)\sin(x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{12a^3}-\frac{\int\frac{1}{\sqrt{\arccos(ax)}}\,dx}{6a^2}\\
&=-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^3}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{6a}+\frac{1}{3}x^3\arccos(ax)^{3/2}\\
&\quad+\frac{\text{Subst}\left(\int\left(\frac{\sin(x)}{4\sqrt{x}}+\frac{\sin(3x)}{4\sqrt{x}}\right)dx,x,\arccos(ax)\right)}{12a^3}+\frac{\text{Subst}\left(\int\frac{\sin(x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{6a^3}\\
&=-\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^3}-\frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{6a}\\
&\quad+\frac{1}{3}x^3\arccos(ax)^{3/2}+\frac{\text{Subst}\left(\int\frac{\sin(x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{48a^3}\\
&\quad+\frac{\text{Subst}\left(\int\frac{\sin(3x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{48a^3}+\frac{\text{Subst}\left(\int\sin\left(x^2\right)dx,x,\sqrt{\arccos(ax)}\right)}{3a^3}
\end{aligned}$$

$$= -\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{6a} + \frac{1}{3}x^3\arccos(ax)^{3/2} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^3} + \frac{\text{Subst}\left(\int\sin\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{24a^3} + \frac{\text{Subst}\left(\int\sin\left(3x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{24a^3}$$

$$= -\frac{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{3a^3} - \frac{x^2\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{6a} + \frac{1}{3}x^3\arccos(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^3} + \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{24a^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int x^2\arccos(ax)^{3/2}\,dx = -\frac{27\sqrt{-i\arccos(ax)}\Gamma\left(\frac{5}{2}, -i\arccos(ax)\right) + 27\sqrt{i\arccos(ax)}\Gamma\left(\frac{5}{2}, i\arccos(ax)\right) + \sqrt{3}\left(\sqrt{-i\arccos(ax)}\Gamma\left(\frac{5}{2}, -\right.\right.}{216a^3\sqrt{\arccos(ax)}}$$

```
[In] Integrate[x^2*ArcCos[a*x]^(3/2),x]
```

```
[Out] -1/216*(27*Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-I)*ArcCos[a*x]] + 27*Sqrt[I*
ArcCos[a*x]]*Gamma[5/2, I*ArcCos[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcCos[a*x]]*Ga
mma[5/2, (-3*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[5/2, (3*I)*ArcCos[
a*x]]))/(a^3*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.88

method	result
default	$\frac{36\arccos(ax)^2 ax+\text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{3}\,\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}+12\arccos(ax)^2\cos(3\arccos(ax))+27\ \text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos}}{\sqrt{\pi}}\right.}{144a^3\sqrt{\arccos(ax)}}$

```
[In] int(x^2*arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/144/a^3*(36*arccos(a*x)^2*a*x+FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*
x)^(1/2))*3^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)+12*arccos(a*x)^2*cos(3
*arccos(a*x))+27*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*arcco
```

```
s(a*x)^(1/2)*Pi^(1/2)-54*arccos(a*x)*(-a^2*x^2+1)^(1/2)-6*arccos(a*x)*sin(3
*arccos(a*x)))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \arccos(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*arccos(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2 \arccos(ax)^{3/2}\,dx = \int x^2 \operatorname{acos}^{\frac{3}{2}}(ax)\,dx$$

```
[In] integrate(x**2*acos(a*x)**(3/2),x)
[Out] Integral(x**2*acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \arccos(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*arccos(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.61

$$\int x^2 \arccos(ax)^{3/2}\,dx = \frac{\arccos(ax)^{\frac{3}{2}}e^{(3i\arccos(ax))}}{24a^3} + \frac{\arccos(ax)^{\frac{3}{2}}e^{(i\arccos(ax))}}{8a^3} + \frac{\arccos(ax)^{\frac{3}{2}}e^{(-i\arccos(ax))}}{8a^3} + \frac{\arccos(ax)^{\frac{3}{2}}e^{(-3i\arccos(ax))}}{24a^3} + \frac{(i-1)\sqrt{6}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{576a^3} - \frac{(i+1)\sqrt{6}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{576a^3} + \frac{(3i-3)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{64a^3} - \frac{(3i+3)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{64a^3} + \frac{i\sqrt{\arccos(ax)}e^{(3i\arccos(ax))}}{48a^3} + \frac{3i\sqrt{\arccos(ax)}e^{(i\arccos(ax))}}{16a^3} - \frac{3i\sqrt{\arccos(ax)}e^{(-i\arccos(ax))}}{16a^3} - \frac{i\sqrt{\arccos(ax)}e^{(-3i\arccos(ax))}}{48a^3}$$

```
[In] integrate(x^2*arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*arccos(a*x)^(3/2)*e^(3*I*arccos(a*x))/a^3 + 1/8*arccos(a*x)^(3/2)*e^(I
*arccos(a*x))/a^3 + 1/8*arccos(a*x)^(3/2)*e^(-I*arccos(a*x))/a^3 + 1/24*arc
cos(a*x)^(3/2)*e^(-3*I*arccos(a*x))/a^3 + (1/576*I - 1/576)*sqrt(6)*sqrt(pi
)*erf((1/2*I - 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^3 - (1/576*I + 1/576)*sqrt
(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^3 + (3/64*I -
3/64)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^3 - (
3/64*I + 3/64)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a*x)
))/a^3 + 1/48*I*sqrt(arccos(a*x))*e^(3*I*arccos(a*x))/a^3 + 3/16*I*sqrt(arc
cos(a*x))*e^(I*arccos(a*x))/a^3 - 3/16*I*sqrt(arccos(a*x))*e^(-I*arccos(a*x
))/a^3 - 1/48*I*sqrt(arccos(a*x))*e^(-3*I*arccos(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^{3/2}\, dx = \int x^2 \operatorname{acos}(a\, x)^{3/2}\, dx$$

```
[In] int(x^2*acos(a*x)^(3/2),x)
[Out] int(x^2*acos(a*x)^(3/2), x)
```

3.83 $\int x \arccos(ax)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 89

$$\int x \arccos(ax)^{3/2}\,dx = -\frac{3x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{8a} - \frac{\arccos(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{3/2} + \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{32a^2}$$

```
[Out] -1/4*arccos(a*x)^(3/2)/a^2+1/2*x^2*arccos(a*x)^(3/2)+3/32*FresnelS(2*arccos
(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2-3/8*x*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/
2)/a
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {4726, 4796, 4738, 4732, 4491, 12, 3386, 3432}

$$\int x \arccos(ax)^{3/2}\,dx = \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{32a^2} - \frac{3x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{8a} - \frac{\arccos(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{3/2}$$

```
[In] Int[x*ArcCos[a*x]^(3/2),x]
```

```
[Out] (-3*x*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(8*a) - ArcCos[a*x]^(3/2)/(4*a^2
) + (x^2*ArcCos[a*x]^(3/2))/2 + (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/
Sqrt[Pi]])/(32*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
```

```
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\arccos(ax)^{3/2} + \frac{1}{4}(3a)\int\frac{x^2\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{8a} + \frac{1}{2}x^2\arccos(ax)^{3/2} - \frac{3}{16}\int\frac{x}{\sqrt{\arccos(ax)}}\,dx + \frac{3\int\frac{\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}\,dx}{8a}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{8a} - \frac{\arccos(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{3/2} + \frac{3\text{Subst}\left(\int\frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{16a^2}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{8a} - \frac{\arccos(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{3/2} + \frac{3\text{Subst}\left(\int\frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \arccos(ax)\right)}{16a^2}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{8a} - \frac{\arccos(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{3/2} + \frac{3\text{Subst}\left(\int\frac{\sin(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{32a^2}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{8a} - \frac{\arccos(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{3/2} + \frac{3\text{Subst}\left(\int\sin\left(2x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{16a^2}$$

$$= -\frac{3x\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{8a} - \frac{\arccos(ax)^{3/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{3/2} + \frac{3\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{32a^2}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

$$\int x \arccos(ax)^{3/2} \, dx = \frac{3\sqrt{\pi} \operatorname{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) - 2\sqrt{\arccos(ax)}(-4\arccos(ax)\cos(2\arccos(ax)) + 3\sin}{32a^2}$$

```
[In] Integrate[x*ArcCos[a*x]^(3/2),x]
```

```
[Out] (3*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] - 2*Sqrt[ArcCos[a*x]]*
(-4*ArcCos[a*x]*Cos[2*ArcCos[a*x]] + 3*Sin[2*ArcCos[a*x]]))/(32*a^2)
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.72

method	result	size
default	$\frac{8\arccos(ax)^2\cos(2\arccos(ax))+3\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\operatorname{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-6\arccos(ax)\sin(2\arccos(ax))}{32a^2\sqrt{\arccos(ax)}}$	64

```
[In] int(x*arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/a^2*(8*arccos(a*x)^2*cos(2*arccos(a*x))+3*arccos(a*x)^(1/2)*Pi^(1/2)*F
resnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))-6*arccos(a*x)*sin(2*arccos(a*x)))/arc
cos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x \arccos(ax)^{3/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x \arccos(ax)^{3/2}\, dx = \int x \operatorname{acos}^{\frac{3}{2}}(ax)\, dx$$

```
[In] integrate(x*acos(a*x)**(3/2),x)
```

```
[Out] Integral(x*acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x \arccos(ax)^{3/2}\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.20

$$\int x \arccos(ax)^{3/2}\, dx = \frac{\arccos(ax)^{\frac{3}{2}} e^{(2i \arccos(ax))}}{8a^2} + \frac{\arccos(ax)^{\frac{3}{2}} e^{(-2i \arccos(ax))}}{8a^2} + \frac{(3i-3)\sqrt{\pi}\operatorname{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{128a^2} - \frac{(3i+3)\sqrt{\pi}\operatorname{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{128a^2} + \frac{3i\sqrt{\arccos(ax)}e^{(2i \arccos(ax))}}{32a^2} - \frac{3i\sqrt{\arccos(ax)}e^{(-2i \arccos(ax))}}{32a^2}$$

```
[In] integrate(x*arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*arccos(a*x)^(3/2)*e^(2*I*arccos(a*x))/a^2 + 1/8*arccos(a*x)^(3/2)*e^(-2
*I*arccos(a*x))/a^2 + (3/128*I - 3/128)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*
x)))/a^2 - (3/128*I + 3/128)*sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^2 +
 3/32*I*sqrt(arccos(a*x))*e^(2*I*arccos(a*x))/a^2 - 3/32*I*sqrt(arccos(a*x)
)*e^(-2*I*arccos(a*x))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x \arccos(ax)^{3/2} \, dx = \int x \operatorname{acos}(a\, x)^{3/2} \, dx$$

```
[In] int(x*acos(a*x)^(3/2),x)
[Out] int(x*acos(a*x)^(3/2), x)
```

3.84 $\int \arccos(ax)^{3/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 75

$$\int \arccos(ax)^{3/2}\,dx = -\frac{3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a} + x\arccos(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a}$$

```
[Out] x*arccos(a*x)^(3/2)+3/4*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2
)*Pi^(1/2)/a-3/2*(-a^2*x^2+1)^(1/2)*arccos(a*x)^(1/2)/a
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4716, 4768, 4720, 3386, 3432}

$$\int \arccos(ax)^{3/2}\,dx = -\frac{3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a} + x\arccos(ax)^{3/2}$$

```
[In] Int[ArcCos[a*x]^(3/2),x]
```

```
[Out] (-3*Sqrt[1 - a^2*x^2]*Sqrt[ArcCos[a*x]])/(2*a) + x*ArcCos[a*x]^(3/2) + (3*S
qrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(2*a)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = x\arccos(ax)^{3/2} + \frac{1}{2}(3a)\int \frac{x\sqrt{\arccos(ax)}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a} + x\arccos(ax)^{3/2} - \frac{3}{4}\int \frac{1}{\sqrt{\arccos(ax)}}\,dx$$

$$= -\frac{3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a} + x\arccos(ax)^{3/2} + \frac{3\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{4a}$$

$$= -\frac{3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a} + x\arccos(ax)^{3/2} + \frac{3\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{2a}$$

$$= -\frac{3\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}{2a} + x\arccos(ax)^{3/2} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \arccos(ax)^{3/2}\,dx = -\frac{\sqrt{-i\arccos(ax)}\Gamma\left(\frac{5}{2},-i\arccos(ax)\right)+\sqrt{i\arccos(ax)}\Gamma\left(\frac{5}{2},i\arccos(ax)\right)}{2a\sqrt{\arccos(ax)}}$$

```
[In] Integrate[ArcCos[a*x]^(3/2),x]
[Out] -1/2*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[5/2, (-I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a
*x]]*Gamma[5/2, I*ArcCos[a*x]])/(a*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{\sqrt{2}\left(-2\arccos(ax)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,ax+3\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\sqrt{-a^{2}x^{2}+1}-3\pi\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{4a\sqrt{\pi}}$	72

```
[In] int(arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4/a*2^(1/2)*(-2*arccos(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a*x+3*2^(1/2)*arccos(
a*x)^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2)-3*Pi*FresnelS(2^(1/2)/Pi^(1/2)*arcco
s(a*x)^(1/2)))/Pi^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \arccos(ax)^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arccos(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \arccos(ax)^{3/2}\,dx = \int \operatorname{acos}^{\frac{3}{2}}(ax)\,dx$$

```
[In] integrate(acos(a*x)**(3/2),x)
[Out] Integral(acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \arccos(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.59

$$\begin{aligned}\int \arccos(ax)^{3/2}\,dx &= \frac{\arccos(ax)^{\frac{3}{2}}e^{(i\arccos(ax))}}{2a} + \frac{\arccos(ax)^{\frac{3}{2}}e^{(-i\arccos(ax))}}{2a}\\ &+ \frac{(3i-3)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{16a}\\ &- \frac{(3i+3)\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{16a}\\ &+ \frac{3i\sqrt{\arccos(ax)}e^{(i\arccos(ax))}}{4a} - \frac{3i\sqrt{\arccos(ax)}e^{(-i\arccos(ax))}}{4a}\end{aligned}$$

```
[In] integrate(arccos(a*x)^(3/2),x, algorithm="giac")
[Out] 1/2*arccos(a*x)^(3/2)*e^(I*arccos(a*x))/a + 1/2*arccos(a*x)^(3/2)*e^(-I*arc
cos(a*x))/a + (3/16*I - 3/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sq
rt(arccos(a*x)))/a - (3/16*I + 3/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sq
rt(2)*sqrt(arccos(a*x)))/a + 3/4*I*sqrt(arccos(a*x))*e^(I*arccos(a*x))/a -
3/4*I*sqrt(arccos(a*x))*e^(-I*arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \arccos(ax)^{3/2}\,dx = \int \operatorname{acos}(a\,x)^{3/2}\,dx$$

```
[In] int(acos(a*x)^(3/2),x)
[Out] int(acos(a*x)^(3/2), x)
```

3.85 $\int \frac{\arccos(ax)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\arccos(ax)^{3/2}}{x} \, dx = \text{Int}\left(\frac{\arccos(ax)^{3/2}}{x}, x\right)$$

```
[Out] Unintegrable(arccos(a*x)^(3/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arccos(ax)^{3/2}}{x} \, dx = \int \frac{\arccos(ax)^{3/2}}{x} \, dx$$

```
[In] Int[ArcCos[a*x]^(3/2)/x,x]
```

```
[Out] Defer[Int][ArcCos[a*x]^(3/2)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arccos(ax)^{3/2}}{x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\arccos(ax)^{3/2}}{x}\,dx = \int \frac{\arccos(ax)^{3/2}}{x}\,dx$$

```
[In] Integrate[ArcCos[a*x]^(3/2)/x,x]
[Out] Integrate[ArcCos[a*x]^(3/2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arccos\left(ax\right)^{\frac{3}{2}}}{x}dx$$

```
[In] int(arccos(a*x)^(3/2)/x,x)
[Out] int(arccos(a*x)^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arccos(a*x)^(3/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.53 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arccos(ax)^{3/2}}{x}\,dx = \int \frac{\operatorname{acos}^{\frac{3}{2}}(ax)}{x}\,dx$$

```
[In] integrate(acos(a*x)**(3/2)/x,x)
[Out] Integral(acos(a*x)**(3/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{3/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^{3/2}}{x}\,dx = \int \frac{\arccos{(ax)}^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate(arccos(a*x)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^(3/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^{3/2}}{x}\,dx = \int \frac{\mathrm{acos}(a\,x)^{3/2}}{x}\,dx$$

```
[In] int(acos(a*x)^(3/2)/x,x)
```

```
[Out] int(acos(a*x)^(3/2)/x, x)
```

3.86 $\int x^4 \arccos(ax)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 298

$$\int x^4 \arccos(ax)^{5/2}\,dx = -\frac{2x\sqrt{\arccos(ax)}}{5a^4} - \frac{x^3\sqrt{\arccos(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\arccos(ax)} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{10a} + \frac{1}{5}x^5\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{60a^5} + \frac{\sqrt{\frac{3\pi}{2}}\,\text{Fresne}}{}$$

```
[Out] 1/5*x^5*arccos(a*x)^(5/2)+3/16000*FresnelC(10^(1/2)/Pi^(1/2)*arccos(a*x)^(1
/2))*10^(1/2)*Pi^(1/2)/a^5+5/1152*FresnelC(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/
2))*6^(1/2)*Pi^(1/2)/a^5+15/64*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))
*2^(1/2)*Pi^(1/2)/a^5-4/15*arccos(a*x)^(3/2)*(-a^2*x^2+1)^(1/2)/a^5-2/15*x^
2*arccos(a*x)^(3/2)*(-a^2*x^2+1)^(1/2)/a^3-1/10*x^4*arccos(a*x)^(3/2)*(-a^2
*x^2+1)^(1/2)/a-2/5*x*arccos(a*x)^(1/2)/a^4-1/15*x^3*arccos(a*x)^(1/2)/a^2-
3/100*x^5*arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 26, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used

= {4726, 4796, 4768, 4716, 4810, 3385, 3433, 3393}

$$\int x^4 \arccos(ax)^{5/2}\,dx = \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{32a^5} + \frac{\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{320a^5} + \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{60a^5} + \frac{3\sqrt{\frac{\pi}{10}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{1600a^5} - \frac{2x\sqrt{\arccos(ax)}}{5a^4} - \frac{x^3\sqrt{\arccos(ax)}}{15a^2} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{10a} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^3} + \frac{1}{5}x^5\arccos(ax)^{5/2} - \frac{3}{100}x^5\sqrt{\arccos(ax)}$$

```
[In] Int[x^4*ArcCos[a*x]^(5/2),x]
```

```
[Out] (-2*x*Sqrt[ArcCos[a*x]])/(5*a^4) - (x^3*Sqrt[ArcCos[a*x]])/(15*a^2) - (3*x^
5*Sqrt[ArcCos[a*x]])/100 - (4*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(15*a^5)
 - (2*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(15*a^3) - (x^4*Sqrt[1 - a^2
*x^2]*ArcCos[a*x]^(3/2))/(10*a) + (x^5*ArcCos[a*x]^(5/2))/5 + (15*Sqrt[Pi/2
]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(32*a^5) + (Sqrt[Pi/6]*FresnelC[S
qrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(60*a^5) + (Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi
]*Sqrt[ArcCos[a*x]]])/(320*a^5) + (3*Sqrt[Pi/10]*FresnelC[Sqrt[10/Pi]*Sqrt[
ArcCos[a*x]]])/(1600*a^5)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
```

```
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{5}x^5\arccos(ax)^{5/2} + \frac{1}{2}a\int \frac{x^5\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{10a} + \frac{1}{5}x^5\arccos(ax)^{5/2} - \frac{3}{20}\int x^4\sqrt{\arccos(ax)}\,dx + \frac{2\int \frac{x^3\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{5a}$$

$$= -\frac{3}{100}x^5\sqrt{\arccos(ax)} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{10a} + \frac{1}{5}x^5\arccos(ax)^{5/2} + \frac{4\int \frac{x\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{15a^3} - \frac{\int x^2\sqrt{\arccos(ax)}\,dx}{5a^2} - \frac{1}{200}(3a)\int \frac{x^5}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx$$

$$= -\frac{x^3\sqrt{\arccos(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\arccos(ax)} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{10a} + \frac{1}{5}x^5\arccos(ax)^{5/2} + \frac{3\text{Subst}\left(\int \frac{\cos^5(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{200a^5} - \frac{2\int \sqrt{\arccos(ax)}\,dx}{5a^4} - \frac{\int \frac{x^3}{\sqrt{1-a^2x^2}\sqrt{\arccos(c}}}{30a}$$

$$= -\frac{2x\sqrt{\arccos(ax)}}{5a^4} - \frac{x^3\sqrt{\arccos(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\arccos(ax)} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{10a} + \frac{1}{5}x^5\arccos(ax)^{5/2} + \frac{3\text{Subst}\left(\int \left(\frac{5\cos(x)}{8\sqrt{x}} + \frac{5\cos(3x)}{16\sqrt{x}} + \frac{\cos(5x)}{16\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{200a^5} + \frac{\text{Subst}\left(\int \frac{\cos^3(x)}{\sqrt{x}}\,d\right.}{30}$$

$$= -\frac{2x\sqrt{\arccos(ax)}}{5a^4} - \frac{x^3\sqrt{\arccos(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\arccos(ax)} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{10a} + \frac{1}{5}x^5\arccos(ax)^{5/2} + \frac{3\text{Subst}\left(\int \frac{\cos(5x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{3200a^5} + \frac{3\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{640a^5} + \frac{3S}{}$$

$$= -\frac{2x\sqrt{\arccos(ax)}}{5a^4} - \frac{x^3\sqrt{\arccos(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\arccos(ax)} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{10a} + \frac{1}{5}x^5\arccos(ax)^{5/2} + \frac{3\text{Subst}\left(\int \cos\left(5x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{1600a^5} + \frac{\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{120a^5}$$

$$= -\frac{2x\sqrt{\arccos(ax)}}{5a^4} - \frac{x^3\sqrt{\arccos(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\arccos(ax)} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{10a} + \frac{1}{5}x^5\arccos(ax)^{5/2} + \frac{3\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{160a^5} + \frac{\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{5a^5} + \cdots$$

$$= -\frac{2x\sqrt{\arccos(ax)}}{5a^4} - \frac{x^3\sqrt{\arccos(ax)}}{15a^2} - \frac{3}{100}x^5\sqrt{\arccos(ax)} - \frac{4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^5} - \frac{2x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{15a^3} - \frac{x^4\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{10a} + \frac{1}{5}x^5\arccos(ax)^{5/2} + \frac{11\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{160a^5} + \frac{\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{5a^5} + \cdots$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.65

$$\int x^4 \arccos(ax)^{5/2}\,dx =$$

$$-\frac{i\left(33750\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{7}{2},-i\arccos(ax)\right) - 33750\sqrt{i\arccos(ax)}\,\Gamma\left(\frac{7}{2},i\arccos(ax)\right) + 625\sqrt{3}\sqrt{-i\arccos\cdots}\right.}{\cdots}$$

```
[In] Integrate[x^4*ArcCos[a*x]^(5/2),x]
```

```
[Out] ((-1/540000*I)*(33750*Sqrt[(-I)*ArcCos[a*x]]*Gamma[7/2, (-I)*ArcCos[a*x]] -
 33750*Sqrt[I*ArcCos[a*x]]*Gamma[7/2, I*ArcCos[a*x]] + 625*Sqrt[3]*Sqrt[(-I
)*ArcCos[a*x]]*Gamma[7/2, (-3*I)*ArcCos[a*x]] - 625*Sqrt[3]*Sqrt[I*ArcCos[a
*x]]*Gamma[7/2, (3*I)*ArcCos[a*x]] + 27*Sqrt[5]*Sqrt[(-I)*ArcCos[a*x]]*Gamm
a[7/2, (-5*I)*ArcCos[a*x]] - 27*Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[7/2, (5*I
)*ArcCos[a*x]]))/(a^5*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.78

method	result
default	$\frac{18000 \arccos(ax)^3 ax+9000 \arccos(ax)^3 \cos(3 \arccos(ax))+1800 \arccos(ax)^3 \cos(5 \arccos(ax))+27\sqrt{5}\,\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\ \mathrm{FresnelC}\left(\right.}{}$

[In] int(x^4*arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/144000/a^5*(18000*arccos(a*x)^3*a*x+9000*arccos(a*x)^3*cos(3*arccos(a*x))+1800*arccos(a*x)^3*cos(5*arccos(a*x))+27*5^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))+625*3^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))-45000*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)-7500*arccos(a*x)^2*sin(3*arccos(a*x))-900*arccos(a*x)^2*sin(5*arccos(a*x))+33750*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))-67500*arccos(a*x)*a*x-3750*arccos(a*x)*cos(3*arccos(a*x))-270*arccos(a*x)*cos(5*arccos(a*x)))/arccos(a*x)^(1/2)

Fricas [F(-2)]

Exception generated.

$$\int x^4 \arccos(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^4*arccos(a*x)^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^{5/2}\,dx = \text{Timed out}$$

[In] integrate(x**4*acos(a*x)**(5/2),x)

[Out] Timed out

Maxima [F(-2)]

Exception generated.

$$\int x^4 \arccos(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4*arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.55

$$\int x^4 \arccos(ax)^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^4*arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/160*arccos(a*x)^(5/2)*e^(5*I*arccos(a*x))/a^5 + 1/32*arccos(a*x)^(5/2)*e^
(3*I*arccos(a*x))/a^5 + 1/16*arccos(a*x)^(5/2)*e^(I*arccos(a*x))/a^5 + 1/16
*arccos(a*x)^(5/2)*e^(-I*arccos(a*x))/a^5 + 1/32*arccos(a*x)^(5/2)*e^(-3*I*
arccos(a*x))/a^5 + 1/160*arccos(a*x)^(5/2)*e^(-5*I*arccos(a*x))/a^5 + 1/320
*I*arccos(a*x)^(3/2)*e^(5*I*arccos(a*x))/a^5 + 5/192*I*arccos(a*x)^(3/2)*e^
(3*I*arccos(a*x))/a^5 + 5/32*I*arccos(a*x)^(3/2)*e^(I*arccos(a*x))/a^5 - 5/
32*I*arccos(a*x)^(3/2)*e^(-I*arccos(a*x))/a^5 - 5/192*I*arccos(a*x)^(3/2)*e
^(-3*I*arccos(a*x))/a^5 - 1/320*I*arccos(a*x)^(3/2)*e^(-5*I*arccos(a*x))/a^
5 - (3/64000*I + 3/64000)*sqrt(10)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(10)*sqrt
(arccos(a*x)))/a^5 + (3/64000*I - 3/64000)*sqrt(10)*sqrt(pi)*erf(-(1/2*I +
1/2)*sqrt(10)*sqrt(arccos(a*x)))/a^5 - (5/4608*I + 5/4608)*sqrt(6)*sqrt(pi)
*erf((1/2*I - 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 + (5/4608*I - 5/4608)*sqr
t(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 - (15/256*I
 + 15/256)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^
5 + (15/256*I - 15/256)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(ar
ccos(a*x)))/a^5 - 3/3200*sqrt(arccos(a*x))*e^(5*I*arccos(a*x))/a^5 - 5/384*
sqrt(arccos(a*x))*e^(3*I*arccos(a*x))/a^5 - 15/64*sqrt(arccos(a*x))*e^(I*ar
ccos(a*x))/a^5 - 15/64*sqrt(arccos(a*x))*e^(-I*arccos(a*x))/a^5 - 5/384*sqr
t(arccos(a*x))*e^(-3*I*arccos(a*x))/a^5 - 3/3200*sqrt(arccos(a*x))*e^(-5*I*
arccos(a*x))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int x^4 \arccos(ax)^{5/2}\,dx = \int x^4\,\mathrm{acos}(a\,x)^{5/2}\,dx$$

```
[In] int(x^4*acos(a*x)^(5/2),x)
```

```
[Out] int(x^4*acos(a*x)^(5/2), x)
```

3.87 $\int x^3 \arccos(ax)^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 205

$$\int x^3 \arccos(ax)^{5/2}\,dx = \frac{225\sqrt{\arccos(ax)}}{2048a^4} - \frac{45x^2\sqrt{\arccos(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\arccos(ax)} - \frac{15x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{32a} - \frac{3\arccos(ax)^{5/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4096a^4} + \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{256a^4}$$

```
[Out] -3/32*arccos(a*x)^(5/2)/a^4+1/4*x^4*arccos(a*x)^(5/2)+15/8192*FresnelC(2*2^
(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4+15/256*FresnelC(2*ar
ccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4-15/64*x*arccos(a*x)^(3/2)*(-a^2*x^2+
1)^(1/2)/a^3-5/32*x^3*arccos(a*x)^(3/2)*(-a^2*x^2+1)^(1/2)/a+225/2048*arcco
s(a*x)^(1/2)/a^4-45/256*x^2*arccos(a*x)^(1/2)/a^2-15/256*x^4*arccos(a*x)^(1
/2)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4726, 4796, 4738, 4810, 3393, 3385, 3433}

$$\int x^3 \arccos(ax)^{5/2} dx = \frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4096a^4} + \frac{15\sqrt{\pi}\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{256a^4} - \frac{3\arccos(ax)^{5/2}}{32a^4} + \frac{225\sqrt{\arccos(ax)}}{2048a^4} - \frac{45x^2\sqrt{\arccos(ax)}}{256a^2} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{32a} - \frac{15x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{64a^3} + \frac{1}{4}x^4\arccos(ax)^{5/2} - \frac{15}{256}x^4\sqrt{\arccos(ax)}$$

```
[In] Int[x^3*ArcCos[a*x]^(5/2),x]
```

```
[Out] (225*Sqrt[ArcCos[a*x]])/(2048*a^4) - (45*x^2*Sqrt[ArcCos[a*x]])/(256*a^2) -
 (15*x^4*Sqrt[ArcCos[a*x]])/256 - (15*x*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2)
)/(64*a^3) - (5*x^3*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(32*a) - (3*ArcCos
[a*x]^(5/2))/(32*a^4) + (x^4*ArcCos[a*x]^(5/2))/4 + (15*Sqrt[Pi/2]*FresnelC
[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(4096*a^4) + (15*Sqrt[Pi]*FresnelC[(2*Sqr
t[ArcCos[a*x]])/Sqrt[Pi]])/(256*a^4)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
```

```
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4\arccos(ax)^{5/2} + \frac{1}{8}(5a)\int \frac{x^4\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx \\
&= -\frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{32a} \\
&\quad + \frac{1}{4}x^4\arccos(ax)^{5/2} - \frac{15}{64}\int x^3\sqrt{\arccos(ax)}\,dx + \frac{15\int \frac{x^2\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{32a} \\
&= -\frac{15}{256}x^4\sqrt{\arccos(ax)} - \frac{15x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{64a^3} \\
&\quad - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{32a} + \frac{1}{4}x^4\arccos(ax)^{5/2} + \frac{15\int \frac{\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{64a^3} \\
&\quad - \frac{45\int x\sqrt{\arccos(ax)}\,dx}{128a^2} - \frac{1}{512}(15a)\int \frac{x^4}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx
\end{aligned}
$$

$$= -\frac{45x^2\sqrt{\arccos(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\arccos(ax)} - \frac{15x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{32a} - \frac{3\arccos(ax)^{5/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{5/2} + \frac{15\text{Subst}\left(\int \frac{\cos^4(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{512a^4} - \frac{45\int \frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx}{512a}$$

$$= -\frac{45x^2\sqrt{\arccos(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\arccos(ax)} - \frac{15x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{32a} - \frac{3\arccos(ax)^{5/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{5/2} + \frac{15\text{Subst}\left(\int \left(\frac{3}{8\sqrt{x}} + \frac{\cos(2x)}{2\sqrt{x}} + \frac{\cos(4x)}{8\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{512a^4} + \frac{45\text{Subst}\left(\int \frac{\cos^2(x)}{\sqrt{x}}\,d\right.}{512a}$$

$$= \frac{45\sqrt{\arccos(ax)}}{2048a^4} - \frac{45x^2\sqrt{\arccos(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\arccos(ax)} - \frac{15x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{32a} - \frac{3\arccos(ax)^{5/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{5/2} + \frac{15\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{4096a^4} + \frac{15\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{1024a^4} +$$

$$= \frac{225\sqrt{\arccos(ax)}}{2048a^4} - \frac{45x^2\sqrt{\arccos(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\arccos(ax)} - \frac{15x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{32a} - \frac{3\arccos(ax)^{5/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{5/2} + \frac{15\text{Subst}\left(\int \cos(4x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{2048a^4} + \frac{15\text{Subst}\left(\int \cos(2x^2)\,dx, x, \sqrt{\text{arc}}\right.}{512a^4}$$

$$= \frac{225\sqrt{\arccos(ax)}}{2048a^4} - \frac{45x^2\sqrt{\arccos(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\arccos(ax)} - \frac{15x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{32a} - \frac{3\arccos(ax)^{5/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4096a^4} + \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{1024a^4} + \frac{45\text{Su}}{}$$

$$= \frac{225\sqrt{\arccos(ax)}}{2048a^4} - \frac{45x^2\sqrt{\arccos(ax)}}{256a^2} - \frac{15}{256}x^4\sqrt{\arccos(ax)} - \frac{15x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{64a^3} - \frac{5x^3\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{32a} - \frac{3\arccos(ax)^{5/2}}{32a^4} + \frac{1}{4}x^4\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4096a^4} + \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{256a^4}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

$$\int x^3 \arccos(ax)^{5/2}\,dx =$$

$$-\frac{i\Big(16\sqrt{2}\sqrt{-i\arccos(ax)}\Gamma\big(\frac{7}{2},-2i\arccos(ax)\big)-16\sqrt{2}\sqrt{i\arccos(ax)}\Gamma\big(\frac{7}{2},2i\arccos(ax)\big)+\sqrt{-i\arccos(ax}}{2048a^4\sqrt{\arccos(ax)}}$$

```
[In] Integrate[x^3*ArcCos[a*x]^(5/2),x]
```

```
[Out] ((-1/2048*I)*(16*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[7/2, (-2*I)*ArcCos[a*
x]] - 16*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[7/2, (2*I)*ArcCos[a*x]] + Sqrt[(
-I)*ArcCos[a*x]]*Gamma[7/2, (-4*I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma
[7/2, (4*I)*ArcCos[a*x]]))/(a^4*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.75

method	result
default	$1024\arccos(ax)^{\frac{5}{2}}\cos(2\arccos(ax))\sqrt{\pi}+256\arccos(ax)^{\frac{5}{2}}\cos(4\arccos(ax))\sqrt{\pi}-1280\arccos(ax)^{\frac{3}{2}}\sin(2\arccos(ax))\sqrt{\pi}-160\arccos($

```
[In] int(x^3*arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8192/a^4/Pi^(1/2)*(1024*arccos(a*x)^(5/2)*cos(2*arccos(a*x))*Pi^(1/2)+256
*arccos(a*x)^(5/2)*cos(4*arccos(a*x))*Pi^(1/2)-1280*arccos(a*x)^(3/2)*sin(2
*arccos(a*x))*Pi^(1/2)-160*arccos(a*x)^(3/2)*sin(4*arccos(a*x))*Pi^(1/2)+15
*Pi*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)-960*cos(2*arccos
(a*x))*arccos(a*x)^(1/2)*Pi^(1/2)+480*Pi*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1
/2))-60*cos(4*arccos(a*x))*arccos(a*x)^(1/2)*Pi^(1/2))
```

Fricas [F(-2)]

Exception generated.

$$\int x^3 \arccos(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3*arccos(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^3 \arccos(ax)^{5/2} \, dx = \int x^3 \operatorname{acos}^{\frac{5}{2}}(ax) \, dx$$

```
[In] integrate(x**3*acos(a*x)**(5/2),x)
[Out] Integral(x**3*acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \arccos(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*arccos(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.45

$$\int x^3 \arccos(ax)^{5/2}\,dx = \frac{\arccos(ax)^{\frac{5}{2}}e^{(4i\arccos(ax))}}{64\,a^4} + \frac{\arccos(ax)^{\frac{5}{2}}e^{(2i\arccos(ax))}}{16\,a^4} + \frac{\arccos(ax)^{\frac{5}{2}}e^{(-2i\arccos(ax))}}{16\,a^4} + \frac{\arccos(ax)^{\frac{5}{2}}e^{(-4i\arccos(ax))}}{64\,a^4} + \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(4i\arccos(ax))}}{512\,a^4} + \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(2i\arccos(ax))}}{64\,a^4} - \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(-2i\arccos(ax))}}{64\,a^4} - \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(-4i\arccos(ax))}}{512\,a^4} - \frac{(15i+15)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{32768\,a^4} + \frac{(15i-15)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{2}\sqrt{\arccos(ax)}\right)}{32768\,a^4} - \frac{(15i+15)\sqrt{\pi}\,\mathrm{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{1024\,a^4} + \frac{(15i-15)\sqrt{\pi}\,\mathrm{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{1024\,a^4} - \frac{15\sqrt{\arccos(ax)}e^{(4i\arccos(ax))}}{4096\,a^4} - \frac{15\sqrt{\arccos(ax)}e^{(2i\arccos(ax))}}{256\,a^4} - \frac{15\sqrt{\arccos(ax)}e^{(-2i\arccos(ax))}}{256\,a^4} - \frac{15\sqrt{\arccos(ax)}e^{(-4i\arccos(ax))}}{4096\,a^4}$$

```
[In] integrate(x^3*arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*arccos(a*x)^(5/2)*e^(4*I*arccos(a*x))/a^4 + 1/16*arccos(a*x)^(5/2)*e^(
2*I*arccos(a*x))/a^4 + 1/16*arccos(a*x)^(5/2)*e^(-2*I*arccos(a*x))/a^4 + 1/
64*arccos(a*x)^(5/2)*e^(-4*I*arccos(a*x))/a^4 + 5/512*I*arccos(a*x)^(3/2)*e
^(4*I*arccos(a*x))/a^4 + 5/64*I*arccos(a*x)^(3/2)*e^(2*I*arccos(a*x))/a^4 -
 5/64*I*arccos(a*x)^(3/2)*e^(-2*I*arccos(a*x))/a^4 - 5/512*I*arccos(a*x)^(3
/2)*e^(-4*I*arccos(a*x))/a^4 - (15/32768*I + 15/32768)*sqrt(2)*sqrt(pi)*erf
((I - 1)*sqrt(2)*sqrt(arccos(a*x)))/a^4 + (15/32768*I - 15/32768)*sqrt(2)*s
qrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arccos(a*x)))/a^4 - (15/1024*I + 15/1024)
*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^4 + (15/1024*I - 15/1024)*sqrt(p
i)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^4 - 15/4096*sqrt(arccos(a*x))*e^(4*I*a
rccos(a*x))/a^4 - 15/256*sqrt(arccos(a*x))*e^(2*I*arccos(a*x))/a^4 - 15/256
*sqrt(arccos(a*x))*e^(-2*I*arccos(a*x))/a^4 - 15/4096*sqrt(arccos(a*x))*e^(
-4*I*arccos(a*x))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^{5/2}\,dx = \int x^3\,\mathrm{acos}(a\,x)^{5/2}\,dx$$

```
[In] int(x^3*acos(a*x)^(5/2),x)
```

```
[Out] int(x^3*acos(a*x)^(5/2), x)
```

3.88 $\int x^2 \arccos(ax)^{5/2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 178

$$\int x^2 \arccos(ax)^{5/2} \, dx = -\frac{5x\sqrt{\arccos(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{18a} + \frac{1}{3}x^3\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^3} + \frac{5\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{144a^3}$$

```
[Out] 1/3*x^3*arccos(a*x)^(5/2)+5/864*FresnelC(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)
)*6^(1/2)*Pi^(1/2)/a^3+15/32*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2
^(1/2)*Pi^(1/2)/a^3-5/9*arccos(a*x)^(3/2)*(-a^2*x^2+1)^(1/2)/a^3-5/18*x^2*a
rccos(a*x)^(3/2)*(-a^2*x^2+1)^(1/2)/a-5/6*x*arccos(a*x)^(1/2)/a^2-5/36*x^3*
arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {4726, 4796, 4768, 4716, 4810, 3385, 3433, 3393}

$$\int x^2 \arccos(ax)^{5/2} \, dx = \frac{15\sqrt{\frac{\pi}{2}}\operatorname{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^3} + \frac{5\sqrt{\frac{\pi}{6}}\operatorname{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{144a^3} - \frac{5x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{18a} - \frac{5x\sqrt{\arccos(ax)}}{6a^2} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{9a^3} + \frac{1}{3}x^3\arccos(ax)^{5/2} - \frac{5}{36}x^3\sqrt{\arccos(ax)}$$

```
[In] Int[x^2*ArcCos[a*x]^(5/2),x]
```

```
[Out] (-5*x*Sqrt[ArcCos[a*x]])/(6*a^2) - (5*x^3*Sqrt[ArcCos[a*x]])/36 - (5*Sqrt[1
 - a^2*x^2]*ArcCos[a*x]^(3/2))/(9*a^3) - (5*x^2*Sqrt[1 - a^2*x^2]*ArcCos[a*
x]^(3/2))/(18*a) + (x^3*ArcCos[a*x]^(5/2))/3 + (15*Sqrt[Pi/2]*FresnelC[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/(16*a^3) + (5*Sqrt[Pi/6]*FresnelC[Sqrt[6/Pi]*Sqr
t[ArcCos[a*x]]])/(144*a^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{3}x^3\arccos(ax)^{5/2} + \frac{1}{6}(5a)\int \frac{x^3\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{5x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{18a} + \frac{1}{3}x^3\arccos(ax)^{5/2} - \frac{5}{12}\int x^2\sqrt{\arccos(ax)}\,dx + \frac{5\int \frac{x\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{9a}$$

$$= -\frac{5}{36}x^3\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{18a} + \frac{1}{3}x^3\arccos(ax)^{5/2} - \frac{5\int \sqrt{\arccos(ax)}\,dx}{6a^2} - \frac{1}{72}(5a)\int \frac{x^3}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx$$

$$= -\frac{5x\sqrt{\arccos(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{18a} + \frac{1}{3}x^3\arccos(ax)^{5/2} + \frac{5\text{Subst}\left(\int \frac{\cos^3(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{72a^3} - \frac{5\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx}{12a}$$

$$= -\frac{5x\sqrt{\arccos(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{18a} + \frac{1}{3}x^3\arccos(ax)^{5/2} + \frac{5\text{Subst}\left(\int \left(\frac{3\cos(x)}{4\sqrt{x}} + \frac{\cos(3x)}{4\sqrt{x}}\right) dx, x, \arccos(ax)\right)}{72a^3} + \frac{5\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \arccos(ax)\right)}{12a^3}$$

$$= -\frac{5x\sqrt{\arccos(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{18a} + \frac{1}{3}x^3\arccos(ax)^{5/2} + \frac{5\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}} dx, x, \arccos(ax)\right)}{288a^3} + \frac{5\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \arccos(ax)\right)}{96a^3} + \frac{5\text{Subst}\left(\int \cos\left(x^2\right) dx, x, \sqrt{\arccos(ax)}\right)}{6a^3}$$

$$= -\frac{5x\sqrt{\arccos(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{18a} + \frac{1}{3}x^3\arccos(ax)^{5/2} + \frac{5\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^3} + \frac{5\text{Subst}\left(\int \cos\left(3x^2\right) dx, x, \sqrt{\arccos(ax)}\right)}{144a^3} + \frac{5\text{Subst}\left(\int \cos\left(x^2\right) dx, x, \sqrt{\arccos(ax)}\right)}{48a^3}$$

$$= -\frac{5x\sqrt{\arccos(ax)}}{6a^2} - \frac{5}{36}x^3\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{9a^3} - \frac{5x^2\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{18a} + \frac{1}{3}x^3\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^3} + \frac{5\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{144a^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.06 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.72

$$\int x^2 \arccos(ax)^{5/2} \, dx =$$

$$-\frac{i\left(81\sqrt{-i\arccos(ax)}\Gamma\left(\frac{7}{2},-i\arccos(ax)\right)-81\sqrt{i\arccos(ax)}\Gamma\left(\frac{7}{2},i\arccos(ax)\right)+\sqrt{3}\left(\sqrt{-i\arccos(ax)}\Gamma\left(\frac{7}{2}\right.\right.\right.}{648a^3\sqrt{\arccos(ax)}}$$

```
[In] Integrate[x^2*ArcCos[a*x]^(5/2),x]
```

```
[Out] ((-1/648*I)*(81*Sqrt[(-I)*ArcCos[a*x]]*Gamma[7/2, (-I)*ArcCos[a*x]] - 81*Sq
rt[I*ArcCos[a*x]]*Gamma[7/2, I*ArcCos[a*x]] + Sqrt[3]*(Sqrt[(-I)*ArcCos[a*x
]]*Gamma[7/2, (-3*I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[7/2, (3*I)*Ar
cCos[a*x]])))/(a^3*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.88

method	result
default	$\frac{216\arccos(ax)^3 ax+72\arccos(ax)^3\cos(3\arccos(ax))+5\sqrt{3}\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{3}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-540\arccos(ax)^2\sqrt{-}}{86}$

```
[In] int(x^2*arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/864/a^3*(216*arccos(a*x)^3*a*x+72*arccos(a*x)^3*cos(3*arccos(a*x))+5*3^(1
/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*ar
ccos(a*x)^(1/2))-540*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)-60*arccos(a*x)^2*sin(
3*arccos(a*x))+405*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(
1/2)*arccos(a*x)^(1/2))-810*arccos(a*x)*a*x-30*arccos(a*x)*cos(3*arccos(a*x
)))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \arccos(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*arccos(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2 \arccos(ax)^{5/2} \, dx = \int x^2 \operatorname{acos}^{\frac{5}{2}}(ax) \, dx$$

```
[In] integrate(x**2*acos(a*x)**(5/2),x)
[Out] Integral(x**2*acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \arccos(ax)^{5/2} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*arccos(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 309, normalized size of antiderivative = 1.74

$$\int x^2 \arccos(ax)^{5/2}\,dx = \frac{\arccos(ax)^{\frac{5}{2}} e^{(3i\arccos(ax))}}{24a^3} + \frac{\arccos(ax)^{\frac{5}{2}} e^{(i\arccos(ax))}}{8a^3} + \frac{\arccos(ax)^{\frac{5}{2}} e^{(-i\arccos(ax))}}{8a^3} + \frac{\arccos(ax)^{\frac{5}{2}} e^{(-3i\arccos(ax))}}{24a^3} + \frac{5i\arccos(ax)^{\frac{3}{2}} e^{(3i\arccos(ax))}}{144a^3} + \frac{5i\arccos(ax)^{\frac{3}{2}} e^{(i\arccos(ax))}}{16a^3} - \frac{5i\arccos(ax)^{\frac{3}{2}} e^{(-i\arccos(ax))}}{16a^3} - \frac{5i\arccos(ax)^{\frac{3}{2}} e^{(-3i\arccos(ax))}}{144a^3} - \frac{(5i+5)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{3456a^3} + \frac{(5i-5)\sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{3456a^3} - \frac{(15i+15)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{128a^3} + \frac{(15i-15)\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{128a^3} - \frac{5\sqrt{\arccos(ax)}e^{(3i\arccos(ax))}}{288a^3} - \frac{15\sqrt{\arccos(ax)}e^{(i\arccos(ax))}}{32a^3} - \frac{15\sqrt{\arccos(ax)}e^{(-i\arccos(ax))}}{32a^3} - \frac{5\sqrt{\arccos(ax)}e^{(-3i\arccos(ax))}}{288a^3}$$

```
[In] integrate(x^2*arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/24*arccos(a*x)^(5/2)*e^(3*I*arccos(a*x))/a^3 + 1/8*arccos(a*x)^(5/2)*e^(I
*arccos(a*x))/a^3 + 1/8*arccos(a*x)^(5/2)*e^(-I*arccos(a*x))/a^3 + 1/24*arc
cos(a*x)^(5/2)*e^(-3*I*arccos(a*x))/a^3 + 5/144*I*arccos(a*x)^(3/2)*e^(3*I*
arccos(a*x))/a^3 + 5/16*I*arccos(a*x)^(3/2)*e^(I*arccos(a*x))/a^3 - 5/16*I*
arccos(a*x)^(3/2)*e^(-I*arccos(a*x))/a^3 - 5/144*I*arccos(a*x)^(3/2)*e^(-3*
I*arccos(a*x))/a^3 - (5/3456*I + 5/3456)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)
*sqrt(6)*sqrt(arccos(a*x)))/a^3 + (5/3456*I - 5/3456)*sqrt(6)*sqrt(pi)*erf(
-(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^3 - (15/128*I + 15/128)*sqrt(2)
*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^3 + (15/128*I - 15
/128)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^3 -
5/288*sqrt(arccos(a*x))*e^(3*I*arccos(a*x))/a^3 - 15/32*sqrt(arccos(a*x))*e
^(I*arccos(a*x))/a^3 - 15/32*sqrt(arccos(a*x))*e^(-I*arccos(a*x))/a^3 - 5/2
88*sqrt(arccos(a*x))*e^(-3*I*arccos(a*x))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^{5/2}\,dx = \int x^2\,\mathrm{acos}(a\,x)^{5/2}\,dx$$

```
[In] int(x^2*acos(a*x)^(5/2),x)
```

```
[Out] int(x^2*acos(a*x)^(5/2), x)
```

3.89 $\int x \arccos(ax)^{5/2} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 119

$$\int x \arccos(ax)^{5/2} \, dx = \frac{15\sqrt{\arccos(ax)}}{64a^2} - \frac{15}{32}x^2\sqrt{\arccos(ax)} - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{8a} - \frac{\arccos(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{5/2} + \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{128a^2}$$

```
[Out] -1/4*arccos(a*x)^(5/2)/a^2+1/2*x^2*arccos(a*x)^(5/2)+15/128*FresnelC(2*arcc
os(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2-5/8*x*arccos(a*x)^(3/2)*(-a^2*x^2+1)^(
1/2)/a+15/64*arccos(a*x)^(1/2)/a^2-15/32*x^2*arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {4726, 4796, 4738, 4810, 3393, 3385, 3433}

$$\int x \arccos(ax)^{5/2} \, dx = \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{128a^2} - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{8a} - \frac{\arccos(ax)^{5/2}}{4a^2} + \frac{15\sqrt{\arccos(ax)}}{64a^2} + \frac{1}{2}x^2\arccos(ax)^{5/2} - \frac{15}{32}x^2\sqrt{\arccos(ax)}$$

```
[In] Int[x*ArcCos[a*x]^(5/2),x]
```

```
[Out] (15*Sqrt[ArcCos[a*x]])/(64*a^2) - (15*x^2*Sqrt[ArcCos[a*x]])/32 - (5*x*Sqrt
[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(8*a) - ArcCos[a*x]^(5/2)/(4*a^2) + (x^2*A
```

```
rcCos[a*x]^(5/2))/2 + (15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]]
)/(128*a^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\arccos(ax)^{5/2} + \frac{1}{4}(5a)\int\frac{x^2\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx$$

$$= -\frac{5x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{8a} + \frac{1}{2}x^2\arccos(ax)^{5/2} - \frac{15}{16}\int x\sqrt{\arccos(ax)}\,dx + \frac{5\int\frac{\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx}{8a}$$

$$= -\frac{15}{32}x^2\sqrt{\arccos(ax)} - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{8a} - \frac{\arccos(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{5/2} - \frac{1}{64}(15a)\int\frac{x^2}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx$$

$$= -\frac{15}{32}x^2\sqrt{\arccos(ax)} - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{8a} - \frac{\arccos(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{5/2} + \frac{15\text{Subst}\left(\int\frac{\cos^2(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{64a^2}$$

$$= -\frac{15}{32}x^2\sqrt{\arccos(ax)} - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{8a} - \frac{\arccos(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{5/2} + \frac{15\text{Subst}\left(\int\left(\frac{1}{2\sqrt{x}} + \frac{\cos(2x)}{2\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{64a^2}$$

$$= \frac{15\sqrt{\arccos(ax)}}{64a^2} - \frac{15}{32}x^2\sqrt{\arccos(ax)} - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{8a} - \frac{\arccos(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{5/2} + \frac{15\text{Subst}\left(\int\frac{\cos(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{128a^2}$$

$$= \frac{15\sqrt{\arccos(ax)}}{64a^2} - \frac{15}{32}x^2\sqrt{\arccos(ax)} - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{8a} - \frac{\arccos(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{5/2} + \frac{15\text{Subst}\left(\int\cos\left(2x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{64a^2}$$

$$= \frac{15\sqrt{\arccos(ax)}}{64a^2} - \frac{15}{32}x^2\sqrt{\arccos(ax)} - \frac{5x\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{8a} - \frac{\arccos(ax)^{5/2}}{4a^2} + \frac{1}{2}x^2\arccos(ax)^{5/2} + \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{128a^2}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

$$\int x \arccos(ax)^{5/2} \, dx = \frac{15\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) - 2\sqrt{\arccos(ax)}((15-16\arccos(ax)^2)\cos(2\arccos(ax}{128a^2}$$

```
[In] Integrate[x*ArcCos[a*x]^(5/2),x]
```

```
[Out] (15*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] - 2*Sqrt[ArcCos[a*x]]
*((15 - 16*ArcCos[a*x]^2)*Cos[2*ArcCos[a*x]] + 20*ArcCos[a*x]*Sin[2*ArcCos[
a*x]]))/(128*a^2)
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.66

method	result
default	$\frac{32\arccos(ax)^{\frac{5}{2}}\cos(2\arccos(ax))\sqrt{\pi}-40\arccos(ax)^{\frac{3}{2}}\sin(2\arccos(ax))\sqrt{\pi}-30\cos(2\arccos(ax))\sqrt{\arccos(ax)}\,\sqrt{\pi}+15\pi\ \text{FresnelC}\left(\frac{2\sqrt{a}}{}\right.}{128a^2\sqrt{\pi}}$

```
[In] int(x*arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/a^2/Pi^(1/2)*(32*arccos(a*x)^(5/2)*cos(2*arccos(a*x))*Pi^(1/2)-40*arc
cos(a*x)^(3/2)*sin(2*arccos(a*x))*Pi^(1/2)-30*cos(2*arccos(a*x))*arccos(a*x
)^(1/2)*Pi^(1/2)+15*Pi*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int x \arccos(ax)^{5/2} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x \arccos(ax)^{5/2}\,dx = \int x \operatorname{acos}^{\frac{5}{2}}(ax)\,dx$$

```
[In] integrate(x*acos(a*x)**(5/2),x)
```

```
[Out] Integral(x*acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int x \arccos(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.20

$$\begin{aligned}\int x \arccos(ax)^{5/2}\,dx &= \frac{\arccos(ax)^{\frac{5}{2}}e^{(2i\arccos(ax))}}{8a^2} \\ &+ \frac{\arccos(ax)^{\frac{5}{2}}e^{(-2i\arccos(ax))}}{8a^2} + \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(2i\arccos(ax))}}{32a^2} \\ &- \frac{5i\arccos(ax)^{\frac{3}{2}}e^{(-2i\arccos(ax))}}{32a^2} - \frac{(15i+15)\sqrt{\pi}\operatorname{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{512a^2} \\ &+ \frac{(15i-15)\sqrt{\pi}\operatorname{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{512a^2} \\ &- \frac{15\sqrt{\arccos(ax)}e^{(2i\arccos(ax))}}{128a^2} - \frac{15\sqrt{\arccos(ax)}e^{(-2i\arccos(ax))}}{128a^2}\end{aligned}$$

```
[In] integrate(x*arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/8*arccos(a*x)^(5/2)*e^(2*I*arccos(a*x))/a^2 + 1/8*arccos(a*x)^(5/2)*e^(-2
*I*arccos(a*x))/a^2 + 5/32*I*arccos(a*x)^(3/2)*e^(2*I*arccos(a*x))/a^2 - 5/
32*I*arccos(a*x)^(3/2)*e^(-2*I*arccos(a*x))/a^2 - (15/512*I + 15/512)*sqrt(
pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^2 + (15/512*I - 15/512)*sqrt(pi)*erf(-
(I + 1)*sqrt(arccos(a*x)))/a^2 - 15/128*sqrt(arccos(a*x))*e^(2*I*arccos(a*x
))/a^2 - 15/128*sqrt(arccos(a*x))*e^(-2*I*arccos(a*x))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int x\arccos(ax)^{5/2}\,dx=\int x\,\mathrm{acos}(a\,x)^{5/2}\,dx$$

```
[In] int(x*acos(a*x)^(5/2),x)
[Out] int(x*acos(a*x)^(5/2), x)
```

3.90 $\int \arccos(ax)^{5/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 88

$$\int \arccos(ax)^{5/2}\,dx = -\frac{15}{4}x\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a} + x\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a}$$

```
[Out] x*arccos(a*x)^(5/2)+15/8*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/
2)*Pi^(1/2)/a-5/2*arccos(a*x)^(3/2)*(-a^2*x^2+1)^(1/2)/a-15/4*x*arccos(a*x)
^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {4716, 4768, 4810, 3385, 3433}

$$\int \arccos(ax)^{5/2}\,dx = -\frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a} + x\arccos(ax)^{5/2} - \frac{15}{4}x\sqrt{\arccos(ax)}$$

```
[In] Int[ArcCos[a*x]^(5/2),x]
```

```
[Out] (-15*x*Sqrt[ArcCos[a*x]])/4 - (5*Sqrt[1 - a^2*x^2]*ArcCos[a*x]^(3/2))/(2*a)
 + x*ArcCos[a*x]^(5/2) + (15*Sqrt[Pi/2]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x
]]])/(4*a)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\arccos(ax)^{5/2}+\frac{1}{2}(5a)\int\frac{x\arccos(ax)^{3/2}}{\sqrt{1-a^2x^2}}\,dx\\
&= -\frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a}+x\arccos(ax)^{5/2}-\frac{15}{4}\int\sqrt{\arccos(ax)}\,dx\\
&= -\frac{15}{4}x\sqrt{\arccos(ax)}-\frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a}\\
&\quad +x\arccos(ax)^{5/2}-\frac{1}{8}(15a)\int\frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx
\end{aligned}$$

$$= -\frac{15}{4}x\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a} + x\arccos(ax)^{5/2} + \frac{15\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{8a}$$

$$= -\frac{15}{4}x\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a} + x\arccos(ax)^{5/2} + \frac{15\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{4a}$$

$$= -\frac{15}{4}x\sqrt{\arccos(ax)} - \frac{5\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}{2a} + x\arccos(ax)^{5/2} + \frac{15\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \arccos(ax)^{5/2}\,dx = -\frac{i\left(\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{7}{2}, -i\arccos(ax)\right) - \sqrt{i\arccos(ax)}\,\Gamma\left(\frac{7}{2}, i\arccos(ax)\right)\right)}{2a\sqrt{\arccos(ax)}}$$

```
[In] Integrate[ArcCos[a*x]^(5/2),x]
[Out] ((-1/2*I)*(Sqrt[(-I)*ArcCos[a*x]]*Gamma[7/2, (-I)*ArcCos[a*x]] - Sqrt[I*Arc
Cos[a*x]]*Gamma[7/2, I*ArcCos[a*x]]))/(a*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00

method	result
default	$-\frac{\sqrt{2}\left(-4\arccos(ax)^{\frac{5}{2}}\sqrt{2}\sqrt{\pi}\,ax+10\arccos(ax)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi}\sqrt{-a^2x^2+1}+15\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\,ax-15\pi\,\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{8a\sqrt{\pi}}$

```
[In] int(arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
[Out] -1/8/a*2^(1/2)*(-4*arccos(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*a*x+10*arccos(a*x)^(3
/2)*2^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2)+15*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/
2)*a*x-15*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)))/Pi^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \arccos(ax)^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \arccos(ax)^{5/2}\,dx = \int \operatorname{acos}^{\frac{5}{2}}(ax)\,dx$$

```
[In] integrate(acos(a*x)**(5/2),x)
```

```
[Out] Integral(acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \arccos(ax)^{5/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.76

$$\int \arccos(ax)^{5/2}\,dx = \frac{\arccos{(ax)}^{\frac{5}{2}}\,e^{(i\arccos(ax))}}{2\,a} + \frac{\arccos{(ax)}^{\frac{5}{2}}\,e^{(-i\arccos(ax))}}{2\,a} + \frac{5i\arccos{(ax)}^{\frac{3}{2}}\,e^{(i\arccos(ax))}}{4\,a} - \frac{5i\arccos{(ax)}^{\frac{3}{2}}\,e^{(-i\arccos(ax))}}{4\,a} - \frac{(15i+15)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos{(ax)}}\right)}{32\,a} + \frac{(15i-15)\,\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos{(ax)}}\right)}{32\,a} - \frac{15\,\sqrt{\arccos{(ax)}}e^{(i\arccos(ax))}}{8\,a} - \frac{15\,\sqrt{\arccos{(ax)}}e^{(-i\arccos(ax))}}{8\,a}$$

```
[In] integrate(arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] 1/2*arccos(a*x)^(5/2)*e^(I*arccos(a*x))/a + 1/2*arccos(a*x)^(5/2)*e^(-I*arc
cos(a*x))/a + 5/4*I*arccos(a*x)^(3/2)*e^(I*arccos(a*x))/a - 5/4*I*arccos(a*
x)^(3/2)*e^(-I*arccos(a*x))/a - (15/32*I + 15/32)*sqrt(2)*sqrt(pi)*erf((1/2
*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a + (15/32*I - 15/32)*sqrt(2)*sqrt(pi)
*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a - 15/8*sqrt(arccos(a*x))*e
^(I*arccos(a*x))/a - 15/8*sqrt(arccos(a*x))*e^(-I*arccos(a*x))/a
```

Mupad [F(-1)]

Timed out.

$$\int \arccos(ax)^{5/2}\,dx = \int \mathrm{acos}(a\,x)^{5/2}\,dx$$

```
[In] int(acos(a*x)^(5/2),x)
```

```
[Out] int(acos(a*x)^(5/2), x)
```

3.91 $\int \frac{\arccos(ax)^{5/2}}{x} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\arccos(ax)^{5/2}}{x} \, dx = \text{Int}\left(\frac{\arccos(ax)^{5/2}}{x}, x\right)$$

`[Out] Unintegrable(arccos(a*x)^(5/2)/x,x)`

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arccos(ax)^{5/2}}{x} \, dx = \int \frac{\arccos(ax)^{5/2}}{x} \, dx$$

`[In] Int[ArcCos[a*x]^(5/2)/x,x]`

`[Out] Defer[Int][ArcCos[a*x]^(5/2)/x, x]`

Rubi steps

$$\text{integral} = \int \frac{\arccos(ax)^{5/2}}{x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\arccos(ax)^{5/2}}{x}\,dx = \int \frac{\arccos(ax)^{5/2}}{x}\,dx$$

```
[In] Integrate[ArcCos[a*x]^(5/2)/x,x]
[Out] Integrate[ArcCos[a*x]^(5/2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.15 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arccos\left(ax\right)^{\frac{5}{2}}}{x}dx$$

```
[In] int(arccos(a*x)^(5/2)/x,x)
[Out] int(arccos(a*x)^(5/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(arccos(a*x)^(5/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 19.29 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\arccos(ax)^{5/2}}{x}\,dx = \int \frac{\operatorname{acos}^{\frac{5}{2}}\left(ax\right)}{x}\,dx$$

```
[In] integrate(acos(a*x)**(5/2)/x,x)
[Out] Integral(acos(a*x)**(5/2)/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^{5/2}}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^(5/2)/x,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^{5/2}}{x}\,dx = \int \frac{\arccos\left(ax\right)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate(arccos(a*x)^(5/2)/x,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^(5/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^{5/2}}{x}\,dx = \int \frac{\mathrm{acos}(a\,x)^{5/2}}{x}\,dx$$

```
[In] int(acos(a*x)^(5/2)/x,x)
```

```
[Out] int(acos(a*x)^(5/2)/x, x)
```

3.92 $\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 106

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{3\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5} - \frac{\sqrt{\frac{\pi}{10}}\operatorname{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5}$$

```
[Out] -1/80*FresnelS(10^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*10^(1/2)*Pi^(1/2)/a^5-1
/8*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^5-1/16*F
resnelS(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a^5
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4732, 4491, 3386, 3432}

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{3\pi}{2}}\operatorname{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5} - \frac{\sqrt{\frac{\pi}{10}}\operatorname{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5}$$

```
[In] Int[x^4/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -1/4*(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^5 - (Sqrt[(3*Pi)
/2]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(8*a^5) - (Sqrt[Pi/10]*FresnelS
[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(8*a^5)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cos^4(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^5}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{8\sqrt{x}} + \frac{3\sin(3x)}{16\sqrt{x}} + \frac{\sin(5x)}{16\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{a^5}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(5x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{16a^5} - \frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{8a^5}\\
&\quad - \frac{3\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{16a^5}\\
&= -\frac{\text{Subst}\left(\int \sin\left(5x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{8a^5} - \frac{\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{4a^5}\\
&\quad - \frac{3\text{Subst}\left(\int \sin\left(3x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{8a^5}
\end{aligned}
$$

$$= -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{3\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5} - \frac{\sqrt{\frac{\pi}{10}}\,\text{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^5}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.81

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = -\frac{-10\sqrt{-i\arccos(ax)}\Gamma\left(\frac{1}{2},-i\arccos(ax)\right) - 10\sqrt{i\arccos(ax)}\Gamma\left(\frac{1}{2},i\arccos(ax)\right) - 5\sqrt{3}\sqrt{-i\arccos(ax)}\Gamma}{}$$

```
[In] Integrate[x^4/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -1/160*(-10*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a*x]] - 10*Sqrt[I
*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] - 5*Sqrt[3]*Sqrt[(-I)*ArcCos[a*x]]*
Gamma[1/2, (-3*I)*ArcCos[a*x]] - 5*Sqrt[3]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (
3*I)*ArcCos[a*x]] - Sqrt[5]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-5*I)*ArcCos
[a*x]] - Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (5*I)*ArcCos[a*x]])/(a^5*Sq
rt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.68

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\sqrt{5}\,\text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+5\sqrt{3}\,\text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+10\,\text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\right)}{80a^5}$	72

```
[In] int(x^4/arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/80/a^5*2^(1/2)*Pi^(1/2)*(5^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)*arcco
s(a*x)^(1/2))+5*3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2)
)+10*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{\arccos(ax)}} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\sqrt{\arccos(ax)}} \, dx = \int \frac{x^4}{\sqrt{\operatorname{acos}(ax)}} \, dx$$

```
[In] integrate(x**4/acos(a*x)**(1/2),x)
```

```
[Out] Integral(x**4/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{\sqrt{\arccos(ax)}} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.31

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = -\frac{(i-1)\sqrt{10}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{10}\sqrt{\arccos(ax)}\right)}{320a^5} + \frac{(i+1)\sqrt{10}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{10}\sqrt{\arccos(ax)}\right)}{320a^5} - \frac{(i-1)\sqrt{6}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{64a^5} + \frac{(i+1)\sqrt{6}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{6}\sqrt{\arccos(ax)}\right)}{64a^5} - \frac{(i-1)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{32a^5} + \frac{(i+1)\sqrt{2}\sqrt{\pi}\,\text{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\sqrt{2}\sqrt{\arccos(ax)}\right)}{32a^5}$$

```
[In] integrate(x^4/arccos(a*x)^(1/2),x, algorithm="giac")
[Out] -(1/320*I - 1/320)*sqrt(10)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(10)*sqrt(arccos
(a*x)))/a^5 + (1/320*I + 1/320)*sqrt(10)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(1
0)*sqrt(arccos(a*x)))/a^5 - (1/64*I - 1/64)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1
/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 + (1/64*I + 1/64)*sqrt(6)*sqrt(pi)*erf(-
(1/2*I + 1/2)*sqrt(6)*sqrt(arccos(a*x)))/a^5 - (1/32*I - 1/32)*sqrt(2)*sqrt
(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^5 + (1/32*I + 1/32)*sqr
t(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x^4}{\sqrt{\mathrm{acos}\,(a\,x)}}\,dx$$

```
[In] int(x^4/acos(a*x)^(1/2),x)
[Out] int(x^4/acos(a*x)^(1/2), x)
```

3.93 $\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^4} - \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{4a^4}$$

```
[Out] -1/16*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-1
/4*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4732, 4491, 3386, 3432}

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^4} - \frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{4a^4}$$

```
[In] Int[x^3/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -1/8*(Sqrt[Pi/2]*FresnelS[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^4 - (Sqrt[Pi]*
FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(4*a^4)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^4}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sin(2x)}{4\sqrt{x}}+\frac{\sin(4x)}{8\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{a^4}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{8a^4}-\frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{4a^4}\\
&= -\frac{\text{Subst}\left(\int \sin\left(4x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{4a^4}-\frac{\text{Subst}\left(\int \sin\left(2x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{2a^4}\\
&= -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^4}-\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{4a^4}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx =$$

$$-\frac{-2\sqrt{2}\sqrt{-i\arccos(ax)}\Gamma\left(\frac{1}{2},-2i\arccos(ax)\right)-2\sqrt{2}\sqrt{i\arccos(ax)}\Gamma\left(\frac{1}{2},2i\arccos(ax)\right)-\sqrt{-i\arccos(ax}}{32a^4\sqrt{\arccos(ax)}}$$

```
[In] Integrate[x^3/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -1/32*(-2*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcCos[a*x]] - 2
*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]] - Sqrt[(-I)*ArcC
os[a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (4
*I)*ArcCos[a*x]])/(a^4*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66

method	result	size
default	$-\frac{\sqrt{\pi}\,\left(\sqrt{2}\ \operatorname{FresnelS}\left(\frac{2 \sqrt{2}\, \sqrt{\arccos\left(a x\right)}}{\sqrt{\pi}}\right)+4\ \operatorname{FresnelS}\left(\frac{2 \sqrt{\arccos\left(a x\right)}}{\sqrt{\pi}}\right)\right)}{16 a^{4}}$	43

```
[In] int(x^3/arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/16/a^4*Pi^(1/2)*(2^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))+
4*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x^{3}}{\sqrt{\operatorname{acos}\left(a x\right)}}\,dx$$

```
[In] integrate(x**3/acos(a*x)**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\begin{aligned}\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = &-\frac{(i-1)\,\sqrt{2}\sqrt{\pi}\operatorname{erf}\left((i-1)\,\sqrt{2}\sqrt{\arccos{(ax)}}\right)}{64\,a^4}\\&+\frac{(i+1)\,\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-(i+1)\,\sqrt{2}\sqrt{\arccos{(ax)}}\right)}{64\,a^4}\\&-\frac{(i-1)\,\sqrt{\pi}\operatorname{erf}\left((i-1)\,\sqrt{\arccos{(ax)}}\right)}{16\,a^4}\\&+\frac{(i+1)\,\sqrt{\pi}\operatorname{erf}\left(-(i+1)\,\sqrt{\arccos{(ax)}}\right)}{16\,a^4}\end{aligned}$$

```
[In] integrate(x^3/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -(1/64*I - 1/64)*sqrt(2)*sqrt(pi)*erf((I - 1)*sqrt(2)*sqrt(arccos(a*x)))/a^
4 + (1/64*I + 1/64)*sqrt(2)*sqrt(pi)*erf(-(I + 1)*sqrt(2)*sqrt(arccos(a*x))
)/a^4 - (1/16*I - 1/16)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^4 + (1/16
*I + 1/16)*sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x^3}{\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

```
[In] int(x^3/acos(a*x)^(1/2),x)
```

```
[Out] int(x^3/acos(a*x)^(1/2), x)
```

3.94 $\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 71

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^3} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^3}$$

```
[Out] -1/12*FresnelS(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a^3-1/4
*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4732, 4491, 3386, 3432}

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^3} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^3}$$

```
[In] Int[x^2/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -1/2*(Sqrt[Pi/2]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^3 - (Sqrt[Pi/6]*
FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(2*a^3)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^3} \\
&= -\frac{\text{Subst}\left(\int \left(\frac{\sin(x)}{4\sqrt{x}}+\frac{\sin(3x)}{4\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{a^3} \\
&= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{4a^3} - \frac{\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{4a^3} \\
&= -\frac{\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{2a^3} - \frac{\text{Subst}\left(\int \sin\left(3x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{2a^3} \\
&= -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^3} - \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.77

$$
\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = -\frac{-3\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2},-i\arccos(ax)\right)-3\sqrt{i\arccos(ax)}\,\Gamma\left(\frac{1}{2},i\arccos(ax)\right)-\sqrt{3}\left(\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2}\right.\right.}{24a^3\sqrt{\arccos(ax)}}
$$

```
[In] Integrate[x^2/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -1/24*(-3*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a*x]] - 3*Sqrt[I*Ar
cCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] - Sqrt[3]*(Sqrt[(-I)*ArcCos[a*x]]*Gamm
a[1/2, (-3*I)*ArcCos[a*x]] + Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*
x]]))/(a^3*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{\sqrt{2}\,\sqrt{\pi}\,\left(\sqrt{3}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos\left(a x\right)}}{\sqrt{\pi}}\right)+3\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos\left(a x\right)}}{\sqrt{\pi}}\right)\right)}{12 a^{3}}$	50

```
[In] int(x^2/arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/12/a^3*2^(1/2)*Pi^(1/2)*(3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arcco
s(a*x)^(1/2))+3*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x^2}{\sqrt{\operatorname{acos}(ax)}}\,dx$$

```
[In] integrate(x**2/acos(a*x)**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.31

$$\begin{aligned}\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = &-\frac{(i-1)\ \sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\ \sqrt{6}\sqrt{\arccos{(ax)}}\right)}{48\,a^3}\\ &+\frac{(i+1)\ \sqrt{6}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{6}\sqrt{\arccos{(ax)}}\right)}{48\,a^3}\\ &-\frac{(i-1)\ \sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\ \sqrt{2}\sqrt{\arccos{(ax)}}\right)}{16\,a^3}\\ &+\frac{(i+1)\ \sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\ \sqrt{2}\sqrt{\arccos{(ax)}}\right)}{16\,a^3}\end{aligned}$$

```
[In] integrate(x^2/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -(1/48*I - 1/48)*sqrt(6)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(6)*sqrt(arccos(a*x
)))/a^3 + (1/48*I + 1/48)*sqrt(6)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(6)*sqrt(
arccos(a*x)))/a^3 - (1/16*I - 1/16)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt
(2)*sqrt(arccos(a*x)))/a^3 + (1/16*I + 1/16)*sqrt(2)*sqrt(pi)*erf(-(1/2*I +
 1/2)*sqrt(2)*sqrt(arccos(a*x)))/a^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x^2}{\sqrt{\mathrm{acos}\,(a\,x)}}\,dx$$

```
[In] int(x^2/acos(a*x)^(1/2),x)
[Out] int(x^2/acos(a*x)^(1/2), x)
```

3.95 $\int \frac{x}{\sqrt{\arccos(ax)}} dx$

Optimal result

Integrand size = 10, antiderivative size = 28

$$\int \frac{x}{\sqrt{\arccos(ax)}} dx = -\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{2a^2}$$

```
[Out] -1/2*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4732, 4491, 12, 3386, 3432}

$$\int \frac{x}{\sqrt{\arccos(ax)}} dx = -\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{2a^2}$$

```
[In] Int[x/Sqrt[ArcCos[a*x]],x]
[Out] -1/2*(Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/a^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{2a^2}\\
&= -\frac{\text{Subst}\left(\int \sin\left(2x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{a^2}\\
&= -\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{2a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = -\frac{\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{2a^2}$$

```
[In] Integrate[x/Sqrt[ArcCos[a*x]],x]
[Out] -1/2*(Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/a^2
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{\pi}}{2a^2}$	21

```
[In] int(x/arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/arccos(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x}{\sqrt{\operatorname{acos}(ax)}}\,dx$$

```
[In] integrate(x/acos(a*x)**(1/2),x)
```

```
[Out] Integral(x/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = -\frac{(i-1)\sqrt{\pi}\operatorname{erf}\left((i-1)\sqrt{\arccos(ax)}\right)}{8a^2} + \frac{(i+1)\sqrt{\pi}\operatorname{erf}\left(-(i+1)\sqrt{\arccos(ax)}\right)}{8a^2}$$

```
[In] integrate(x/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -(1/8*I - 1/8)*sqrt(pi)*erf((I - 1)*sqrt(arccos(a*x)))/a^2 + (1/8*I + 1/8)*
sqrt(pi)*erf(-(I + 1)*sqrt(arccos(a*x)))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\arccos(ax)}}\,dx = \int \frac{x}{\sqrt{\mathrm{acos}\,(a\,x)}}\,dx$$

```
[In] int(x/acos(a*x)^(1/2),x)
[Out] int(x/acos(a*x)^(1/2), x)
```

3.96 $\int \frac{1}{\sqrt{\arccos(ax)}} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int \frac{1}{\sqrt{\arccos(ax)}} \, dx = -\frac{\sqrt{2\pi} \operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

```
[Out] -FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4720, 3386, 3432}

$$\int \frac{1}{\sqrt{\arccos(ax)}} \, dx = -\frac{\sqrt{2\pi} \operatorname{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

```
[In] Int[1/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -((Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/a)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a}\\
&= -\frac{2\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{a}\\
&= -\frac{\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = -\frac{-\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2}, -i\arccos(ax)\right) - \sqrt{i\arccos(ax)}\,\Gamma\left(\frac{1}{2}, i\arccos(ax)\right)}{2a\sqrt{\arccos(ax)}}$$

```
[In] Integrate[1/Sqrt[ArcCos[a*x]],x]
```

```
[Out] -1/2*(-(Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a*x]]) - Sqrt[I*ArcCo
s[a*x]]*Gamma[1/2, I*ArcCos[a*x]])/(a*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.84

method	result	size
default	$-\frac{\text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{2}\,\sqrt{\pi}}{a}$	26

```
[In] int(1/arccos(a*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/arccos(a*x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{\sqrt{\operatorname{acos}(ax)}}\,dx$$

```
[In] integrate(1/acos(a*x)**(1/2),x)
```

```
[Out] Integral(1/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.52

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = -\frac{(i-1)\,\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(\left(\frac{1}{2}i-\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos(ax)}\right)}{4\,a} + \frac{(i+1)\,\sqrt{2}\sqrt{\pi}\operatorname{erf}\left(-\left(\frac{1}{2}i+\frac{1}{2}\right)\,\sqrt{2}\sqrt{\arccos(ax)}\right)}{4\,a}$$

```
[In] integrate(1/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] -(1/4*I - 1/4)*sqrt(2)*sqrt(pi)*erf((1/2*I - 1/2)*sqrt(2)*sqrt(arccos(a*x))
)/a + (1/4*I + 1/4)*sqrt(2)*sqrt(pi)*erf(-(1/2*I + 1/2)*sqrt(2)*sqrt(arccos
(a*x)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

```
[In] int(1/acos(a*x)^(1/2),x)
```

```
[Out] int(1/acos(a*x)^(1/2), x)
```

3.97 $\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \operatorname{Int}\left(\frac{1}{x\sqrt{\arccos(ax)}},x\right)$$

```
[Out] Unintegrable(1/x/arccos(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x\sqrt{\arccos(ax)}}\,dx$$

```
[In] Int[1/(x*Sqrt[ArcCos[a*x]]),x]
[Out] Defer[Int][1/(x*Sqrt[ArcCos[a*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\sqrt{\arccos(ax)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x\sqrt{\arccos(ax)}}\,dx$$

```
[In] Integrate[1/(x*Sqrt[ArcCos[a*x]]),x]
[Out] Integrate[1/(x*Sqrt[ArcCos[a*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.99 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x\sqrt{\arccos\left(ax\right)}}dx$$

```
[In] int(1/x/arccos(a*x)^(1/2),x)
[Out] int(1/x/arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/arccos(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x\sqrt{\operatorname{acos}\left(ax\right)}}\,dx$$

```
[In] integrate(1/x/acos(a*x)**(1/2),x)
[Out] Integral(1/(x*sqrt(acos(a*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/x/arccos(a*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x\sqrt{\arccos(ax)}}\,dx$$

```
[In] integrate(1/x/arccos(a*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*sqrt(arccos(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x\,\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

```
[In] int(1/(x*acos(a*x)^(1/2)),x)
```

```
[Out] int(1/(x*acos(a*x)^(1/2)), x)
```

3.98 $\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \operatorname{Int}\left(\frac{1}{x^2\sqrt{\arccos(ax)}},x\right)$$

```
[Out] Unintegrable(1/x^2/arccos(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx$$

```
[In] Int[1/(x^2*Sqrt[ArcCos[a*x]]),x]
[Out] Defer[Int][1/(x^2*Sqrt[ArcCos[a*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx$$

```
[In] Integrate[1/(x^2*Sqrt[ArcCos[a*x]]),x]
[Out] Integrate[1/(x^2*Sqrt[ArcCos[a*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.08 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x^2\sqrt{\arccos\left(ax\right)}}dx$$

```
[In] int(1/x^2/arccos(a*x)^(1/2),x)
[Out] int(1/x^2/arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/arccos(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.53 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\operatorname{acos}\left(ax\right)}}\,dx$$

```
[In] integrate(1/x**2/acos(a*x)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(acos(a*x))), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/x^2/arccos(a*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x^2\sqrt{\arccos{(ax)}}}\,dx$$

```
[In] integrate(1/x^2/arccos(a*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(x^2*sqrt(arccos(a*x))), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{\arccos(ax)}}\,dx = \int \frac{1}{x^2\,\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

```
[In] int(1/(x^2*acos(a*x)^(1/2)),x)
[Out] int(1/(x^2*acos(a*x)^(1/2)), x)
```

3.99 $\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 171

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{5\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{9\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{\sqrt{\frac{7\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{14}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7}$$

```
[Out] -5/32*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^7-9/3
2*FresnelC(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a^7-5/32*Fr
esnelC(10^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*10^(1/2)*Pi^(1/2)/a^7-1/32*Fres
nelC(14^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*14^(1/2)*Pi^(1/2)/a^7+2*x^6*(-a^2
*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used

= {4728, 3385, 3433}

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = -\frac{5\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{9\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{\sqrt{\frac{7\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{14}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} + \frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

```
[In] Int[x^6/ArcCos[a*x]^(3/2),x]
```

```
[Out] (2*x^6*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (5*Sqrt[Pi/2]*FresnelC[Sq
rt[2/Pi]*Sqrt[ArcCos[a*x]]])/(16*a^7) - (9*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/P
i]*Sqrt[ArcCos[a*x]]])/(16*a^7) - (5*Sqrt[(5*Pi)/2]*FresnelC[Sqrt[10/Pi]*Sq
rt[ArcCos[a*x]]])/(16*a^7) - (Sqrt[(7*Pi)/2]*FresnelC[Sqrt[14/Pi]*Sqrt[ArcC
os[a*x]]])/(16*a^7)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} + \frac{2\text{Subst}\left(\int\left(-\frac{5\cos(x)}{64\sqrt{x}} - \frac{27\cos(3x)}{64\sqrt{x}} - \frac{25\cos(5x)}{64\sqrt{x}} - \frac{7\cos(7x)}{64\sqrt{x}}\right)dx, x, \arccos(ax)\right)}{a^7}$$

$$= \frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{5\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{32a^7} - \frac{7\text{Subst}\left(\int \frac{\cos(7x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{32a^7} - \frac{25\text{Subst}\left(\int \frac{\cos(5x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{32a^7} - \frac{27\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{32a^7}$$

$$= \frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{5\text{Subst}\left(\int \cos(x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{16a^7} - \frac{7\text{Subst}\left(\int \cos(7x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{16a^7} - \frac{25\text{Subst}\left(\int \cos(5x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{16a^7} - \frac{27\text{Subst}\left(\int \cos(3x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{16a^7}$$

$$= \frac{2x^6\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{5\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{9\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7} - \frac{\sqrt{\frac{7\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{14}{\pi}}\sqrt{\arccos(ax)}\right)}{16a^7}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.22 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.79

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \frac{i\left(-10i\sqrt{1-a^2x^2} + 5\sqrt{-i\arccos(ax)}\Gamma\left(\frac{1}{2}, -i\arccos(ax)\right) - 5\sqrt{i\arccos(ax)}\Gamma\left(\frac{1}{2}, i\arccos\right.\right.}{}$$

```
[In] Integrate[x^6/ArcCos[a*x]^(3/2),x]
```

```
[Out] ((I/64)*((-10*I)*Sqrt[1 - a^2*x^2] + 5*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-
I)*ArcCos[a*x]] - 5*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] + 9*Sqrt[
3]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - 9*Sqrt[3]*Sqrt[I
*ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*x]] + 5*Sqrt[5]*Sqrt[(-I)*ArcCos[a*
x]]*Gamma[1/2, (-5*I)*ArcCos[a*x]] - 5*Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[1/
2, (5*I)*ArcCos[a*x]] + Sqrt[7]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-7*I)*Ar
cCos[a*x]] - Sqrt[7]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (7*I)*ArcCos[a*x]] - (1
8*I)*Sin[3*ArcCos[a*x]] - (10*I)*Sin[5*ArcCos[a*x]] - (2*I)*Sin[7*ArcCos[a*
x]]))/(a^7*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.06

method	result
default	$\frac{-\sqrt{2}\,\sqrt{\pi}\,\sqrt{7}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{7}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{\arccos(ax)}-9\sqrt{3}\,\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-5\sqrt{5}\,\sqrt{2}\,\sqrt{\mathrm{arcco}}}{}$

[In] int(x^6/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/32/a^7*(-2^(1/2)*Pi^(1/2)*7^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*7^(1/2)*arcco s(a*x)^(1/2))*arccos(a*x)^(1/2)-9*3^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))-5*5^(1/2)*2^(1/2)*ar ccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))-5*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a* x)^(1/2))+5*(-a^2*x^2+1)^(1/2)+9*sin(3*arccos(a*x))+5*sin(5*arccos(a*x))+si n(7*arccos(a*x)))/arccos(a*x)^(1/2)

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^6/arccos(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^6}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

[In] integrate(x**6/acos(a*x)**(3/2),x)

[Out] Integral(x**6/acos(a*x)**(3/2), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^6/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^6}{\arccos\left(ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^6/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^6}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^6}{\mathrm{acos}\left(a\,x\right)^{3/2}}\,dx$$

```
[In] int(x^6/acos(a*x)^(3/2),x)
```

```
[Out] int(x^6/acos(a*x)^(3/2), x)
```

3.100 $\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 127

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^6} - \frac{\sqrt{3\pi}\,\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^6} - \frac{5\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^6}$$

```
[Out] -1/2*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^6-5/
8*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^6-1/8*FresnelC(2*3^(1/2
)/Pi^(1/2)*arccos(a*x)^(1/2))*3^(1/2)*Pi^(1/2)/a^6+2*x^5*(-a^2*x^2+1)^(1/2)
/a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4728, 3385, 3433}

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^6} - \frac{\sqrt{3\pi}\,\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^6} - \frac{5\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^6} + \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

```
[In] Int[x^5/ArcCos[a*x]^(3/2),x]
```

```
[Out] (2*x^5*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[2*Sq
rt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^6 - (Sqrt[3*Pi]*FresnelC[2*Sqrt[3/Pi]*Sqrt[A
rcCos[a*x]]])/(8*a^6) - (5*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]
])/(8*a^6)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} + \frac{2\text{Subst}\left(\int \left(-\frac{5\cos(2x)}{16\sqrt{x}} - \frac{\cos(4x)}{2\sqrt{x}} - \frac{3\cos(6x)}{16\sqrt{x}}\right) dx, x, \arccos(ax)\right)}{a^6}$$

$$= \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{3\text{Subst}\left(\int \frac{\cos(6x)}{\sqrt{x}} dx, x, \arccos(ax)\right)}{8a^6} - \frac{5\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}} dx, x, \arccos(ax)\right)}{8a^6} - \frac{\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}} dx, x, \arccos(ax)\right)}{a^6}$$

$$= \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{3\text{Subst}\left(\int \cos(6x^2)\, dx, x, \sqrt{\arccos(ax)}\right)}{4a^6} - \frac{5\text{Subst}\left(\int \cos(2x^2)\, dx, x, \sqrt{\arccos(ax)}\right)}{4a^6} - \frac{2\text{Subst}\left(\int \cos(4x^2)\, dx, x, \sqrt{\arccos(ax)}\right)}{a^6}$$

$$= \frac{2x^5\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^6} - \frac{\sqrt{3\pi}\,\text{FresnelC}\left(2\sqrt{\frac{3}{\pi}}\sqrt{\arccos(ax)}\right)}{8a^6} - \frac{5\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{8a^6}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.78

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \frac{i\left(5\sqrt{2}\sqrt{-i\arccos(ax)}\Gamma\left(\frac{1}{2},-2i\arccos(ax)\right)-5\sqrt{2}\sqrt{i\arccos(ax)}\Gamma\left(\frac{1}{2},2i\arccos(ax)\right)\right.}{}$$

```
[In] Integrate[x^5/ArcCos[a*x]^(3/2),x]
```

```
[Out] ((I/32)*(5*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcCos[a*x]] -
5*Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]] + 8*Sqrt[(-I)*A
rcCos[a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - 8*Sqrt[I*ArcCos[a*x]]*Gamma[1/
2, (4*I)*ArcCos[a*x]] + Sqrt[6]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-6*I)*Ar
cCos[a*x]] - Sqrt[6]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (6*I)*ArcCos[a*x]] - (1
0*I)*Sin[2*ArcCos[a*x]] - (8*I)*Sin[4*ArcCos[a*x]] - (2*I)*Sin[6*ArcCos[a*x
]]))/(a^6*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.95

method	result
default	$\frac{-8\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-2\sqrt{\pi}\,\sqrt{3}\,\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{6}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\sqrt{\arccos(ax)}-10\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\text{Fre}}{16a^6\sqrt{\arccos(ax)}}$

```
[In] int(x^5/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/16/a^6/arccos(a*x)^(1/2)*(-8*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(
2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))-2*Pi^(1/2)*3^(1/2)*FresnelC(2^(1/2)/P
i^(1/2)*6^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(1/2)-10*arccos(a*x)^(1/2)*P
i^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))+5*sin(2*arccos(a*x))+4*sin(4
*arccos(a*x))+sin(6*arccos(a*x)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^5/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^5}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(x**5/acos(a*x)**(3/2),x)
```

```
[Out] Integral(x**5/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^5/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^5/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^5}{\mathrm{acos}\left(a\,x\right)^{3/2}} \,d x$$

```
[In] int(x^5/acos(a*x)^(3/2),x)
[Out] int(x^5/acos(a*x)^(3/2), x)
```

3.101 $\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 136

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^5} - \frac{3\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5}$$

```
[Out] -1/4*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^5-3/8*
FresnelC(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a^5-1/8*Fresn
elC(10^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*10^(1/2)*Pi^(1/2)/a^5+2*x^4*(-a^2*
x^2+1)^(1/2)/a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4728, 3385, 3433}

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^5} - \frac{3\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5} + \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

```
[In] Int[x^4/ArcCos[a*x]^(3/2),x]
```

```
[Out] (2*x^4*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/(2*a^5) - (3*Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]*
Sqrt[ArcCos[a*x]]])/(4*a^5) - (Sqrt[(5*Pi)/2]*FresnelC[Sqrt[10/Pi]*Sqrt[Arc
Cos[a*x]]])/(4*a^5)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} + \frac{2\text{Subst}\left(\int \left(-\frac{\cos(x)}{8\sqrt{x}} - \frac{9\cos(3x)}{16\sqrt{x}} - \frac{5\cos(5x)}{16\sqrt{x}}\right) dx, x, \arccos(ax)\right)}{a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}} dx, x, \arccos(ax)\right)}{4a^5} - \frac{5\text{Subst}\left(\int \frac{\cos(5x)}{\sqrt{x}} dx, x, \arccos(ax)\right)}{8a^5} - \frac{9\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}} dx, x, \arccos(ax)\right)}{8a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\text{Subst}\left(\int \cos\left(x^2\right) dx, x, \sqrt{\arccos(ax)}\right)}{2a^5} - \frac{5\text{Subst}\left(\int \cos\left(5x^2\right) dx, x, \sqrt{\arccos(ax)}\right)}{4a^5} - \frac{9\text{Subst}\left(\int \cos\left(3x^2\right) dx, x, \sqrt{\arccos(ax)}\right)}{4a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^5} - \frac{3\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5} - \frac{\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{4a^5}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.71

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \frac{i\Big(-4i\sqrt{1-a^2x^2} + 2\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2}, -i\arccos(ax)\right) - 2\sqrt{i\arccos(ax)}\,\Gamma\left(\frac{1}{2}, i\,\text{arcc}\right.}{}$$

```
[In] Integrate[x^4/ArcCos[a*x]^(3/2),x]
```

```
[Out] ((I/16)*((-4*I)*Sqrt[1 - a^2*x^2] + 2*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I
)*ArcCos[a*x]] - 2*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] + 3*Sqrt[3
]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - 3*Sqrt[3]*Sqrt[I*
ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*x]] + Sqrt[5]*Sqrt[(-I)*ArcCos[a*x]]
*Gamma[1/2, (-5*I)*ArcCos[a*x]] - Sqrt[5]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (5
*I)*ArcCos[a*x]] - (6*I)*Sin[3*ArcCos[a*x]] - (2*I)*Sin[5*ArcCos[a*x]]))/(a
^5*Sqrt[ArcCos[a*x]])
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.02

method	result
default	$\frac{-3\sqrt{3}\,\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-\sqrt{5}\,\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-2\sqrt{2}\,\sqrt{\arccos(ax}}{8a^5\sqrt{\arccos(ax)}}$

```
[In] int(x^4/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/a^5*(-3*3^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^
(1/2)*3^(1/2)*arccos(a*x)^(1/2))-5^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)
*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)*arccos(a*x)^(1/2))-2*2^(1/2)*arccos(a*x)
^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))+2*(-a^2*x^2+1)
^(1/2)+3*sin(3*arccos(a*x))+sin(5*arccos(a*x)))/arccos(a*x)^(1/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^4/arccos(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ rate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^4}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

[In] integrate(x**4/acos(a*x)**(3/2),x)

[Out] Integral(x**4/acos(a*x)**(3/2), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

[In] integrate(x^4/arccos(a*x)^(3/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ e exponent.

Giac [F]

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^4}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

[In] integrate(x^4/arccos(a*x)^(3/2),x, algorithm="giac")

[Out] integrate(x^4/arccos(a*x)^(3/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^4}{\mathrm{acos}\left(a\,x\right)^{3/2}}\,dx$$

```
[In] int(x^4/acos(a*x)^(3/2),x)
[Out] int(x^4/acos(a*x)^(3/2), x)
```

3.102 $\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 91

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx=\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^4}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^4}$$

```
[Out] -1/2*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4-Fr
esnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4+2*x^3*(-a^2*x^2+1)^(1/2)/
a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4728, 3385, 3433}

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx=-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^4}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^4}+\frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

```
[In] Int[x^3/ArcCos[a*x]^(3/2),x]
```

```
[Out] (2*x^3*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[2*Sq
rt[2/Pi]*Sqrt[ArcCos[a*x]]])/a^4 - (Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])
/Sqrt[Pi]])/a^4
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}+\frac{2\text{Subst}\left(\int\left(-\frac{\cos(2x)}{2\sqrt{x}}-\frac{\cos(4x)}{2\sqrt{x}}\right)dx,x,\arccos(ax)\right)}{a^4}\\
&= \frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{\text{Subst}\left(\int\frac{\cos(2x)}{\sqrt{x}}dx,x,\arccos(ax)\right)}{a^4}-\frac{\text{Subst}\left(\int\frac{\cos(4x)}{\sqrt{x}}dx,x,\arccos(ax)\right)}{a^4}\\
&= \frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\text{Subst}\left(\int\cos\left(2x^2\right)dx,x,\sqrt{\arccos(ax)}\right)}{a^4}\\
&\quad-\frac{2\text{Subst}\left(\int\cos\left(4x^2\right)dx,x,\sqrt{\arccos(ax)}\right)}{a^4}\\
&= \frac{2x^3\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^4}-\frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^4}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.69

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \frac{i\sqrt{2}\sqrt{-i\arccos(ax)}\Gamma\left(\frac{1}{2},-2i\arccos(ax)\right)-i\sqrt{2}\sqrt{i\arccos(ax)}\Gamma\left(\frac{1}{2},2i\arccos(ax)\right)+\ldots}{}$$

```
[In] Integrate[x^3/ArcCos[a*x]^(3/2),x]
```

```
[Out] (I*Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)*ArcCos[a*x]] - I*Sqrt[2
]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]] + I*Sqrt[(-I)*ArcCos[a*
x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - I*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (4*I)
*ArcCos[a*x]] + 2*Sin[2*ArcCos[a*x]] + Sin[4*ArcCos[a*x]])/(4*a^4*Sqrt[ArcC
os[a*x]])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

method	result
default	$\frac{-2\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-4\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+2\sin(2\arccos(ax))+\sin(4\arccos\ldots}{4a^4\sqrt{\arccos(ax)}}$

```
[In] int(x^3/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^4/arccos(a*x)^(1/2)*(-2*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2
*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))-4*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(
2*arccos(a*x)^(1/2)/Pi^(1/2))+2*sin(2*arccos(a*x))+sin(4*arccos(a*x)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^3}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(x**3/acos(a*x)**(3/2),x)
[Out] Integral(x**3/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3/arccos(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3/arccos(a*x)^(3/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^3}{\operatorname{acos}(a\,x)^{3/2}}\,dx$$

```
[In] int(x^3/acos(a*x)^(3/2),x)
[Out] int(x^3/acos(a*x)^(3/2), x)
```

3.103 $\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 97

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3} - \frac{\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3}$$

```
[Out] -1/2*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3-1/2*
FresnelC(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a^3+2*x^2*(-a
^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4728, 3385, 3433}

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3} - \frac{\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3} + \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}$$

```
[In] Int[x^2/ArcCos[a*x]^(3/2),x]
```

```
[Out] (2*x^2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (Sqrt[Pi/2]*FresnelC[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/a^3 - (Sqrt[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]*Sqrt[A
rcCos[a*x]]])/a^3
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} + \frac{2\text{Subst}\left(\int \left(-\frac{\cos(x)}{4\sqrt{x}} - \frac{3\cos(3x)}{4\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{a^3}\\
&= \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{2a^3} - \frac{3\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{2a^3}\\
&= \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\text{Subst}\left(\int \cos\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{a^3}\\
&\quad - \frac{3\text{Subst}\left(\int \cos\left(3x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{a^3}\\
&= \frac{2x^2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3} - \frac{\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.64

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \frac{i\left(-2i\sqrt{1-a^2x^2}+\sqrt{-i\arccos(ax)}\Gamma\left(\frac{1}{2},-i\arccos(ax)\right)-\sqrt{i\arccos(ax)}\Gamma\left(\frac{1}{2},i\arccos\right.\right.}{}$$

[In] Integrate[x^2/ArcCos[a*x]^(3/2),x]

[Out] ((I/4)*((-2*I)*Sqrt[1 - a^2*x^2] + Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a*x]] - Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]] + Sqrt[3]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - Sqrt[3]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*x]] - (2*I)*Sin[3*ArcCos[a*x]]))/(a^3*Sqrt[ArcCos[a*x]])

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

method	result
default	$\frac{-\sqrt{3}\,\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sqrt{-a^2x^2+1}+\sin(3\arccos}{2a^3\sqrt{\arccos(ax)}}$

[In] int(x^2/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)

[Out] 1/2/a^3*(-3^(1/2)*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))-2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))+(-a^2*x^2+1)^(1/2)+sin(3*arccos(a*x)))/arccos(a*x)^(1/2)

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^2/arccos(a*x)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^2}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(x**2/acos(a*x)**(3/2),x)
[Out] Integral(x**2/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2/arccos(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^2}{\arccos(ax)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/arccos(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x^2/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx = \int \frac{x^2}{\operatorname{acos}(a\,x)^{3/2}}\,dx$$

```
[In] int(x^2/acos(a*x)^(3/2),x)
[Out] int(x^2/acos(a*x)^(3/2), x)
```

3.104 $\int \frac{x}{\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 55

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx=\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^2}$$

```
[Out] -2*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2+2*x*(-a^2*x^2+1)^(1/
2)/a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4728, 3385, 3433}

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx=\frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^2}$$

```
[In] Int[x/ArcCos[a*x]^(3/2),x]
```

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[Pi]*FresnelC[(2*Sqr
t[ArcCos[a*x]])/Sqrt[Pi]])/a^2
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{2\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^2} \\
&= \frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{4\text{Subst}\left(\int \cos\left(2x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{a^2} \\
&= \frac{2x\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{a^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$
\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \frac{-2\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) + \frac{\sin(2\arccos(ax))}{\sqrt{\arccos(ax)}}}{a^2}
$$

```
[In] Integrate[x/ArcCos[a*x]^(3/2),x]
[Out] (-2*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + Sin[2*ArcCos[a*x]]/
Sqrt[ArcCos[a*x]])/a^2
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.76

method	result	size
default	$\frac{-2\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\sin(2\arccos(ax))}{a^2\sqrt{\arccos(ax)}}$	42

```
[In] int(x/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2/arccos(a*x)^(1/2)*(-2*arccos(a*x)^(1/2)*Pi^(1/2)*FresnelC(2*arccos(a*
x)^(1/2)/Pi^(1/2))+sin(2*arccos(a*x)))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \int \frac{x}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(x/acos(a*x)**(3/2),x)
```

```
[Out] Integral(x/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \int \frac{x}{\arccos\left(ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/arccos(a*x)^(3/2),x, algorithm="giac")
[Out] integrate(x/arccos(a*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^{3/2}}\,dx = \int \frac{x}{\mathrm{acos}\left(a\,x\right)^{3/2}}\,dx$$

```
[In] int(x/acos(a*x)^(3/2),x)
[Out] int(x/acos(a*x)^(3/2), x)
```

3.105 $\int \frac{1}{\arccos(ax)^{3/2}} \, dx$

Optimal result

Integrand size = 8, antiderivative size = 59

$$\int \frac{1}{\arccos(ax)^{3/2}} \, dx = \frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

```
[Out] -2*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a+2*(-a^2*
x^2+1)^(1/2)/a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4718, 4810, 3385, 3433}

$$\int \frac{1}{\arccos(ax)^{3/2}} \, dx = \frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}} - \frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}$$

```
[In] Int[ArcCos[a*x]^(-3/2),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(a*Sqrt[ArcCos[a*x]]) - (2*Sqrt[2*Pi]*FresnelC[Sqrt[2
/Pi]*Sqrt[ArcCos[a*x]]])/a
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}+(2a)\int\frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx\\
&= \frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\text{Subst}\left(\int\frac{\cos(x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{a}\\
&= \frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{4\text{Subst}\left(\int\cos\left(x^2\right)\,dx,x,\sqrt{\arccos(ax)}\right)}{a}\\
&= \frac{2\sqrt{1-a^2x^2}}{a\sqrt{\arccos(ax)}}-\frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int\frac{1}{\arccos(ax)^{3/2}}\,dx = -\frac{-2\sqrt{1-a^2x^2}-i\sqrt{-i\arccos(ax)}\,\Gamma\left(\frac{1}{2},-i\arccos(ax)\right)+i\sqrt{i\arccos(ax)}\,\Gamma\left(\frac{1}{2},i\arccos(ax)\right)}{a\sqrt{\arccos(ax)}}$$

```
[In] Integrate[ArcCos[a*x]^(-3/2),x]
```

```
[Out] -((-2*Sqrt[1 - a^2*x^2] - I*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-I)*ArcCos[a
*x]] + I*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, I*ArcCos[a*x]])/(a*Sqrt[ArcCos[a*x]
]))
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

method	result	size
default	$-\frac{\sqrt{2}\left(2\arccos(ax)\pi\ \mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-\sqrt{2}\,\sqrt{\arccos(ax)}\,\sqrt{\pi}\,\sqrt{-a^2x^2+1}\right)}{a\sqrt{\pi}\,\arccos(ax)}$	66

```
[In] int(1/arccos(a*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*2^(1/2)/Pi^(1/2)/arccos(a*x)*(2*arccos(a*x)*Pi*FresnelC(2^(1/2)/Pi^(1/
2)*arccos(a*x)^(1/2))-2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2)
)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/arccos(a*x)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = \int \frac{1}{\operatorname{acos}^{\frac{3}{2}}(ax)}\,dx$$

```
[In] integrate(1/acos(a*x)**(3/2),x)
```

```
[Out] Integral(acos(a*x)**(-3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = \int \frac{1}{\arccos{(ax)}^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^{3/2}}\,dx = \int \frac{1}{{\mathrm{acos}\left(a\,x\right)}^{3/2}}\,dx$$

```
[In] int(1/acos(a*x)^(3/2),x)
```

```
[Out] int(1/acos(a*x)^(3/2), x)
```

3.106 $\int \frac{1}{x \arccos(ax)^{3/2}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x \arccos(ax)^{3/2}} \, dx = \operatorname{Int}\left(\frac{1}{x \arccos(ax)^{3/2}}, x\right)$$

```
[Out] Unintegrable(1/x/arccos(a*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \arccos(ax)^{3/2}} \, dx = \int \frac{1}{x \arccos(ax)^{3/2}} \, dx$$

```
[In] Int[1/(x*ArcCos[a*x]^(3/2)),x]
[Out] Defer[Int][1/(x*ArcCos[a*x]^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \arccos(ax)^{3/2}} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x \arccos(ax)^{3/2}}\,dx = \int \frac{1}{x \arccos(ax)^{3/2}}\,dx$$

```
[In] Integrate[1/(x*ArcCos[a*x]^(3/2)),x]
[Out] Integrate[1/(x*ArcCos[a*x]^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \arccos\left(ax\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/x/arccos(a*x)^(3/2),x)
[Out] int(1/x/arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/arccos(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{3/2}}\,dx = \int \frac{1}{x \operatorname{acos}^{\frac{3}{2}}\left(ax\right)}\,dx$$

```
[In] integrate(1/x/acos(a*x)**(3/2),x)
[Out] Integral(1/(x*acos(a*x)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{3/2}} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/x/arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{3/2}} \, dx = \int \frac{1}{x \arccos\left(ax\right)^{\frac{3}{2}}} \, dx$$

```
[In] integrate(1/x/arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{3/2}} \, dx = \int \frac{1}{x \,\mathrm{acos}\left(a\,x\right)^{3/2}} \, dx$$

```
[In] int(1/(x*acos(a*x)^(3/2)),x)
```

```
[Out] int(1/(x*acos(a*x)^(3/2)), x)
```

3.107 $\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 235

$$\begin{aligned}\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx &= \frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}-\frac{16x^3}{3a^2\sqrt{\arccos(ax)}}\\&\quad+\frac{20x^5}{3\sqrt{\arccos(ax)}}+\frac{25\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^5}\\&\quad-\frac{4\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5}+\frac{25\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^5}\\&\quad-\frac{4\sqrt{\frac{2\pi}{3}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5}+\frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^5}\end{aligned}$$

```
[Out] 3/4*FresnelS(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a^5+1/6*F
resnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^5+5/12*Fresn
elS(10^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*10^(1/2)*Pi^(1/2)/a^5+2/3*x^4*(-a^
2*x^2+1)^(1/2)/a/arccos(a*x)^(3/2)-16/3*x^3/a^2/arccos(a*x)^(1/2)+20/3*x^5/
arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used

= {4730, 4808, 4732, 4491, 3386, 3432}

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = -\frac{4\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5} + \frac{25\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^5} - \frac{4\sqrt{\frac{2\pi}{3}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5} + \frac{25\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^5} + \frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^5} - \frac{16x^3}{3a^2\sqrt{\arccos(ax)}} + \frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} + \frac{20x^5}{3\sqrt{\arccos(ax)}}$$

```
[In] Int[x^4/ArcCos[a*x]^(5/2),x]
```

```
[Out] (2*x^4*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (16*x^3)/(3*a^2*Sqrt[Ar
cCos[a*x]]) + (20*x^5)/(3*Sqrt[ArcCos[a*x]]) + (25*Sqrt[Pi/2]*FresnelS[Sqrt
[2/Pi]*Sqrt[ArcCos[a*x]]])/(3*a^5) - (4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt
[ArcCos[a*x]]])/a^5 + (25*Sqrt[Pi/6]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]]
)/(2*a^5) - (4*Sqrt[(2*Pi)/3]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/a^5 +
 (5*Sqrt[(5*Pi)/2]*FresnelS[Sqrt[10/Pi]*Sqrt[ArcCos[a*x]]])/(6*a^5)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
```

```
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

integral

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{8\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}\,dx}{3a} + \frac{1}{3}(10a)\int \frac{x^5}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}\,dx$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\arccos(ax)}} + \frac{20x^5}{3\sqrt{\arccos(ax)}} - \frac{100}{3}\int \frac{x^4}{\sqrt{\arccos(ax)}}\,dx + \frac{16\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx}{a^2}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\arccos(ax)}} + \frac{20x^5}{3\sqrt{\arccos(ax)}} - \frac{16\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^5} + \frac{100\text{Subst}\left(\int \frac{\cos^4(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{3a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\arccos(ax)}} + \frac{20x^5}{3\sqrt{\arccos(ax)}} - \frac{16\text{Subst}\left(\int \left(\frac{\sin(x)}{4\sqrt{x}} + \frac{\sin(3x)}{4\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{a^5} + \frac{100\text{Subst}\left(\int \left(\frac{\sin(x)}{8\sqrt{x}} + \frac{3\sin(3x)}{16\sqrt{x}} + \frac{\sin(5x)}{16\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{3a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\arccos(ax)}} + \frac{20x^5}{3\sqrt{\arccos(ax)}} + \frac{25\text{Subst}\left(\int \frac{\sin(5x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{12a^5} - \frac{4\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^5} - \frac{4\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^5} + \frac{25\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{6a^5} + \frac{25\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{4a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\arccos(ax)}} + \frac{20x^5}{3\sqrt{\arccos(ax)}} + \frac{25\text{Subst}\left(\int \sin(5x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{6a^5} - \frac{8\text{Subst}\left(\int \sin(x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{a^5} - \frac{8\text{Subst}\left(\int \sin(3x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{a^5} + \frac{25\text{Subst}\left(\int \sin(x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{3a^5} + \frac{25\text{Subst}\left(\int \sin(3x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{2a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{16x^3}{3a^2\sqrt{\arccos(ax)}} + \frac{20x^5}{3\sqrt{\arccos(ax)}} + \frac{25\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^5} - \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5} + \frac{25\sqrt{\frac{\pi}{6}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{2a^5} - \frac{4\sqrt{\frac{2\pi}{3}}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5} + \frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelS}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{6a^5}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.13 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.37

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx =$$

$$-\frac{2\left(-\sqrt{1-a^2x^2} - e^{-i\arccos(ax)}\arccos(ax) - e^{i\arccos(ax)}\arccos(ax) + \sqrt{-i\arccos(ax)}\arccos(ax)\Gamma\left(\frac{1}{2}, -i\arc\right.}{}$$

```
[In] Integrate[x^4/ArcCos[a*x]^(5/2),x]
```

```
[Out] -1/24*(2*(-Sqrt[1 - a^2*x^2] - ArcCos[a*x]/E^(I*ArcCos[a*x]) - E^(I*ArcCos[
a*x])*ArcCos[a*x] + Sqrt[(-I)*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, (-I)*ArcC
os[a*x]] + Sqrt[I*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, I*ArcCos[a*x]]) - 5*A
```

```
rcCos[a*x]*(E^((-5*I)*ArcCos[a*x]) + E^((5*I)*ArcCos[a*x]) - Sqrt[5]*Sqrt[(
-I)*ArcCos[a*x]]*Gamma[1/2, (-5*I)*ArcCos[a*x]] - Sqrt[5]*Sqrt[I*ArcCos[a*x
]]*Gamma[1/2, (5*I)*ArcCos[a*x]]) - 3*(3*ArcCos[a*x]*(E^((-3*I)*ArcCos[a*x]
) + E^((3*I)*ArcCos[a*x]) - Sqrt[3]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-3*I
)*ArcCos[a*x]] - Sqrt[3]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (3*I)*ArcCos[a*x]])
 + Sin[3*ArcCos[a*x]]) - Sin[5*ArcCos[a*x]])/(a^5*ArcCos[a*x]^(3/2))
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.74

method	result
default	$\frac{10\sqrt{2}\,\sqrt{\pi}\,\sqrt{5}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+18\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\ \mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+4\sqrt{2}\,\sqrt{\pi}\ \mathrm{FresnelS}}{}$

```
[In] int(x^4/arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/24/a^5*(10*2^(1/2)*Pi^(1/2)*5^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*5^(1/2)*arc
cos(a*x)^(1/2))*arccos(a*x)^(3/2)+18*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelS(2^(1
/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(3/2)+4*2^(1/2)*Pi^(1/2
)*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(3/2)+4*arccos(a
*x)*a*x+10*arccos(a*x)*cos(5*arccos(a*x))+18*arccos(a*x)*cos(3*arccos(a*x))
+2*(-a^2*x^2+1)^(1/2)+3*sin(3*arccos(a*x))+sin(5*arccos(a*x)))/arccos(a*x)^
(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4/arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^4}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

```
[In] integrate(x**4/acos(a*x)**(5/2),x)
[Out] Integral(x**4/acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4/arccos(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^4}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^4/arccos(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(x^4/arccos(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^4}{\operatorname{acos}(a\,x)^{5/2}}\,dx$$

```
[In] int(x^4/acos(a*x)^(5/2),x)
[Out] int(x^4/acos(a*x)^(5/2), x)
```

3.108 $\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 126

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\arccos(ax)}} + \frac{16x^4}{3\sqrt{\arccos(ax)}} + \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^4} + \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{3a^4}$$

```
[Out] 4/3*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4+4/3*FresnelS(2*2^(1
/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4+2/3*x^3*(-a^2*x^2+1)^(
1/2)/a/arccos(a*x)^(3/2)-4*x^2/a^2/arccos(a*x)^(1/2)+16/3*x^4/arccos(a*x)^(
1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4730, 4808, 4732, 4491, 3386, 3432, 12}

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^4} + \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{3a^4} - \frac{4x^2}{a^2\sqrt{\arccos(ax)}} + \frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} + \frac{16x^4}{3\sqrt{\arccos(ax)}}$$

```
[In] Int[x^3/ArcCos[a*x]^(5/2),x]
```

```
[Out] (2*x^3*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (4*x^2)/(a^2*Sqrt[ArcCo
s[a*x]]) + (16*x^4)/(3*Sqrt[ArcCos[a*x]]) + (4*Sqrt[2*Pi]*FresnelS[2*Sqrt[2
```

```
/Pi]*Sqrt[ArcCos[a*x]]])/(3*a^4) + (4*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]
])/Sqrt[Pi]])/(3*a^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
```

```
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{2\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}\,dx}{a} + \frac{1}{3}(8a)\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}\,dx \\
&= \frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\arccos(ax)}} + \frac{16x^4}{3\sqrt{\arccos(ax)}} \\
&\quad - \frac{64}{3}\int \frac{x^3}{\sqrt{\arccos(ax)}}\,dx + \frac{8\int \frac{x}{\sqrt{\arccos(ax)}}\,dx}{a^2} \\
&= \frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\arccos(ax)}} + \frac{16x^4}{3\sqrt{\arccos(ax)}} \\
&\quad - \frac{8\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^4} \\
&\quad + \frac{64\text{Subst}\left(\int \frac{\cos^3(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{3a^4} \\
&= \frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\arccos(ax)}} + \frac{16x^4}{3\sqrt{\arccos(ax)}} \\
&\quad - \frac{8\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^4} \\
&\quad + \frac{64\text{Subst}\left(\int \left(\frac{\sin(2x)}{4\sqrt{x}} + \frac{\sin(4x)}{8\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{3a^4} \\
&= \frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\arccos(ax)}} + \frac{16x^4}{3\sqrt{\arccos(ax)}} + \frac{8\text{Subst}\left(\int \frac{\sin(4x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{3a^4} \\
&\quad - \frac{4\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^4} + \frac{16\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{3a^4} \\
&= \frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\arccos(ax)}} + \frac{16x^4}{3\sqrt{\arccos(ax)}} \\
&\quad + \frac{16\text{Subst}\left(\int \sin(4x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{3a^4} \\
&\quad - \frac{8\text{Subst}\left(\int \sin(2x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{a^4} \\
&\quad + \frac{32\text{Subst}\left(\int \sin(2x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{3a^4}
\end{aligned}$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4x^2}{a^2\sqrt{\arccos(ax)}} + \frac{16x^4}{3\sqrt{\arccos(ax)}} + \frac{4\sqrt{2\pi}\,\text{FresnelS}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^4} + \frac{4\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{3a^4}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.62 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.61

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = -\frac{-4\arccos(ax)\left(e^{-4i\arccos(ax)} + e^{4i\arccos(ax)} - 2\sqrt{-i\arccos(ax)}\Gamma\left(\frac{1}{2}, -4i\arccos(ax)\right) - 2\sqrt{i\arccos(ax)}\Gamma\left(\frac{1}{2}\right.\right.}{}$$

```
[In] Integrate[x^3/ArcCos[a*x]^(5/2),x]
```

```
[Out] -1/12*(-4*ArcCos[a*x]*(E^((-4*I)*ArcCos[a*x]) + E^((4*I)*ArcCos[a*x]) - 2*S
qrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-4*I)*ArcCos[a*x]] - 2*Sqrt[I*ArcCos[a*x]
]*Gamma[1/2, (4*I)*ArcCos[a*x]]) - 2*(2*ArcCos[a*x]*(E^((-2*I)*ArcCos[a*x])
 + E^((2*I)*ArcCos[a*x]) - Sqrt[2]*Sqrt[(-I)*ArcCos[a*x]]*Gamma[1/2, (-2*I)
*ArcCos[a*x]] - Sqrt[2]*Sqrt[I*ArcCos[a*x]]*Gamma[1/2, (2*I)*ArcCos[a*x]])
+ Sin[2*ArcCos[a*x]]) - Sin[4*ArcCos[a*x]])/(a^4*ArcCos[a*x]^(3/2))
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.85

method	result
default	$\frac{16\sqrt{2}\,\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+16\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+8\arccos(ax)\cos(2\arccos(ax))+8}{12a^4\arccos(ax)^{\frac{3}{2}}}$

```
[In] int(x^3/arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12/a^4*(16*2^(1/2)*Pi^(1/2)*FresnelS(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2)
)*arccos(a*x)^(3/2)+16*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*arcc
os(a*x)^(3/2)+8*arccos(a*x)*cos(2*arccos(a*x))+8*arccos(a*x)*cos(4*arccos(a
*x))+2*sin(2*arccos(a*x))+sin(4*arccos(a*x)))/arccos(a*x)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^3}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

```
[In] integrate(x**3/acos(a*x)**(5/2),x)
```

```
[Out] Integral(x**3/acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3/arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3/arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^3}{\mathrm{acos}\left(a\,x\right)^{5/2}}\,dx$$

```
[In] int(x^3/acos(a*x)^(5/2),x)
[Out] int(x^3/acos(a*x)^(5/2), x)
```

3.109 $\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 125

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\arccos(ax)}} + \frac{4x^3}{\sqrt{\arccos(ax)}} + \frac{\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^3} + \frac{\sqrt{6\pi}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3}$$

```
[Out] 1/3*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3+Fresn
elS(6^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a^3+2/3*x^2*(-a^2*
x^2+1)^(1/2)/a/arccos(a*x)^(3/2)-8/3*x/a^2/arccos(a*x)^(1/2)+4*x^3/arccos(a
*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4730, 4808, 4732, 4491, 3386, 3432, 4720}

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \frac{\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^3} + \frac{\sqrt{6\pi}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3} + \frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\arccos(ax)}} + \frac{4x^3}{\sqrt{\arccos(ax)}}$$

```
[In] Int[x^2/ArcCos[a*x]^(5/2),x]
```

```
[Out] (2*x^2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - (8*x)/(3*a^2*Sqrt[ArcCo
s[a*x]]) + (4*x^3)/Sqrt[ArcCos[a*x]] + (Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt
[ArcCos[a*x]]])/(3*a^3) + (Sqrt[6*Pi]*FresnelS[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]
])/a^3
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
```

```
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}\,dx}{3a} + (2a)\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}\,dx$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\arccos(ax)}} + \frac{4x^3}{\sqrt{\arccos(ax)}} - 12\int \frac{x^2}{\sqrt{\arccos(ax)}}\,dx + \frac{8\int \frac{1}{\sqrt{\arccos(ax)}}\,dx}{3a^2}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\arccos(ax)}} + \frac{4x^3}{\sqrt{\arccos(ax)}} - \frac{8\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{3a^3} + \frac{12\text{Subst}\left(\int \frac{\cos^2(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^3}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\arccos(ax)}} + \frac{4x^3}{\sqrt{\arccos(ax)}} - \frac{16\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{3a^3} + \frac{12\text{Subst}\left(\int \left(\frac{\sin(x)}{4\sqrt{x}} + \frac{\sin(3x)}{4\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{a^3}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\arccos(ax)}} + \frac{4x^3}{\sqrt{\arccos(ax)}} - \frac{8\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^3} + \frac{3\text{Subst}\left(\int \frac{\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^3} + \frac{3\text{Subst}\left(\int \frac{\sin(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{a^3}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\arccos(ax)}} + \frac{4x^3}{\sqrt{\arccos(ax)}} - \frac{8\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^3} + \frac{6\text{Subst}\left(\int \sin\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{a^3} + \frac{6\text{Subst}\left(\int \sin\left(3x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{a^3}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{8x}{3a^2\sqrt{\arccos(ax)}} + \frac{4x^3}{\sqrt{\arccos(ax)}} + \frac{\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^3} + \frac{\sqrt{6\pi}\,\text{FresnelS}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.58 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.76

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = -\frac{-\sqrt{1-a^2x^2} - e^{-i\arccos(ax)}\arccos(ax) - e^{i\arccos(ax)}\arccos(ax) + \sqrt{-i\arccos(ax)}\arccos(ax)\Gamma\left(\frac{1}{2}, -i\arcco\right.}{}$$

```
[In] Integrate[x^2/ArcCos[a*x]^(5/2),x]
```

```
[Out] -1/6*(-Sqrt[1 - a^2*x^2] - ArcCos[a*x]/E^(I*ArcCos[a*x]) - E^(I*ArcCos[a*x]
)*ArcCos[a*x] + Sqrt[(-I)*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos[a
*x]] + Sqrt[I*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, I*ArcCos[a*x]] - 3*ArcCos
[a*x]*(E^((-3*I)*ArcCos[a*x]) + E^((3*I)*ArcCos[a*x]) - Sqrt[3]*Sqrt[(-I)*A
rcCos[a*x]]*Gamma[1/2, (-3*I)*ArcCos[a*x]] - Sqrt[3]*Sqrt[I*ArcCos[a*x]]*Ga
mma[1/2, (3*I)*ArcCos[a*x]]) - Sin[3*ArcCos[a*x]])/(a^3*ArcCos[a*x]^(3/2))
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.92

method	result
default	$\frac{6\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+2\sqrt{2}\,\sqrt{\pi}\,\text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+2\arccos(ax)ax+6\arccos}{6a^3\arccos(ax)^{\frac{3}{2}}}$

```
[In] int(x^2/arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/a^3*(6*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelS(2^(1/2)/Pi^(1/2)*3^(1/2)*arcco
s(a*x)^(1/2))*arccos(a*x)^(3/2)+2*2^(1/2)*Pi^(1/2)*FresnelS(2^(1/2)/Pi^(1/2
)*arccos(a*x)^(1/2))*arccos(a*x)^(3/2)+2*arccos(a*x)*a*x+6*arccos(a*x)*cos(
3*arccos(a*x))+(-a^2*x^2+1)^(1/2)+sin(3*arccos(a*x)))/arccos(a*x)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2/arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^2}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

```
[In] integrate(x**2/acos(a*x)**(5/2),x)
```

```
[Out] Integral(x**2/acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2/arccos(a*x)^(5/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^2}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x^2/arccos(a*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/arccos(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^{5/2}}\,dx = \int \frac{x^2}{\mathrm{acos}\left(a\,x\right)^{5/2}}\,dx$$

```
[In] int(x^2/acos(a*x)^(5/2),x)
[Out] int(x^2/acos(a*x)^(5/2), x)
```

3.110 $\int \frac{x}{\arccos(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx=\frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\arccos(ax)}}+\frac{8x^2}{3\sqrt{\arccos(ax)}}+\frac{8\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{3a^2}$$

```
[Out] 8/3*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2+2/3*x*(-a^2*x^2+1)^
(1/2)/a/arccos(a*x)^(3/2)-4/3/a^2/arccos(a*x)^(1/2)+8/3*x^2/arccos(a*x)^(1/
2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {4730, 4808, 4732, 4491, 12, 3386, 3432, 4738}

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx=\frac{8\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{3a^2}+\frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}-\frac{4}{3a^2\sqrt{\arccos(ax)}}+\frac{8x^2}{3\sqrt{\arccos(ax)}}$$

```
[In] Int[x/ArcCos[a*x]^(5/2),x]
```

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) - 4/(3*a^2*Sqrt[ArcCos[a*x]
]) + (8*x^2)/(3*Sqrt[ArcCos[a*x]]) + (8*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*
x]])/Sqrt[Pi]])/(3*a^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{2\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}\,dx}{3a} + \frac{1}{3}(4a)\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}\,dx \\
&= \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\arccos(ax)}} + \frac{8x^2}{3\sqrt{\arccos(ax)}} - \frac{16}{3}\int \frac{x}{\sqrt{\arccos(ax)}}\,dx \\
&= \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\arccos(ax)}} + \frac{8x^2}{3\sqrt{\arccos(ax)}} \\
&\quad + \frac{16\text{Subst}\left(\int \frac{\cos(x)\sin(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{3a^2} \\
&= \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\arccos(ax)}} + \frac{8x^2}{3\sqrt{\arccos(ax)}} + \frac{16\text{Subst}\left(\int \frac{\sin(2x)}{2\sqrt{x}}\,dx, x, \arccos(ax)\right)}{3a^2} \\
&= \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\arccos(ax)}} + \frac{8x^2}{3\sqrt{\arccos(ax)}} + \frac{8\text{Subst}\left(\int \frac{\sin(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{3a^2} \\
&= \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\arccos(ax)}} + \frac{8x^2}{3\sqrt{\arccos(ax)}} \\
&\quad + \frac{16\text{Subst}\left(\int \sin\left(2x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{3a^2} \\
&= \frac{2x\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}} - \frac{4}{3a^2\sqrt{\arccos(ax)}} + \frac{8x^2}{3\sqrt{\arccos(ax)}} + \frac{8\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{3a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \frac{8\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+\frac{4\arccos(ax)\cos(2\arccos(ax))+\sin(2\arccos(ax))}{\arccos(ax)^{3/2}}}{3a^2}$$

```
[In] Integrate[x/ArcCos[a*x]^(5/2),x]
```

```
[Out] (8*Sqrt[Pi]*FresnelS[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]] + (4*ArcCos[a*x]*Cos[2
*ArcCos[a*x]] + Sin[2*ArcCos[a*x]])/ArcCos[a*x]^(3/2))/(3*a^2)
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

method	result	size
default	$\frac{8\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{3}{2}}+4\arccos(ax)\cos(2\arccos(ax))+\sin(2\arccos(ax))}{3a^2\arccos(ax)^{\frac{3}{2}}}$	56

```
[In] int(x/arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a^2*(8*Pi^(1/2)*FresnelS(2*arccos(a*x)^(1/2)/Pi^(1/2))*arccos(a*x)^(3/2
)+4*arccos(a*x)*cos(2*arccos(a*x))+sin(2*arccos(a*x)))/arccos(a*x)^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

```
[In] integrate(x/acos(a*x)**(5/2),x)
[Out] Integral(x/acos(a*x)**(5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/arccos(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \int \frac{x}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/arccos(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(x/arccos(a*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^{5/2}}\,dx = \int \frac{x}{\operatorname{acos}(a\,x)^{5/2}}\,dx$$

```
[In] int(x/acos(a*x)^(5/2),x)
[Out] int(x/acos(a*x)^(5/2), x)
```

3.111 $\int \frac{1}{\arccos(ax)^{5/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 76

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}+\frac{4x}{3\sqrt{\arccos(ax)}}+\frac{4\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a}$$

```
[Out] 4/3*FresnelS(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a+2/3*(-a
^2*x^2+1)^(1/2)/a/arccos(a*x)^(3/2)+4/3*x/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4718, 4808, 4720, 3386, 3432}

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}+\frac{4\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a}+\frac{4x}{3\sqrt{\arccos(ax)}}$$

```
[In] Int[ArcCos[a*x]^(-5/2),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(3*a*ArcCos[a*x]^(3/2)) + (4*x)/(3*Sqrt[ArcCos[a*x]])
 + (4*Sqrt[2*Pi]*FresnelS[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(3*a)
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}+\frac{1}{3}(2a)\int\frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^{3/2}}\,dx\\
&= \frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}+\frac{4x}{3\sqrt{\arccos(ax)}}-\frac{4}{3}\int\frac{1}{\sqrt{\arccos(ax)}}\,dx\\
&= \frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}+\frac{4x}{3\sqrt{\arccos(ax)}}+\frac{4\text{Subst}\left(\int\frac{\sin(x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{3a}\\
&= \frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}+\frac{4x}{3\sqrt{\arccos(ax)}}+\frac{8\text{Subst}\left(\int\sin\left(x^2\right)\,dx,x,\sqrt{\arccos(ax)}\right)}{3a}\\
&= \frac{2\sqrt{1-a^2x^2}}{3a\arccos(ax)^{3/2}}+\frac{4x}{3\sqrt{\arccos(ax)}}+\frac{4\sqrt{2\pi}\,\text{FresnelS}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{3a}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.61

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx =$$
$$-\frac{2\left(-\sqrt{1-a^2x^2}-e^{-i\arccos(ax)}\arccos(ax)-e^{i\arccos(ax)}\arccos(ax)+\sqrt{-i\arccos(ax)}\arccos(ax)\Gamma\left(\frac{1}{2},-i\,\mathrm{arc}\right.}{3a\arccos(ax)^{3/2}}$$

```
[In] Integrate[ArcCos[a*x]^(-5/2),x]
```

```
[Out] (-2*(-Sqrt[1 - a^2*x^2] - ArcCos[a*x]/E^(I*ArcCos[a*x]) - E^(I*ArcCos[a*x])
*ArcCos[a*x] + Sqrt[(-I)*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos[a*
x]] + Sqrt[I*ArcCos[a*x]]*ArcCos[a*x]*Gamma[1/2, I*ArcCos[a*x]]))/(3*a*ArcC
os[a*x]^(3/2))
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

method	result	size
default	$\frac{\sqrt{2}\left(4\arccos(ax)^2\pi\,\mathrm{FresnelS}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)+2\arccos(ax)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi}\,ax+\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\sqrt{-a^2x^2+1}\right)}{3a\sqrt{\pi}\arccos(ax)^2}$	83

```
[In] int(1/arccos(a*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3/a*2^(1/2)/Pi^(1/2)*(4*arccos(a*x)^2*Pi*FresnelS(2^(1/2)/Pi^(1/2)*arccos
(a*x)^(1/2))+2*arccos(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a*x+2^(1/2)*arccos(a*x)^(
1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2))/arccos(a*x)^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/arccos(a*x)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \int \frac{1}{\operatorname{acos}^{\frac{5}{2}}(ax)}\,dx$$

```
[In] integrate(1/acos(a*x)**(5/2),x)
[Out] Integral(acos(a*x)**(-5/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/arccos(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \int \frac{1}{\arccos(ax)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/arccos(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(arccos(a*x)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^{5/2}}\,dx = \int \frac{1}{\operatorname{acos}(a\,x)^{5/2}}\,dx$$

```
[In] int(1/acos(a*x)^(5/2),x)
[Out] int(1/acos(a*x)^(5/2), x)
```

3.112 $\int \frac{1}{x \arccos(ax)^{5/2}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \text{Int}\left(\frac{1}{x \arccos(ax)^{5/2}}, x\right)$$

```
[Out] Unintegrable(1/x/arccos(a*x)^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \int \frac{1}{x \arccos(ax)^{5/2}} \, dx$$

```
[In] Int[1/(x*ArcCos[a*x]^(5/2)),x]
[Out] Defer[Int][1/(x*ArcCos[a*x]^(5/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \arccos(ax)^{5/2}} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \int \frac{1}{x \arccos(ax)^{5/2}} \, dx$$

```
[In] Integrate[1/(x*ArcCos[a*x]^(5/2)),x]
[Out] Integrate[1/(x*ArcCos[a*x]^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \arccos\left(ax\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/x/arccos(a*x)^(5/2),x)
[Out] int(1/x/arccos(a*x)^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/arccos(a*x)^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 7.78 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \int \frac{1}{x \operatorname{acos}^{\frac{5}{2}}\left(ax\right)}\, dx$$

```
[In] integrate(1/x/acos(a*x)**(5/2),x)
[Out] Integral(1/(x*acos(a*x)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/x/arccos(a*x)^(5/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \int \frac{1}{x \arccos\left(ax\right)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/x/arccos(a*x)^(5/2),x, algorithm="giac")
[Out] integrate(1/(x*arccos(a*x)^(5/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{5/2}} \, dx = \int \frac{1}{x \, \mathrm{acos}\left(a\,x\right)^{5/2}} \, dx$$

```
[In] int(1/(x*acos(a*x)^(5/2)),x)
[Out] int(1/(x*acos(a*x)^(5/2)), x)
```

3.113 $\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 264

$$\begin{aligned}\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx &= \frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{16x^3}{15a^2\arccos(ax)^{3/2}} \\ &+ \frac{4x^5}{3\arccos(ax)^{3/2}} + \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} - \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\arccos(ax)}} \\ &+ \frac{\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^5} + \frac{5\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5} \\ &- \frac{8\sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{5a^5} + \frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^5}\end{aligned}$$

```
[Out] -16/15*x^3/a^2/arccos(a*x)^(3/2)+4/3*x^5/arccos(a*x)^(3/2)+9/10*FresnelC(6^
(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a^5+1/15*FresnelC(2^(1/2
)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^5+5/6*FresnelC(10^(1/2)/Pi
^(1/2)*arccos(a*x)^(1/2))*10^(1/2)*Pi^(1/2)/a^5+2/5*x^4*(-a^2*x^2+1)^(1/2)/
a/arccos(a*x)^(5/2)+32/5*x^2*(-a^2*x^2+1)^(1/2)/a^3/arccos(a*x)^(1/2)-40/3*
x^4*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {4730, 4808, 4728, 3385, 3433}

$$\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx = \frac{\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^5} - \frac{8\sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{5a^5} + \frac{5\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5} + \frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^5} - \frac{16x^3}{15a^2\arccos(ax)^{3/2}} - \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\arccos(ax)}} + \frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} + \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} + \frac{4x^5}{3\arccos(ax)^{3/2}}$$

```
[In] Int[x^4/ArcCos[a*x]^(7/2),x]
```

```
[Out] (2*x^4*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (16*x^3)/(15*a^2*ArcCos
[a*x]^(3/2)) + (4*x^5)/(3*ArcCos[a*x]^(3/2)) + (32*x^2*Sqrt[1 - a^2*x^2])/(
5*a^3*Sqrt[ArcCos[a*x]]) - (40*x^4*Sqrt[1 - a^2*x^2])/(3*a*Sqrt[ArcCos[a*x]
]) + (Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a^5) + (5*Sqrt
[(3*Pi)/2]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/a^5 - (8*Sqrt[6*Pi]*Fres
nelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(5*a^5) + (5*Sqrt[(5*Pi)/2]*FresnelC[Sq
rt[10/Pi]*Sqrt[ArcCos[a*x]]])/(3*a^5)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{8\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}\,dx}{5a} + (2a)\int \frac{x^5}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}\,dx \\
&= \frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{16x^3}{15a^2\arccos(ax)^{3/2}} + \frac{4x^5}{3\arccos(ax)^{3/2}} \\
&\quad - \frac{20}{3}\int \frac{x^4}{\arccos(ax)^{3/2}}\,dx + \frac{16\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx}{5a^2} \\
&= \frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{16x^3}{15a^2\arccos(ax)^{3/2}} + \frac{4x^5}{3\arccos(ax)^{3/2}} + \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} \\
&\quad - \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\arccos(ax)}} + \frac{32\text{Subst}\left(\int\left(-\frac{\cos(x)}{4\sqrt{x}} - \frac{3\cos(3x)}{4\sqrt{x}}\right)dx, x, \arccos(ax)\right)}{5a^5} \\
&\quad - \frac{40\text{Subst}\left(\int\left(-\frac{\cos(x)}{8\sqrt{x}} - \frac{9\cos(3x)}{16\sqrt{x}} - \frac{5\cos(5x)}{16\sqrt{x}}\right)dx, x, \arccos(ax)\right)}{3a^5}
\end{aligned}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{16x^3}{15a^2\arccos(ax)^{3/2}} + \frac{4x^5}{3\arccos(ax)^{3/2}} + \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} - \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\arccos(ax)}} - \frac{8\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{5a^5} + \frac{5\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{3a^5} + \frac{25\text{Subst}\left(\int \frac{\cos(5x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{6a^5} - \frac{24\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{5a^5} + \frac{15\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{2a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{16x^3}{15a^2\arccos(ax)^{3/2}} + \frac{4x^5}{3\arccos(ax)^{3/2}} + \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} - \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\arccos(ax)}} - \frac{16\text{Subst}\left(\int \cos(x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{5a^5} + \frac{10\text{Subst}\left(\int \cos(x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{3a^5} + \frac{25\text{Subst}\left(\int \cos(5x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{3a^5} - \frac{48\text{Subst}\left(\int \cos(3x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{5a^5} + \frac{15\text{Subst}\left(\int \cos(3x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{a^5}$$

$$= \frac{2x^4\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{16x^3}{15a^2\arccos(ax)^{3/2}} + \frac{4x^5}{3\arccos(ax)^{3/2}} + \frac{32x^2\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} - \frac{40x^4\sqrt{1-a^2x^2}}{3a\sqrt{\arccos(ax)}} + \frac{\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^5} + \frac{5\sqrt{\frac{3\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{a^5} - \frac{8\sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{5a^5} + \frac{5\sqrt{\frac{5\pi}{2}}\,\text{FresnelC}\left(\sqrt{\frac{10}{\pi}}\sqrt{\arccos(ax)}\right)}{3a^5}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.37 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.58

$$\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx =$$
$$-\frac{2\left(-6\sqrt{1-a^2x^2}-2ie^{i\arccos(ax)}\arccos(ax)(-i+2\arccos(ax))-4(-i\arccos(ax))^{3/2}\arccos(ax)\Gamma\left(\frac{1}{2},-i\arccos\right.\right.}{}$$

```
[In] Integrate[x^4/ArcCos[a*x]^(7/2),x]
```

```
[Out] -1/240*(2*(-6*Sqrt[1 - a^2*x^2] - (2*I)*E^(I*ArcCos[a*x])*ArcCos[a*x]*(-I +
 2*ArcCos[a*x]) - 4*((-I)*ArcCos[a*x])^(3/2)*ArcCos[a*x]*Gamma[1/2, (-I)*Ar
cCos[a*x]] + (ArcCos[a*x]*(-2 + (4*I)*ArcCos[a*x] - 4*E^(I*ArcCos[a*x])*(I*
ArcCos[a*x])^(3/2)*Gamma[1/2, I*ArcCos[a*x]]))/E^(I*ArcCos[a*x])) - 5*ArcCo
s[a*x]*(2*E^((5*I)*ArcCos[a*x])*(1 + (10*I)*ArcCos[a*x]) + 20*Sqrt[5]*((-I)
*ArcCos[a*x])^(3/2)*Gamma[1/2, (-5*I)*ArcCos[a*x]] + (2 - (20*I)*ArcCos[a*x
] + 20*Sqrt[5]*E^((5*I)*ArcCos[a*x])*(I*ArcCos[a*x])^(3/2)*Gamma[1/2, (5*I)
*ArcCos[a*x]])/E^((5*I)*ArcCos[a*x])) + 9*(-2*ArcCos[a*x]*(E^((3*I)*ArcCos[
a*x])*(1 + (6*I)*ArcCos[a*x]) + 6*Sqrt[3]*((-I)*ArcCos[a*x])^(3/2)*Gamma[1/
2, (-3*I)*ArcCos[a*x]] + (1 - (6*I)*ArcCos[a*x] + 6*Sqrt[3]*E^((3*I)*ArcCos
[a*x])*(I*ArcCos[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcCos[a*x]])/E^((3*I)*ArcCos
[a*x])) - 2*Sin[3*ArcCos[a*x]]) - 6*Sin[5*ArcCos[a*x]])/(a^5*ArcCos[a*x]^(5
/2))
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.85

method	result
default	$-\frac{-100\sqrt{2}\,\sqrt{\pi}\,\sqrt{5}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{5}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}-108\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}-8\sqrt{2}\,\sqrt{\pi}\,\mathrm{F}}{}$

```
[In] int(x^4/arccos(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/120/a^5*(-100*2^(1/2)*Pi^(1/2)*5^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*5^(1/2)
*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)-108*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelC
(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)-8*2^(1/2)*Pi
^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)+8*arc
cos(a*x)^2*(-a^2*x^2+1)^(1/2)+100*arccos(a*x)^2*sin(5*arccos(a*x))+108*arcc
os(a*x)^2*sin(3*arccos(a*x))-4*arccos(a*x)*a*x-10*arccos(a*x)*cos(5*arccos(
a*x))-18*arccos(a*x)*cos(3*arccos(a*x))-6*(-a^2*x^2+1)^(1/2)-3*sin(5*arccos
(a*x))-9*sin(3*arccos(a*x)))/arccos(a*x)^(5/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{7/2}} dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^4/arccos(a*x)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^4}{\arccos(ax)^{7/2}} dx = \int \frac{x^4}{\operatorname{acos}^{\frac{7}{2}}(ax)} dx$$

```
[In] integrate(x**4/acos(a*x)**(7/2),x)
[Out] Integral(x**4/acos(a*x)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^4}{\arccos(ax)^{7/2}} dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^4/arccos(a*x)^(7/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x^4}{\arccos(ax)^{7/2}} dx = \int \frac{x^4}{\arccos(ax)^{\frac{7}{2}}} dx$$

```
[In] integrate(x^4/arccos(a*x)^(7/2),x, algorithm="giac")
[Out] integrate(x^4/arccos(a*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^4}{\mathrm{acos}\left(a\,x\right)^{7/2}}\,dx$$

```
[In] int(x^4/acos(a*x)^(7/2),x)
[Out] int(x^4/acos(a*x)^(7/2), x)
```

3.114 $\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 190

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4x^2}{5a^2\arccos(ax)^{3/2}} + \frac{16x^4}{15\arccos(ax)^{3/2}} + \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} - \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{32\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^4} + \frac{16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{15a^4}$$

```
[Out] -4/5*x^2/a^2/arccos(a*x)^(3/2)+16/15*x^4/arccos(a*x)^(3/2)+16/15*FresnelC(2
*arccos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^4+32/15*FresnelC(2*2^(1/2)/Pi^(1/2)
*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^4+2/5*x^3*(-a^2*x^2+1)^(1/2)/a/arcco
s(a*x)^(5/2)+16/5*x*(-a^2*x^2+1)^(1/2)/a^3/arccos(a*x)^(1/2)-128/15*x^3*(-a
^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4730, 4808, 4728, 3385, 3433}

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \frac{32\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^4} + \frac{16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{15a^4} - \frac{4x^2}{5a^2\arccos(ax)^{3/2}} - \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} + \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} + \frac{16x^4}{15\arccos(ax)^{3/2}}$$

```
[In] Int[x^3/ArcCos[a*x]^(7/2),x]
```

```
[Out] (2*x^3*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (4*x^2)/(5*a^2*ArcCos[a
*x]^(3/2)) + (16*x^4)/(15*ArcCos[a*x]^(3/2)) + (16*x*Sqrt[1 - a^2*x^2])/(5*
a^3*Sqrt[ArcCos[a*x]]) - (128*x^3*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[ArcCos[a*x]
]) + (32*Sqrt[2*Pi]*FresnelC[2*Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a^4) + (1
6*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(15*a^4)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{6\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(8a)\int \frac{x^4}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}\,dx \\
&= \frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4x^2}{5a^2\arccos(ax)^{3/2}} + \frac{16x^4}{15\arccos(ax)^{3/2}} \\
&\quad - \frac{64}{15}\int \frac{x^3}{\arccos(ax)^{3/2}}\,dx + \frac{8\int \frac{x}{\arccos(ax)^{3/2}}\,dx}{5a^2} \\
&= \frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4x^2}{5a^2\arccos(ax)^{3/2}} + \frac{16x^4}{15\arccos(ax)^{3/2}} + \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} \\
&\quad - \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} - \frac{16\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{5a^4} \\
&\quad - \frac{128\text{Subst}\left(\int \left(-\frac{\cos(2x)}{2\sqrt{x}} - \frac{\cos(4x)}{2\sqrt{x}}\right)\,dx, x, \arccos(ax)\right)}{15a^4} \\
&= \frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4x^2}{5a^2\arccos(ax)^{3/2}} + \frac{16x^4}{15\arccos(ax)^{3/2}} + \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} \\
&\quad - \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{64\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{15a^4} \\
&\quad + \frac{64\text{Subst}\left(\int \frac{\cos(4x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{15a^4} \\
&\quad - \frac{32\text{Subst}\left(\int \cos\left(2x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{5a^4} \\
&= \frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4x^2}{5a^2\arccos(ax)^{3/2}} + \frac{16x^4}{15\arccos(ax)^{3/2}} \\
&\quad + \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} - \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} - \frac{16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{5a^4} \\
&\quad + \frac{128\text{Subst}\left(\int \cos\left(2x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{15a^4} \\
&\quad + \frac{128\text{Subst}\left(\int \cos\left(4x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{15a^4}
\end{aligned}
$$

$$= \frac{2x^3\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4x^2}{5a^2\arccos(ax)^{3/2}} + \frac{16x^4}{15\arccos(ax)^{3/2}} + \frac{16x\sqrt{1-a^2x^2}}{5a^3\sqrt{\arccos(ax)}} - \frac{128x^3\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{32\sqrt{2\pi}\,\text{FresnelC}\left(2\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^4} + \frac{16\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{15a^4}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.09 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.39

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = -\frac{-4e^{-2i\arccos(ax)}\left(1+e^{4i\arccos(ax)}(1+4i\arccos(ax))-4i\arccos(ax)\right)\arccos(ax)+\frac{16\sqrt{2}\arccos(ax)^3\Gamma\left(\frac{1}{2},-2i\arccos(a\right.}{\sqrt{-i\arccos(ax)}}}{}$$

```
[In] Integrate[x^3/ArcCos[a*x]^(7/2),x]
```

```
[Out] -1/60*((-4*(1 + E^((4*I)*ArcCos[a*x])*(1 + (4*I)*ArcCos[a*x]) - (4*I)*ArcCo
s[a*x])*ArcCos[a*x])/E^((2*I)*ArcCos[a*x]) + (16*Sqrt[2]*ArcCos[a*x]^3*Gamm
a[1/2, (-2*I)*ArcCos[a*x]])/Sqrt[(-I)*ArcCos[a*x]] + (16*I)*Sqrt[2]*(I*ArcC
os[a*x])^(5/2)*Gamma[1/2, (2*I)*ArcCos[a*x]] - 2*ArcCos[a*x]*(2*E^((4*I)*Ar
cCos[a*x])*(1 + (8*I)*ArcCos[a*x]) + 32*((-I)*ArcCos[a*x])^(3/2)*Gamma[1/2,
 (-4*I)*ArcCos[a*x]] + (2*(1 - (8*I)*ArcCos[a*x] + 16*E^((4*I)*ArcCos[a*x])
*(I*ArcCos[a*x])^(3/2)*Gamma[1/2, (4*I)*ArcCos[a*x]]))/E^((4*I)*ArcCos[a*x]
)) - 6*Sin[2*ArcCos[a*x]] - 3*Sin[4*ArcCos[a*x]])/(a^4*ArcCos[a*x]^(5/2))
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.73

method	result
default	$-\frac{-128\sqrt{2}\,\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}-64\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}+32\sin(2\arccos(ax))\arccos(a}{60a}$

```
[In] int(x^3/arccos(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/a^4*(-128*2^(1/2)*Pi^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)*arccos(a*x)^(1
/2))*arccos(a*x)^(5/2)-64*Pi^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*a
rccos(a*x)^(5/2)+32*sin(2*arccos(a*x))*arccos(a*x)^2+64*sin(4*arccos(a*x))*
```

```
arccos(a*x)^2-8*arccos(a*x)*cos(2*arccos(a*x))-8*arccos(a*x)*cos(4*arccos(a
*x))-6*sin(2*arccos(a*x))-3*sin(4*arccos(a*x)))/arccos(a*x)^(5/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^3/arccos(a*x)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^3}{\operatorname{acos}^{\frac{7}{2}}(ax)}\,dx$$

```
[In] integrate(x**3/acos(a*x)**(7/2),x)
[Out] Integral(x**3/acos(a*x)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3/arccos(a*x)^(7/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3/arccos(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^3}{\mathrm{acos}\left(a\,x\right)^{7/2}}\,dx$$

```
[In] int(x^3/acos(a*x)^(7/2),x)
```

```
[Out] int(x^3/acos(a*x)^(7/2), x)
```

3.115 $\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 191

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{8x}{15a^2\arccos(ax)^{3/2}} + \frac{4x^3}{5\arccos(ax)^{3/2}} + \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\arccos(ax)}} - \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\arccos(ax)}} + \frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^3} + \frac{6\sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{5a^3}$$

```
[Out] -8/15*x/a^2/arccos(a*x)^(3/2)+4/5*x^3/arccos(a*x)^(3/2)+2/15*FresnelC(2^(1/
2)/Pi^(1/2)*arccos(a*x)^(1/2))*2^(1/2)*Pi^(1/2)/a^3+6/5*FresnelC(6^(1/2)/Pi
^(1/2)*arccos(a*x)^(1/2))*6^(1/2)*Pi^(1/2)/a^3+2/5*x^2*(-a^2*x^2+1)^(1/2)/a
/arccos(a*x)^(5/2)+16/15*(-a^2*x^2+1)^(1/2)/a^3/arccos(a*x)^(1/2)-24/5*x^2*
(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4730, 4808, 4728, 3385, 3433, 4718, 4810}

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^3} + \frac{6\sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{5a^3} - \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\arccos(ax)}} + \frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{8x}{15a^2\arccos(ax)^{3/2}} + \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\arccos(ax)}} + \frac{4x^3}{5\arccos(ax)^{3/2}}$$

```
[In] Int[x^2/ArcCos[a*x]^(7/2),x]
```

```
[Out] (2*x^2*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - (8*x)/(15*a^2*ArcCos[a*
x]^(3/2)) + (4*x^3)/(5*ArcCos[a*x]^(3/2)) + (16*Sqrt[1 - a^2*x^2])/(15*a^3*
Sqrt[ArcCos[a*x]]) - (24*x^2*Sqrt[1 - a^2*x^2])/(5*a*Sqrt[ArcCos[a*x]]) + (
2*Sqrt[2*Pi]*FresnelC[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a^3) + (6*Sqrt[6*P
i]*FresnelC[Sqrt[6/Pi]*Sqrt[ArcCos[a*x]]])/(5*a^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
```

```
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4\int \frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(6a)\int \frac{x^3}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}\,dx \\
&= \frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{8x}{15a^2\arccos(ax)^{3/2}} + \frac{4x^3}{5\arccos(ax)^{3/2}} \\
&\quad - \frac{12}{5}\int \frac{x^2}{\arccos(ax)^{3/2}}\,dx + \frac{8\int \frac{1}{\arccos(ax)^{3/2}}\,dx}{15a^2} \\
&= \frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{8x}{15a^2\arccos(ax)^{3/2}} + \frac{4x^3}{5\arccos(ax)^{3/2}} \\
&\quad + \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\arccos(ax)}} - \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\arccos(ax)}} \\
&\quad - \frac{24\text{Subst}\left(\int \left(-\frac{\cos(x)}{4\sqrt{x}} - \frac{3\cos(3x)}{4\sqrt{x}}\right)dx, x, \arccos(ax)\right)}{5a^3} + \frac{16\int \frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx}{15a} \\
&= \frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{8x}{15a^2\arccos(ax)^{3/2}} + \frac{4x^3}{5\arccos(ax)^{3/2}} + \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\arccos(ax)}} \\
&\quad - \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\arccos(ax)}} - \frac{16\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{15a^3} \\
&\quad + \frac{6\text{Subst}\left(\int \frac{\cos(x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{5a^3} + \frac{18\text{Subst}\left(\int \frac{\cos(3x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{5a^3}
\end{aligned}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{8x}{15a^2\arccos(ax)^{3/2}} + \frac{4x^3}{5\arccos(ax)^{3/2}} + \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\arccos(ax)}} - \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\arccos(ax)}} - \frac{32\text{Subst}\left(\int\cos\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{15a^3} + \frac{12\text{Subst}\left(\int\cos\left(x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{5a^3} + \frac{36\text{Subst}\left(\int\cos\left(3x^2\right)\,dx, x, \sqrt{\arccos(ax)}\right)}{5a^3}$$

$$= \frac{2x^2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{8x}{15a^2\arccos(ax)^{3/2}} + \frac{4x^3}{5\arccos(ax)^{3/2}} + \frac{16\sqrt{1-a^2x^2}}{15a^3\sqrt{\arccos(ax)}} - \frac{24x^2\sqrt{1-a^2x^2}}{5a\sqrt{\arccos(ax)}} + \frac{2\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a^3} + \frac{6\sqrt{6\pi}\,\text{FresnelC}\left(\sqrt{\frac{6}{\pi}}\sqrt{\arccos(ax)}\right)}{5a^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.47

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = -\frac{-6\sqrt{1-a^2x^2} - 2ie^{i\arccos(ax)}\arccos(ax)(-i+2\arccos(ax)) - 4(-i\arccos(ax))^{3/2}\arccos(ax)\Gamma\left(\frac{1}{2}, -i\arccos\right.}{}$$

```
[In] Integrate[x^2/ArcCos[a*x]^(7/2),x]
```

```
[Out] -1/60*(-6*Sqrt[1 - a^2*x^2] - (2*I)*E^(I*ArcCos[a*x])*ArcCos[a*x]*(-I + 2*A
rcCos[a*x]) - 4*((-I)*ArcCos[a*x])^(3/2)*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos
[a*x]] + (ArcCos[a*x]*(-2 + (4*I)*ArcCos[a*x] - 4*E^(I*ArcCos[a*x])*(I*ArcC
os[a*x])^(3/2)*Gamma[1/2, I*ArcCos[a*x]]))/E^(I*ArcCos[a*x]) - 6*ArcCos[a*x
]*(E^((3*I)*ArcCos[a*x])*(1 + (6*I)*ArcCos[a*x]) + 6*Sqrt[3]*((-I)*ArcCos[a
*x])^(3/2)*Gamma[1/2, (-3*I)*ArcCos[a*x]] + (1 - (6*I)*ArcCos[a*x] + 6*Sqrt
[3]*E^((3*I)*ArcCos[a*x])*(I*ArcCos[a*x])^(3/2)*Gamma[1/2, (3*I)*ArcCos[a*x
]])/E^((3*I)*ArcCos[a*x])) - 6*Sin[3*ArcCos[a*x]])/(a^3*ArcCos[a*x]^(5/2))
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.81

method	result
default	$-\frac{-36\sqrt{2}\,\sqrt{\pi}\,\sqrt{3}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{3}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}-4\sqrt{2}\,\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}+4\arccos(ax)^{2}\sqrt{-a}}{30a^{3}\arccos}$

[In] int(x^2/arccos(a*x)^(7/2),x,method=_RETURNVERBOSE)

[Out] -1/30/a^3*(-36*2^(1/2)*Pi^(1/2)*3^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*3^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)-4*2^(1/2)*Pi^(1/2)*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*arccos(a*x)^(5/2)+4*arccos(a*x)^2*(-a^2*x^2+1)^(1/2)+36*arccos(a*x)^2*sin(3*arccos(a*x))-2*arccos(a*x)*a*x-6*arccos(a*x)*cos(3*arccos(a*x))-3*(-a^2*x^2+1)^(1/2)-3*sin(3*arccos(a*x)))/arccos(a*x)^(5/2)

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^2/arccos(a*x)^(7/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^2}{\operatorname{acos}^{\frac{7}{2}}(ax)}\,dx$$

[In] integrate(x**2/acos(a*x)**(7/2),x)

[Out] Integral(x**2/acos(a*x)**(7/2), x)

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2/arccos(a*x)^(7/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^2}{\arccos\left(ax\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x^2/arccos(a*x)^(7/2),x, algorithm="giac")
[Out] integrate(x^2/arccos(a*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\arccos(ax)^{7/2}}\,dx = \int \frac{x^2}{\mathrm{acos}\left(a\,x\right)^{7/2}}\,dx$$

```
[In] int(x^2/acos(a*x)^(7/2),x)
[Out] int(x^2/acos(a*x)^(7/2), x)
```

3.116 $\int \frac{x}{\arccos(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 119

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4}{15a^2\arccos(ax)^{3/2}} + \frac{8x^2}{15\arccos(ax)^{3/2}} - \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{32\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{15a^2}$$

```
[Out] -4/15/a^2/arccos(a*x)^(3/2)+8/15*x^2/arccos(a*x)^(3/2)+32/15*FresnelC(2*arc
cos(a*x)^(1/2)/Pi^(1/2))*Pi^(1/2)/a^2+2/5*x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x
)^(5/2)-32/15*x*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4730, 4808, 4728, 3385, 3433, 4738}

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \frac{32\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{15a^2} - \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4}{15a^2\arccos(ax)^{3/2}} + \frac{8x^2}{15\arccos(ax)^{3/2}}$$

```
[In] Int[x/ArcCos[a*x]^(7/2),x]
```

```
[Out] (2*x*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) - 4/(15*a^2*ArcCos[a*x]^(3/
2)) + (8*x^2)/(15*ArcCos[a*x]^(3/2)) - (32*x*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[
```

```
ArcCos[a*x]]) + (32*Sqrt[Pi]*FresnelC[(2*Sqrt[ArcCos[a*x]])/Sqrt[Pi]])/(15*
a^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{2\int \frac{1}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}\,dx}{5a} + \frac{1}{5}(4a)\int \frac{x^2}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}\,dx$$

$$= \frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4}{15a^2\arccos(ax)^{3/2}} + \frac{8x^2}{15\arccos(ax)^{3/2}} - \frac{16}{15}\int \frac{x}{\arccos(ax)^{3/2}}\,dx$$

$$= \frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4}{15a^2\arccos(ax)^{3/2}} + \frac{8x^2}{15\arccos(ax)^{3/2}} - \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{32\text{Subst}\left(\int \frac{\cos(2x)}{\sqrt{x}}\,dx, x, \arccos(ax)\right)}{15a^2}$$

$$= \frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4}{15a^2\arccos(ax)^{3/2}} + \frac{8x^2}{15\arccos(ax)^{3/2}} - \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{64\text{Subst}\left(\int \cos(2x^2)\,dx, x, \sqrt{\arccos(ax)}\right)}{15a^2}$$

$$= \frac{2x\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} - \frac{4}{15a^2\arccos(ax)^{3/2}} + \frac{8x^2}{15\arccos(ax)^{3/2}} - \frac{32x\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{32\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)}{15a^2}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.63

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \frac{\frac{4\cos(2\arccos(ax))}{\arccos(ax)^{3/2}} + 32\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right) - \frac{(-3+16\arccos(ax)^2)\sin(2\arccos(ax))}{\arccos(ax)^{5/2}}}{15a^2}$$

```
[In] Integrate[x/ArcCos[a*x]^(7/2),x]
[Out] ((4*Cos[2*ArcCos[a*x]])/ArcCos[a*x]^(3/2) + 32*Sqrt[Pi]*FresnelC[(2*Sqrt[Ar
cCos[a*x]])/Sqrt[Pi]] - ((-3 + 16*ArcCos[a*x]^2)*Sin[2*ArcCos[a*x]])/ArcCos
[a*x]^(5/2))/(15*a^2)
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

method	result
default	$-\frac{-32\sqrt{\pi}\,\mathrm{FresnelC}\left(\frac{2\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)\arccos(ax)^{\frac{5}{2}}+16\sin(2\arccos(ax))\arccos(ax)^{2}-4\arccos(ax)\cos(2\arccos(ax))-3\sin(2\arccos(ax)}{15a^{2}\arccos(ax)^{\frac{5}{2}}}$

```
[In] int(x/arccos(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15/a^2*(-32*Pi^(1/2)*FresnelC(2*arccos(a*x)^(1/2)/Pi^(1/2))*arccos(a*x)^
(5/2)+16*sin(2*arccos(a*x))*arccos(a*x)^2-4*arccos(a*x)*cos(2*arccos(a*x))-
3*sin(2*arccos(a*x)))/arccos(a*x)^(5/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/arccos(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \int \frac{x}{\operatorname{acos}^{\frac{7}{2}}(ax)}\,dx$$

```
[In] integrate(x/acos(a*x)**(7/2),x)
```

```
[Out] Integral(x/acos(a*x)**(7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x/arccos(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \int \frac{x}{\arccos(ax)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(x/arccos(a*x)^(7/2),x, algorithm="giac")
[Out] integrate(x/arccos(a*x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\arccos(ax)^{7/2}}\,dx = \int \frac{x}{\mathrm{acos}(a\,x)^{7/2}}\,dx$$

```
[In] int(x/acos(a*x)^(7/2),x)
[Out] int(x/acos(a*x)^(7/2), x)
```

3.117 $\int \frac{1}{\arccos(ax)^{7/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 105

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} + \frac{4x}{15\arccos(ax)^{3/2}} - \frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{8\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a}$$

```
[Out] 4/15*x/arccos(a*x)^(3/2)+8/15*FresnelC(2^(1/2)/Pi^(1/2)*arccos(a*x)^(1/2))*
2^(1/2)*Pi^(1/2)/a+2/5*(-a^2*x^2+1)^(1/2)/a/arccos(a*x)^(5/2)-8/15*(-a^2*x^
2+1)^(1/2)/a/arccos(a*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4718, 4808, 4810, 3385, 3433}

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = -\frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}} + \frac{8\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a} + \frac{4x}{15\arccos(ax)^{3/2}}$$

```
[In] Int[ArcCos[a*x]^(-7/2),x]
```

```
[Out] (2*Sqrt[1 - a^2*x^2])/(5*a*ArcCos[a*x]^(5/2)) + (4*x)/(15*ArcCos[a*x]^(3/2)
) - (8*Sqrt[1 - a^2*x^2])/(15*a*Sqrt[ArcCos[a*x]]) + (8*Sqrt[2*Pi]*FresnelC
[Sqrt[2/Pi]*Sqrt[ArcCos[a*x]]])/(15*a)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{1}{5}(2a)\int\frac{x}{\sqrt{1-a^2x^2}\arccos(ax)^{5/2}}\,dx\\ &= \frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{4x}{15\arccos(ax)^{3/2}}-\frac{4}{15}\int\frac{1}{\arccos(ax)^{3/2}}\,dx\\ &= \frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{4x}{15\arccos(ax)^{3/2}}-\frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}}\\ &\quad-\frac{1}{15}(8a)\int\frac{x}{\sqrt{1-a^2x^2}\sqrt{\arccos(ax)}}\,dx\end{aligned}$$

$$= \frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{4x}{15\arccos(ax)^{3/2}}-\frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}}+\frac{8\text{Subst}\left(\int\frac{\cos(x)}{\sqrt{x}}\,dx,x,\arccos(ax)\right)}{15a}$$

$$= \frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{4x}{15\arccos(ax)^{3/2}}-\frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{16\text{Subst}\left(\int\cos\left(x^2\right)\,dx,x,\sqrt{\arccos(ax)}\right)}{15a}$$

$$= \frac{2\sqrt{1-a^2x^2}}{5a\arccos(ax)^{5/2}}+\frac{4x}{15\arccos(ax)^{3/2}}-\frac{8\sqrt{1-a^2x^2}}{15a\sqrt{\arccos(ax)}} + \frac{8\sqrt{2\pi}\,\text{FresnelC}\left(\sqrt{\frac{2}{\pi}}\sqrt{\arccos(ax)}\right)}{15a}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.44

$$\int\frac{1}{\arccos(ax)^{7/2}}\,dx = -\frac{-6\sqrt{1-a^2x^2}-2ie^{i\arccos(ax)}\arccos(ax)(-i+2\arccos(ax))-4(-i\arccos(ax))^{3/2}\arccos(ax)\Gamma\left(\frac{1}{2},-i\arccos\right.}{15a\arccos(}$$

```
[In] Integrate[ArcCos[a*x]^(-7/2),x]
```

```
[Out] -1/15*(-6*Sqrt[1 - a^2*x^2] - (2*I)*E^(I*ArcCos[a*x])*ArcCos[a*x]*(-I + 2*A
rcCos[a*x]) - 4*((-I)*ArcCos[a*x])^(3/2)*ArcCos[a*x]*Gamma[1/2, (-I)*ArcCos
[a*x]] + (ArcCos[a*x]*(-2 + (4*I)*ArcCos[a*x] - 4*E^(I*ArcCos[a*x])*(I*ArcC
os[a*x])^(3/2)*Gamma[1/2, I*ArcCos[a*x]]))/E^(I*ArcCos[a*x]))/(a*ArcCos[a*x
]^(5/2))
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.05

method	result
default	$\frac{\sqrt{2}\left(8\arccos(ax)^3\pi\,\text{FresnelC}\left(\frac{\sqrt{2}\sqrt{\arccos(ax)}}{\sqrt{\pi}}\right)-4\arccos(ax)^{\frac{5}{2}}\sqrt{2}\sqrt{\pi}\sqrt{-a^2x^2+1}+2\arccos(ax)^{\frac{3}{2}}\sqrt{2}\sqrt{\pi}\,ax+3\sqrt{2}\sqrt{\arccos(ax)}\sqrt{\pi}\right.}{15a\sqrt{\pi}\arccos(ax)^3}$

```
[In] int(1/arccos(a*x)^(7/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15/a*2^(1/2)/Pi^(1/2)*(8*arccos(a*x)^3*Pi*FresnelC(2^(1/2)/Pi^(1/2)*arcco
s(a*x)^(1/2))-4*arccos(a*x)^(5/2)*2^(1/2)*Pi^(1/2)*(-a^2*x^2+1)^(1/2)+2*arc
cos(a*x)^(3/2)*2^(1/2)*Pi^(1/2)*a*x+3*2^(1/2)*arccos(a*x)^(1/2)*Pi^(1/2)*(-
a^2*x^2+1)^(1/2))/arccos(a*x)^3
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/arccos(a*x)^(7/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \int \frac{1}{\operatorname{acos}^{\frac{7}{2}}(ax)}\,dx$$

```
[In] integrate(1/acos(a*x)**(7/2),x)
```

```
[Out] Integral(acos(a*x)**(-7/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/arccos(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \int \frac{1}{\arccos\left(ax\right)^{\frac{7}{2}}}\,dx$$

```
[In] integrate(1/arccos(a*x)^(7/2),x, algorithm="giac")
[Out] integrate(arccos(a*x)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\arccos(ax)^{7/2}}\,dx = \int \frac{1}{\mathrm{acos}\left(a\,x\right)^{7/2}}\,dx$$

```
[In] int(1/acos(a*x)^(7/2),x)
[Out] int(1/acos(a*x)^(7/2), x)
```

3.118 $\int \frac{1}{x \arccos(ax)^{7/2}} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx = \text{Int}\left(\frac{1}{x \arccos(ax)^{7/2}}, x\right)$$

```
[Out] Unintegrable(1/x/arccos(a*x)^(7/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx = \int \frac{1}{x \arccos(ax)^{7/2}} \, dx$$

```
[In] Int[1/(x*ArcCos[a*x]^(7/2)),x]
[Out] Defer[Int][1/(x*ArcCos[a*x]^(7/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x \arccos(ax)^{7/2}} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx = \int \frac{1}{x \arccos(ax)^{7/2}} \, dx$$

```
[In] Integrate[1/(x*ArcCos[a*x]^(7/2)),x]
[Out] Integrate[1/(x*ArcCos[a*x]^(7/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1}{x \arccos\left(ax\right)^{\frac{7}{2}}}dx$$

```
[In] int(1/x/arccos(a*x)^(7/2),x)
[Out] int(1/x/arccos(a*x)^(7/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/arccos(a*x)^(7/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 68.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx = \int \frac{1}{x \operatorname{acos}^{\frac{7}{2}}{\left(ax \right)}}\, dx$$

```
[In] integrate(1/x/acos(a*x)**(7/2),x)
[Out] Integral(1/(x*acos(a*x)**(7/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/x/arccos(a*x)^(7/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx = \int \frac{1}{x \arccos(ax)^{\frac{7}{2}}} \, dx$$

```
[In] integrate(1/x/arccos(a*x)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/(x*arccos(a*x)^(7/2)), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{1}{x \arccos(ax)^{7/2}} \, dx = \int \frac{1}{x \operatorname{acos}(a x)^{7/2}} \, dx$$

```
[In] int(1/(x*acos(a*x)^(7/2)),x)
```

```
[Out] int(1/(x*acos(a*x)^(7/2)), x)
```

3.119 $\int (bx)^m \arccos(ax)^4 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int (bx)^m \arccos(ax)^4 \, dx = \frac{(bx)^{1+m} \arccos(ax)^4}{b(1+m)} + \frac{4a\text{Int}\left(\frac{(bx)^{1+m} \arccos(ax)^3}{\sqrt{1-a^2x^2}}, x\right)}{b(1+m)}$$

```
[Out] (b*x)^(1+m)*arccos(a*x)^4/b/(1+m)+4*a*Unintegrable((b*x)^(1+m)*arccos(a*x)^
3/(-a^2*x^2+1)^(1/2),x)/b/(1+m)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (bx)^m \arccos(ax)^4 \, dx = \int (bx)^m \arccos(ax)^4 \, dx$$

```
[In] Int[(b*x)^m*ArcCos[a*x]^4,x]
```

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x]^4)/(b*(1 + m)) + (4*a*Defer[Int][((b*x)^(1 + m)*
ArcCos[a*x]^3)/Sqrt[1 - a^2*x^2], x])/(b*(1 + m))
```

Rubi steps

$$\text{integral} = \frac{(bx)^{1+m} \arccos(ax)^4}{b(1+m)} + \frac{(4a) \int \frac{(bx)^{1+m} \arccos(ax)^3}{\sqrt{1-a^2x^2}} \, dx}{b(1+m)}$$

Mathematica [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^4 \, dx = \int (bx)^m \arccos(ax)^4 \, dx$$

```
[In] Integrate[(b*x)^m*ArcCos[a*x]^4,x]
[Out] Integrate[(b*x)^m*ArcCos[a*x]^4, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos\left(ax\right)^4 dx$$

```
[In] int((b*x)^m*arccos(a*x)^4,x)
[Out] int((b*x)^m*arccos(a*x)^4,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^4 \, dx = \int (bx)^m \arccos\left(ax\right)^4 \, dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^4,x, algorithm="fricas")
[Out] integral((b*x)^m*arccos(a*x)^4, x)
```

Sympy [N/A]

Not integrable

Time = 5.39 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^4 \, dx = \int (bx)^m \operatorname{acos}^4\left(ax\right) dx$$

```
[In] integrate((b*x)**m*acos(a*x)**4,x)
[Out] Integral((b*x)**m*acos(a*x)**4, x)
```

Maxima [N/A]

Not integrable

Time = 0.70 (sec) , antiderivative size = 115, normalized size of antiderivative = 9.58

$$\int (bx)^m \arccos(ax)^4 \, dx = \int (bx)^m \arccos(ax)^4 \, dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^4,x, algorithm="maxima")
```

```
[Out] (b^m*x*x^m*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^4 - 4*(a*b^m*m + a*b^
m)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x*x^m*arctan2(sqrt(a*x + 1)*sqrt(
-a*x + 1), a*x)^3/((a^2*m + a^2)*x^2 - m - 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^4 \, dx = \int (bx)^m \arccos(ax)^4 \, dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^4,x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*arccos(a*x)^4, x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^4 \, dx = \int \mathrm{acos}(a\,x)^4\,(b\,x)^m \, dx$$

```
[In] int(acos(a*x)^4*(b*x)^m,x)
```

```
[Out] int(acos(a*x)^4*(b*x)^m, x)
```

3.120 $\int (bx)^m \arccos(ax)^3 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int (bx)^m \arccos(ax)^3 \, dx = \frac{(bx)^{1+m} \arccos(ax)^3}{b(1+m)} + \frac{3a\mathrm{Int}\left(\frac{(bx)^{1+m} \arccos(ax)^2}{\sqrt{1-a^2x^2}}, x\right)}{b(1+m)}$$

```
[Out] (b*x)^(1+m)*arccos(a*x)^3/b/(1+m)+3*a*Unintegrable((b*x)^(1+m)*arccos(a*x)^
2/(-a^2*x^2+1)^(1/2),x)/b/(1+m)
```

Rubi [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (bx)^m \arccos(ax)^3 \, dx = \int (bx)^m \arccos(ax)^3 \, dx$$

```
[In] Int[(b*x)^m*ArcCos[a*x]^3,x]
```

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x]^3)/(b*(1 + m)) + (3*a*Defer[Int][((b*x)^(1 + m)*
ArcCos[a*x]^2)/Sqrt[1 - a^2*x^2], x])/(b*(1 + m))
```

Rubi steps

$$\text{integral} = \frac{(bx)^{1+m} \arccos(ax)^3}{b(1+m)} + \frac{(3a) \int \frac{(bx)^{1+m} \arccos(ax)^2}{\sqrt{1-a^2x^2}} \, dx}{b(1+m)}$$

Mathematica [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^3\,dx = \int (bx)^m \arccos(ax)^3\,dx$$

```
[In] Integrate[(b*x)^m*ArcCos[a*x]^3,x]
[Out] Integrate[(b*x)^m*ArcCos[a*x]^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.86 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos\left(ax\right)^3 dx$$

```
[In] int((b*x)^m*arccos(a*x)^3,x)
[Out] int((b*x)^m*arccos(a*x)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^3\,dx = \int (bx)^m \arccos\left(ax\right)^3\,dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^3,x, algorithm="fricas")
[Out] integral((b*x)^m*arccos(a*x)^3, x)
```

Sympy [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^3\,dx = \int (bx)^m \operatorname{acos}^3\left(ax\right) dx$$

```
[In] integrate((b*x)**m*acos(a*x)**3,x)
[Out] Integral((b*x)**m*acos(a*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 115, normalized size of antiderivative = 9.58

$$\int (bx)^m \arccos(ax)^3 \, dx = \int (bx)^m \arccos{(ax)}^3 \, dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^3,x, algorithm="maxima")
```

```
[Out] (b^m*x*x^m*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^3 - 3*(a*b^m*m + a*b^
m)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x*x^m*arctan2(sqrt(a*x + 1)*sqrt(
-a*x + 1), a*x)^2/((a^2*m + a^2)*x^2 - m - 1), x))/(m + 1)
```

Giac [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^3 \, dx = \int (bx)^m \arccos{(ax)}^3 \, dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^3,x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*arccos(a*x)^3, x)
```

Mupad [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^3 \, dx = \int \mathrm{acos}(a\,x)^3\,{(b\,x)}^m \,dx$$

```
[In] int(acos(a*x)^3*(b*x)^m,x)
```

```
[Out] int(acos(a*x)^3*(b*x)^m, x)
```

3.121 $\int (bx)^m \arccos(ax)^2 \, dx$

Optimal result

Integrand size = 12, antiderivative size = 150

$$\int (bx)^m \arccos(ax)^2 \, dx = \frac{(bx)^{1+m}\arccos(ax)^2}{b(1+m)} + \frac{2a(bx)^{2+m}\arccos(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{b^2(1+m)(2+m)} + \frac{2a^2(bx)^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};a^2x^2\right)}{b^3(1+m)(2+m)(3+m)}$$

```
[Out] (b*x)^(1+m)*arccos(a*x)^2/b/(1+m)+2*a*(b*x)^(2+m)*arccos(a*x)*hypergeom([1/
2, 1+1/2*m],[2+1/2*m],a^2*x^2)/b^2/(1+m)/(2+m)+2*a^2*(b*x)^(3+m)*hypergeom(
[1, 3/2+1/2*m, 3/2+1/2*m],[2+1/2*m, 5/2+1/2*m],a^2*x^2)/b^3/(3+m)/(m^2+3*m+
2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4724, 4806}

$$\int (bx)^m \arccos(ax)^2 \, dx = \frac{2a^2(bx)^{m+3}\,{}_3F_2\left(1,\frac{m}{2}+\frac{3}{2},\frac{m}{2}+\frac{3}{2};\frac{m}{2}+2,\frac{m}{2}+\frac{5}{2};a^2x^2\right)}{b^3(m+1)(m+2)(m+3)} + \frac{2a\arccos(ax)(bx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{b^2(m+1)(m+2)} + \frac{\arccos(ax)^2(bx)^{m+1}}{b(m+1)}$$

```
[In] Int[(b*x)^m*ArcCos[a*x]^2,x]
```

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x]^2)/(b*(1 + m)) + (2*a*(b*x)^(2 + m)*ArcCos[a*x]*
Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(b^2*(1 + m)*(2 + m)
) + (2*a^2*(b*x)^(3 + m)*HypergeometricPFQ[{1, 3/2 + m/2, 3/2 + m/2}, {2 +
m/2, 5/2 + m/2}, a^2*x^2])/(b^3*(1 + m)*(2 + m)*(3 + m))
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4806

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{(bx)^{1+m}\arccos(ax)^2}{b(1+m)} + \frac{(2a)\int\frac{(bx)^{1+m}\arccos(ax)}{\sqrt{1-a^2x^2}}\,dx}{b(1+m)} \\
&= \frac{(bx)^{1+m}\arccos(ax)^2}{b(1+m)} \\
&\quad + \frac{2a(bx)^{2+m}\arccos(ax)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{b^2(1+m)(2+m)} \\
&\quad + \frac{2a^2(bx)^{3+m}\,{}_3F_2\left(1,\frac{3}{2}+\frac{m}{2},\frac{3}{2}+\frac{m}{2};2+\frac{m}{2},\frac{5}{2}+\frac{m}{2};a^2x^2\right)}{b^3(1+m)(2+m)(3+m)}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5 in optimal.

Time = 1.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.88

$$\int (bx)^m\arccos(ax)^2\,dx$$

$$= \frac{x(bx)^m\left(4\arccos(ax)^2 + ax\left(\frac{8\sqrt{1-a^2x^2}\arccos(ax)\,\text{Hypergeometric2F1}\left(1,\frac{3+m}{2},\frac{4+m}{2},a^2x^2\right)}{2+m} + 2^{-m}a\sqrt{\pi}x\,\text{Gamma}(2+m)\right.\right.}{4(1+m)}$$

```
[In] Integrate[(b*x)^m*ArcCos[a*x]^2,x]
```

```
[Out] (x*(b*x)^m*(4*ArcCos[a*x]^2 + a*x*((8*Sqrt[1 - a^2*x^2]*ArcCos[a*x]*Hyperge
ometric2F1[1, (3 + m)/2, (4 + m)/2, a^2*x^2])/(2 + m) + (a*Sqrt[Pi]*x*Gamma
[2 + m]*HypergeometricPFQRegularized[{1, (3 + m)/2, (3 + m)/2}, {(4 + m)/2,
 (5 + m)/2}, a^2*x^2])/2^m)))/(4*(1 + m))
```

Maple [F]

$$\int (bx)^m \arccos{(ax)}^2 \, dx$$

```
[In] int((b*x)^m*arccos(a*x)^2,x)
```

```
[Out] int((b*x)^m*arccos(a*x)^2,x)
```

Fricas [F]

$$\int (bx)^m \arccos(ax)^2 \, dx = \int (bx)^m \arccos{(ax)}^2 \, dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^2,x, algorithm="fricas")
```

```
[Out] integral((b*x)^m*arccos(a*x)^2, x)
```

Sympy [F]

$$\int (bx)^m \arccos(ax)^2 \, dx = \int (bx)^m \operatorname{acos}^2{(ax)} \, dx$$

```
[In] integrate((b*x)**m*acos(a*x)**2,x)
```

```
[Out] Integral((b*x)**m*acos(a*x)**2, x)
```

Maxima [F]

$$\int (bx)^m \arccos(ax)^2 \, dx = \int (bx)^m \arccos{(ax)}^2 \, dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] (b^m*x*x^m*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)^2 - 2*(a*b^m*m + a*b^
m)*integrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x*x^m*arctan2(sqrt(a*x + 1)*sqrt(
-a*x + 1), a*x)/((a^2*m + a^2)*x^2 - m - 1), x))/(m + 1)
```

Giac [F]

$$\int (bx)^m \arccos(ax)^2 \, dx = \int (bx)^m \arccos(ax)^2 \, dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^2,x, algorithm="giac")
[Out] integrate((b*x)^m*arccos(a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (bx)^m \arccos(ax)^2 \, dx = \int \operatorname{acos}(a\,x)^2\,(b\,x)^m \, dx$$

```
[In] int(acos(a*x)^2*(b*x)^m,x)
[Out] int(acos(a*x)^2*(b*x)^m, x)
```

3.122 $\int (bx)^m \arccos(ax)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 68

$$\int (bx)^m \arccos(ax)\,dx = \frac{(bx)^{1+m}\arccos(ax)}{b(1+m)} + \frac{a(bx)^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{b^2(1+m)(2+m)}$$

```
[Out] (b*x)^(1+m)*arccos(a*x)/b/(1+m)+a*(b*x)^(2+m)*hypergeom([1/2, 1+1/2*m],[2+1
/2*m],a^2*x^2)/b^2/(1+m)/(2+m)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{4724, 371\}$

$$\int (bx)^m \arccos(ax)\,dx = \frac{a(bx)^{m+2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+2}{2},\frac{m+4}{2},a^2x^2\right)}{b^2(m+1)(m+2)} + \frac{\arccos(ax)(bx)^{m+1}}{b(m+1)}$$

```
[In] Int[(b*x)^m*ArcCos[a*x],x]
```

```
[Out] ((b*x)^(1 + m)*ArcCos[a*x])/(b*(1 + m)) + (a*(b*x)^(2 + m)*Hypergeometric2F
1[1/2, (2 + m)/2, (4 + m)/2, a^2*x^2])/(b^2*(1 + m)*(2 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
```

```
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{(bx)^{1+m}\arccos(ax)}{b(1+m)} + \frac{a\int \frac{(bx)^{1+m}}{\sqrt{1-a^2x^2}}\,dx}{b(1+m)} \\ &= \frac{(bx)^{1+m}\arccos(ax)}{b(1+m)} + \frac{a(bx)^{2+m}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{2+m}{2},\frac{4+m}{2},a^2x^2\right)}{b^2(1+m)(2+m)}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (bx)^m \arccos(ax)\,dx = \frac{x(bx)^m\left((2+m)\arccos(ax) + ax\,\text{Hypergeometric2F1}\left(\frac{1}{2},1+\frac{m}{2},2+\frac{m}{2},a^2x^2\right)\right)}{(1+m)(2+m)}$$

```
[In] Integrate[(b*x)^m*ArcCos[a*x],x]
[Out] (x*(b*x)^m*((2 + m)*ArcCos[a*x] + a*x*Hypergeometric2F1[1/2, 1 + m/2, 2 + m
/2, a^2*x^2]))/((1 + m)*(2 + m))
```

Maple [F]

$$\int (bx)^m \arccos(ax)\,dx$$

```
[In] int((b*x)^m*arccos(a*x),x)
[Out] int((b*x)^m*arccos(a*x),x)
```

Fricas [F]

$$\int (bx)^m \arccos(ax)\, dx = \int (bx)^m \arccos(ax)\, dx$$

```
[In] integrate((b*x)^m*arccos(a*x),x, algorithm="fricas")
[Out] integral((b*x)^m*arccos(a*x), x)
```

Sympy [F]

$$\int (bx)^m \arccos(ax)\, dx = \int (bx)^m \operatorname{acos}(ax)\, dx$$

```
[In] integrate((b*x)**m*acos(a*x),x)
[Out] Integral((b*x)**m*acos(a*x), x)
```

Maxima [F]

$$\int (bx)^m \arccos(ax)\, dx = \int (bx)^m \arccos(ax)\, dx$$

```
[In] integrate((b*x)^m*arccos(a*x),x, algorithm="maxima")
[Out] (b^m*x*x^m*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x) - (a*b^m*m + a*b^m)*i
ntegrate(sqrt(a*x + 1)*sqrt(-a*x + 1)*x*x^m/((a^2*m + a^2)*x^2 - m - 1), x)
)/(m + 1)
```

Giac [F]

$$\int (bx)^m \arccos(ax)\, dx = \int (bx)^m \arccos(ax)\, dx$$

```
[In] integrate((b*x)^m*arccos(a*x),x, algorithm="giac")
[Out] integrate((b*x)^m*arccos(a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int (bx)^m \arccos(ax)\,dx = \int \mathrm{acos}(a\,x)\,\left(b\,x\right)^m\,dx$$

```
[In] int(acos(a*x)*(b*x)^m,x)
[Out] int(acos(a*x)*(b*x)^m, x)
```

3.123 $\int \frac{(bx)^m}{\arccos(ax)} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{(bx)^m}{\arccos(ax)} \, dx = \text{Int}\left(\frac{(bx)^m}{\arccos(ax)}, x\right)$$

```
[Out] Unintegrable((b*x)^m/arccos(a*x),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(bx)^m}{\arccos(ax)} \, dx = \int \frac{(bx)^m}{\arccos(ax)} \, dx$$

```
[In] Int[(b*x)^m/ArcCos[a*x],x]
```

```
[Out] Defer[Int][(b*x)^m/ArcCos[a*x], x]
```

Rubi steps

$$\text{integral} = \int \frac{(bx)^m}{\arccos(ax)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{(bx)^m}{\arccos(ax)}\,dx$$

```
[In] Integrate[(b*x)^m/ArcCos[a*x],x]
[Out] Integrate[(b*x)^m/ArcCos[a*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.14 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos\left(ax\right)}dx$$

```
[In] int((b*x)^m/arccos(a*x),x)
[Out] int((b*x)^m/arccos(a*x),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{(bx)^m}{\arccos\left(ax\right)}\,dx$$

```
[In] integrate((b*x)^m/arccos(a*x),x, algorithm="fricas")
[Out] integral((b*x)^m/arccos(a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{(bx)^m}{\operatorname{acos}\left(ax\right)}\,dx$$

```
[In] integrate((b*x)**m/acos(a*x),x)
[Out] Integral((b*x)**m/acos(a*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{(bx)^m}{\arccos{(ax)}}\,dx$$

```
[In] integrate((b*x)^m/arccos(a*x),x, algorithm="maxima")
[Out] integrate((b*x)^m/arccos(a*x), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{(bx)^m}{\arccos{(ax)}}\,dx$$

```
[In] integrate((b*x)^m/arccos(a*x),x, algorithm="giac")
[Out] integrate((b*x)^m/arccos(a*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)}\,dx = \int \frac{(b\,x)^m}{\mathrm{acos}\left(a\,x\right)}\,dx$$

```
[In] int((b*x)^m/acos(a*x),x)
[Out] int((b*x)^m/acos(a*x), x)
```

3.124 $\int \frac{(bx)^m}{\arccos(ax)^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = \operatorname{Int}\left(\frac{(bx)^m}{\arccos(ax)^2},x\right)$$

```
[Out] Unintegrable((b*x)^m/arccos(a*x)^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = \int \frac{(bx)^m}{\arccos(ax)^2}\,dx$$

```
[In] Int[(b*x)^m/ArcCos[a*x]^2,x]
[Out] Defer[Int][(b*x)^m/ArcCos[a*x]^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(bx)^m}{\arccos(ax)^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = \int \frac{(bx)^m}{\arccos(ax)^2}\,dx$$

```
[In] Integrate[(b*x)^m/ArcCos[a*x]^2,x]
[Out] Integrate[(b*x)^m/ArcCos[a*x]^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.29 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos\left(ax\right)^2}dx$$

```
[In] int((b*x)^m/arccos(a*x)^2,x)
[Out] int((b*x)^m/arccos(a*x)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = \int \frac{(bx)^m}{\arccos\left(ax\right)^2}\,dx$$

```
[In] integrate((b*x)^m/arccos(a*x)^2,x, algorithm="fricas")
[Out] integral((b*x)^m/arccos(a*x)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = \int \frac{(bx)^m}{\operatorname{acos}^2\left(ax\right)}\,dx$$

```
[In] integrate((b*x)**m/acos(a*x)**2,x)
[Out] Integral((b*x)**m/acos(a*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 156, normalized size of antiderivative = 13.00

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = \int \frac{(bx)^m}{\arccos{(ax)}^2}\,dx$$

```
[In] integrate((b*x)^m/arccos(a*x)^2,x, algorithm="maxima")
```

```
[Out] (sqrt(a*x + 1)*sqrt(-a*x + 1)*b^m*x^m - a*arctan2(sqrt(a*x + 1)*sqrt(-a*x +
 1), a*x)*integrate(((a^2*b^m*m + a^2*b^m)*x^2 - b^m*m)*sqrt(a*x + 1)*sqrt(
-a*x + 1)*x^m/((a^3*x^3 - a*x)*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x)),
 x))/(a*arctan2(sqrt(a*x + 1)*sqrt(-a*x + 1), a*x))
```

Giac [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = \int \frac{(bx)^m}{\arccos{(ax)}^2}\,dx$$

```
[In] integrate((b*x)^m/arccos(a*x)^2,x, algorithm="giac")
```

```
[Out] integrate((b*x)^m/arccos(a*x)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{(bx)^m}{\arccos(ax)^2}\,dx = \int \frac{{(b\,x)}^m}{{\mathrm{acos}\left(a\,x\right)}^2}\,dx$$

```
[In] int((b*x)^m/acos(a*x)^2,x)
```

```
[Out] int((b*x)^m/acos(a*x)^2, x)
```

3.125 $\int (bx)^m \arccos(ax)^{3/2}\, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (bx)^m \arccos(ax)^{3/2}\, dx = \operatorname{Int}\left((bx)^m \arccos(ax)^{3/2}, x\right)$$

```
[Out] Unintegrable((b*x)^m*arccos(a*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (bx)^m \arccos(ax)^{3/2}\, dx = \int (bx)^m \arccos(ax)^{3/2}\, dx$$

```
[In] Int[(b*x)^m*ArcCos[a*x]^(3/2),x]
```

```
[Out] Defer[Int][(b*x)^m*ArcCos[a*x]^(3/2), x]
```

Rubi steps

$$\text{integral} = \int (bx)^m \arccos(ax)^{3/2}\, dx$$

Mathematica [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (bx)^m \arccos(ax)^{3/2}\, dx = \int (bx)^m \arccos(ax)^{3/2}\, dx$$

```
[In] Integrate[(b*x)^m*ArcCos[a*x]^(3/2),x]
[Out] Integrate[(b*x)^m*ArcCos[a*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.54 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (bx)^m \arccos{(ax)}^{\frac{3}{2}}\, dx$$

```
[In] int((b*x)^m*arccos(a*x)^(3/2),x)
[Out] int((b*x)^m*arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (bx)^m \arccos(ax)^{3/2}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x)^m*arccos(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 64.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^{3/2}\, dx = \int (bx)^m \operatorname{acos}^{\frac{3}{2}}(ax)\, dx$$

```
[In] integrate((b*x)**m*acos(a*x)**(3/2),x)
[Out] Integral((b*x)**m*acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (bx)^m \arccos(ax)^{3/2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*x)^m*arccos(a*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 1.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^{3/2}\,dx = \int (bx)^m \arccos{(ax)}^{\frac{3}{2}}\,dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*x)^m*arccos(a*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^{3/2}\,dx = \int \mathrm{acos}(a\,x)^{3/2}\,{(b\,x)}^m\,dx$$

```
[In] int(acos(a*x)^(3/2)*(b*x)^m,x)
```

```
[Out] int(acos(a*x)^(3/2)*(b*x)^m, x)
```

3.126 $\int (bx)^m \sqrt{\arccos(ax)}\, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (bx)^m \sqrt{\arccos(ax)}\, dx = \operatorname{Int}\left((bx)^m \sqrt{\arccos(ax)}, x\right)$$

```
[Out] Unintegrable((b*x)^m*arccos(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (bx)^m \sqrt{\arccos(ax)}\, dx = \int (bx)^m \sqrt{\arccos(ax)}\, dx$$

```
[In] Int[(b*x)^m*Sqrt[ArcCos[a*x]],x]
[Out] Defer[Int][(b*x)^m*Sqrt[ArcCos[a*x]], x]
```

Rubi steps

$$\text{integral} = \int (bx)^m \sqrt{\arccos(ax)}\, dx$$

Mathematica [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (bx)^m \sqrt{\arccos(ax)}\, dx = \int (bx)^m \sqrt{\arccos(ax)}\, dx$$

```
[In] Integrate[(b*x)^m*Sqrt[ArcCos[a*x]],x]
[Out] Integrate[(b*x)^m*Sqrt[ArcCos[a*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.52 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (bx)^m \sqrt{\arccos (ax)}dx$$

```
[In] int((b*x)^m*arccos(a*x)^(1/2),x)
[Out] int((b*x)^m*arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (bx)^m \sqrt{\arccos(ax)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x)^m*arccos(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \sqrt{\arccos(ax)}\, dx = \int (bx)^m \sqrt{\operatorname{acos}(ax)}\, dx$$

```
[In] integrate((b*x)**m*acos(a*x)**(1/2),x)
[Out] Integral((b*x)**m*sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int (bx)^m \sqrt{\arccos(ax)}\, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*x)^m*arccos(a*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 1.00 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \sqrt{\arccos(ax)}\, dx = \int (bx)^m \sqrt{\arccos(ax)}\, dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^(1/2),x, algorithm="giac")
[Out] integrate((b*x)^m*sqrt(arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^m \sqrt{\arccos(ax)}\, dx = \int \sqrt{\mathrm{acos}\,(a\,x)}\,(b\,x)^m\, dx$$

```
[In] int(acos(a*x)^(1/2)*(b*x)^m,x)
[Out] int(acos(a*x)^(1/2)*(b*x)^m, x)
```

3.127 $\int \frac{(bx)^m}{\sqrt{\arccos(ax)}} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}} dx = \text{Int}\left(\frac{(bx)^m}{\sqrt{\arccos(ax)}}, x\right)$$

```
[Out] Unintegrable((b*x)^m/arccos(a*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}} dx = \int \frac{(bx)^m}{\sqrt{\arccos(ax)}} dx$$

```
[In] Int[(b*x)^m/Sqrt[ArcCos[a*x]],x]
[Out] Defer[Int][(b*x)^m/Sqrt[ArcCos[a*x]], x]
```

Rubi steps

$$\text{integral} = \int \frac{(bx)^m}{\sqrt{\arccos(ax)}} dx$$

Mathematica [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx = \int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx$$

```
[In] Integrate[(b*x)^m/Sqrt[ArcCos[a*x]],x]
[Out] Integrate[(b*x)^m/Sqrt[ArcCos[a*x]], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(bx)^m}{\sqrt{\arccos\left(ax\right)}}dx$$

```
[In] int((b*x)^m/arccos(a*x)^(1/2),x)
[Out] int((b*x)^m/arccos(a*x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x)^m/arccos(a*x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx = \int \frac{(bx)^m}{\sqrt{\operatorname{acos}\left(ax\right)}}\,dx$$

```
[In] integrate((b*x)**m/acos(a*x)**(1/2),x)
[Out] Integral((b*x)**m/sqrt(acos(a*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*x)^m/arccos(a*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.87 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx = \int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx$$

```
[In] integrate((b*x)^m/arccos(a*x)^(1/2),x, algorithm="giac")
[Out] integrate((b*x)^m/sqrt(arccos(a*x)), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\sqrt{\arccos(ax)}}\,dx = \int \frac{(b\,x)^m}{\sqrt{\mathrm{acos}\left(a\,x\right)}}\,dx$$

```
[In] int((b*x)^m/acos(a*x)^(1/2),x)
[Out] int((b*x)^m/acos(a*x)^(1/2), x)
```

3.128 $\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \text{Int}\left(\frac{(bx)^m}{\arccos(ax)^{3/2}},x\right)$$

```
[Out] Unintegrable((b*x)^m/arccos(a*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx$$

```
[In] Int[(b*x)^m/ArcCos[a*x]^(3/2),x]
[Out] Defer[Int][(b*x)^m/ArcCos[a*x]^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx$$

```
[In] Integrate[(b*x)^m/ArcCos[a*x]^(3/2),x]
[Out] Integrate[(b*x)^m/ArcCos[a*x]^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{(bx)^m}{\arccos\left(ax\right)^{\frac{3}{2}}}dx$$

```
[In] int((b*x)^m/arccos(a*x)^(3/2),x)
[Out] int((b*x)^m/arccos(a*x)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((b*x)^m/arccos(a*x)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 4.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \int \frac{(bx)^m}{\operatorname{acos}^{\frac{3}{2}}\left(ax\right)}\,dx$$

```
[In] integrate((b*x)**m/acos(a*x)**(3/2),x)
[Out] Integral((b*x)**m/acos(a*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*x)^m/arccos(a*x)^(3/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.78 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \int \frac{(bx)^m}{\arccos\left(ax\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((b*x)^m/arccos(a*x)^(3/2),x, algorithm="giac")
[Out] integrate((b*x)^m/arccos(a*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{(bx)^m}{\arccos(ax)^{3/2}}\,dx = \int \frac{(b\,x)^m}{\mathrm{acos}\left(a\,x\right)^{3/2}}\,dx$$

```
[In] int((b*x)^m/acos(a*x)^(3/2),x)
[Out] int((b*x)^m/acos(a*x)^(3/2), x)
```

3.129 $\int (bx)^m \arccos(ax)^n \, dx$

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int (bx)^m \arccos(ax)^n \, dx = \text{Int}((bx)^m \arccos(ax)^n, x)$$

```
[Out] Unintegrable((b*x)^m*arccos(a*x)^n,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (bx)^m \arccos(ax)^n \, dx = \int (bx)^m \arccos(ax)^n \, dx$$

```
[In] Int[(b*x)^m*ArcCos[a*x]^n,x]
[Out] Defer[Int][(b*x)^m*ArcCos[a*x]^n, x]
```

Rubi steps

$$\text{integral} = \int (bx)^m \arccos(ax)^n \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^n \, dx = \int (bx)^m \arccos(ax)^n \, dx$$

```
[In] Integrate[(b*x)^m*ArcCos[a*x]^n,x]
[Out] Integrate[(b*x)^m*ArcCos[a*x]^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.87 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos (ax)^n \, dx$$

```
[In] int((b*x)^m*arccos(a*x)^n,x)
[Out] int((b*x)^m*arccos(a*x)^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^n \, dx = \int (bx)^m \arccos (ax)^n \, dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^n,x, algorithm="fricas")
[Out] integral((b*x)^m*arccos(a*x)^n, x)
```

Sympy [N/A]

Not integrable

Time = 4.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int (bx)^m \arccos(ax)^n \, dx = \int (bx)^m \operatorname{acos}^n (ax) \, dx$$

```
[In] integrate((b*x)**m*acos(a*x)**n,x)
[Out] Integral((b*x)**m*acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (bx)^m \arccos(ax)^n \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*x)^m*arccos(a*x)^n,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^n \, dx = \int (bx)^m \arccos{(ax)}^n \, dx$$

```
[In] integrate((b*x)^m*arccos(a*x)^n,x, algorithm="giac")
[Out] integrate((b*x)^m*arccos(a*x)^n, x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int (bx)^m \arccos(ax)^n \, dx = \int \mathrm{acos}(a\,x)^n\,(b\,x)^m \, dx$$

```
[In] int(acos(a*x)^n*(b*x)^m,x)
[Out] int(acos(a*x)^n*(b*x)^m, x)
```

3.130 $\int x^3 \arccos(ax)^n \, dx$

Optimal result

Integrand size = 10, antiderivative size = 165

$$\begin{aligned}\int x^3 \arccos(ax)^n \, dx = {} & \frac{2^{-4-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-2i\arccos(ax))}{a^4} \\ & + \frac{2^{-4-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,2i\arccos(ax))}{a^4} \\ & + \frac{2^{-2(3+n)}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-4i\arccos(ax))}{a^4} \\ & + \frac{2^{-2(3+n)}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,4i\arccos(ax))}{a^4}\end{aligned}$$

```
[Out] 2^(-4-n)*arccos(a*x)^n*GAMMA(1+n,-2*I*arccos(a*x))/a^4/((-I*arccos(a*x))^n)
+2^(-4-n)*arccos(a*x)^n*GAMMA(1+n,2*I*arccos(a*x))/a^4/((I*arccos(a*x))^n)+
arccos(a*x)^n*GAMMA(1+n,-4*I*arccos(a*x))/(2^(6+2*n))/a^4/((-I*arccos(a*x))
^n)+arccos(a*x)^n*GAMMA(1+n,4*I*arccos(a*x))/(2^(6+2*n))/a^4/((I*arccos(a*x
))^n)
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used

$= \{4732, 4491, 3389, 2212\}$

$$\int x^3 \arccos(ax)^n\,dx = \frac{2^{-n-4}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-2i\arccos(ax))}{a^4} + \frac{2^{-2(n+3)}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-4i\arccos(ax))}{a^4} + \frac{2^{-n-4}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(n+1,2i\arccos(ax))}{a^4} + \frac{2^{-2(n+3)}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(n+1,4i\arccos(ax))}{a^4}$$

```
[In] Int[x^3*ArcCos[a*x]^n,x]
```

```
[Out] (2^(-4 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-2*I)*ArcCos[a*x]])/(a^4*((-I)*ArcC
os[a*x])^n) + (2^(-4 - n)*ArcCos[a*x]^n*Gamma[1 + n, (2*I)*ArcCos[a*x]])/(a
^4*(I*ArcCos[a*x])^n) + (ArcCos[a*x]^n*Gamma[1 + n, (-4*I)*ArcCos[a*x]])/(2
^(2*(3 + n))*a^4*((-I)*ArcCos[a*x])^n) + (ArcCos[a*x]^n*Gamma[1 + n, (4*I)*
ArcCos[a*x]])/(2^(2*(3 + n))*a^4*(I*ArcCos[a*x])^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = -\frac{\text{Subst}\left(\int x^n \cos^3(x)\sin(x)\,dx, x, \arccos(ax)\right)}{a^4}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{4}x^n \sin(2x) + \frac{1}{8}x^n \sin(4x)\right)\,dx, x, \arccos(ax)\right)}{a^4}$$
$$= -\frac{\text{Subst}\left(\int x^n \sin(4x)\,dx, x, \arccos(ax)\right)}{8a^4} - \frac{\text{Subst}\left(\int x^n \sin(2x)\,dx, x, \arccos(ax)\right)}{4a^4}$$
$$= -\frac{i\text{Subst}\left(\int e^{-4ix}x^n\,dx, x, \arccos(ax)\right)}{16a^4} + \frac{i\text{Subst}\left(\int e^{4ix}x^n\,dx, x, \arccos(ax)\right)}{16a^4} - \frac{i\text{Subst}\left(\int e^{-2ix}x^n\,dx, x, \arccos(ax)\right)}{8a^4} + \frac{i\text{Subst}\left(\int e^{2ix}x^n\,dx, x, \arccos(ax)\right)}{8a^4}$$
$$= \frac{2^{-4-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n, -2i\arccos(ax))}{a^4} + \frac{2^{-4-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n, 2i\arccos(ax))}{a^4} + \frac{4^{-3-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n, -4i\arccos(ax))}{a^4} + \frac{4^{-3-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n, 4i\arccos(ax))}{a^4}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.79

$$\int x^3 \arccos(ax)^n\,dx$$
$$= \frac{2^{-2(3+n)}\arccos(ax)^n\left(\arccos(ax)^2\right)^{-n}\left(2^{2+n}(i\arccos(ax))^n\Gamma(1+n, -2i\arccos(ax)) + 2^{2+n}(-i\arccos(ax))\right.}{}$$

```
[In] Integrate[x^3*ArcCos[a*x]^n,x]
```

```
[Out] (ArcCos[a*x]^n*(2^(2 + n)*(I*ArcCos[a*x])^n*Gamma[1 + n, (-2*I)*ArcCos[a*x]
] + 2^(2 + n)*((-I)*ArcCos[a*x])^n*Gamma[1 + n, (2*I)*ArcCos[a*x]] + (I*Arc
Cos[a*x])^n*Gamma[1 + n, (-4*I)*ArcCos[a*x]] + ((-I)*ArcCos[a*x])^n*Gamma[1
 + n, (4*I)*ArcCos[a*x]]))/(2^(2*(3 + n))*a^4*(ArcCos[a*x]^2)^n)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.90 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.74

method	result
default	$-\frac{\sqrt{\pi}\left(\frac{2\arccos(ax)^{1+n}\sin(2\arccos(ax))}{\sqrt{\pi}\,(2+n)}-\frac{2^{\frac{1}{2}-n}\sqrt{\arccos(ax)}\,\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},2\arccos(ax)\right)\sin(2\arccos(ax))}{\sqrt{\pi}\,(2+n)}-\frac{3\,2^{-\frac{3}{2}-n}\left(\frac{4}{3}+\frac{2n}{3}\right)(2\arccos(ax}{}}{8a^4}$

[In] int(x^3*arccos(a*x)^n,x,method=_RETURNVERBOSE)

[Out] -1/8*Pi^(1/2)/a^4*(2/Pi^(1/2)/(2+n)*arccos(a*x)^(1+n)*sin(2*arccos(a*x))-2^(1/2-n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*LommelS1(n+3/2,3/2,2*arccos(a*x))*sin(2*arccos(a*x))-3*2^(-3/2-n)/Pi^(1/2)/(2+n)/arccos(a*x)^(1/2)*(4/3+2/3*n)*(2*arccos(a*x)*cos(2*arccos(a*x))-sin(2*arccos(a*x)))*LommelS1(n+1/2,1/2,2*arccos(a*x)))-2^(-5-n)*Pi^(1/2)/a^4*(2^(2+n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1+n)*sin(4*arccos(a*x))-2^(1-n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*LommelS1(n+3/2,3/2,4*arccos(a*x))*sin(4*arccos(a*x))-3*2^(-2-n)/Pi^(1/2)/(2+n)/arccos(a*x)^(1/2)*(4/3+2/3*n)*(4*arccos(a*x)*cos(4*arccos(a*x))-sin(4*arccos(a*x)))*LommelS1(n+1/2,1/2,4*arccos(a*x)))

Fricas [F]

$$\int x^3 \arccos(ax)^n \, dx = \int x^3 \arccos{(ax)}^n \, dx$$

[In] integrate(x^3*arccos(a*x)^n,x, algorithm="fricas")

[Out] integral(x^3*arccos(a*x)^n, x)

Sympy [F]

$$\int x^3 \arccos(ax)^n \, dx = \int x^3 \operatorname{acos}^n{(ax)} \, dx$$

[In] integrate(x**3*acos(a*x)**n,x)

[Out] Integral(x**3*acos(a*x)**n, x)

Maxima [F(-2)]

Exception generated.

$$\int x^3 \arccos(ax)^n\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^3*arccos(a*x)^n,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x^3 \arccos(ax)^n\,dx = \int x^3 \arccos\left(ax\right)^n\,dx$$

```
[In] integrate(x^3*arccos(a*x)^n,x, algorithm="giac")
[Out] integrate(x^3*arccos(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \arccos(ax)^n\,dx = \int x^3\,\mathrm{acos}(a\,x)^n\,dx$$

```
[In] int(x^3*acos(a*x)^n,x)
[Out] int(x^3*acos(a*x)^n, x)
```

3.131 $\int x^2 \arccos(ax)^n \, dx$

Optimal result

Integrand size = 10, antiderivative size = 163

$$\int x^2 \arccos(ax)^n \, dx = \frac{(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-i\arccos(ax))}{8a^3} + \frac{(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,i\arccos(ax))}{8a^3} + \frac{3^{-1-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-3i\arccos(ax))}{8a^3} + \frac{3^{-1-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,3i\arccos(ax))}{8a^3}$$

```
[Out] 1/8*arccos(a*x)^n*GAMMA(1+n,-I*arccos(a*x))/a^3/((-I*arccos(a*x))^n)+1/8*ar
ccos(a*x)^n*GAMMA(1+n,I*arccos(a*x))/a^3/((I*arccos(a*x))^n)+1/8*3^(-1-n)*a
rccos(a*x)^n*GAMMA(1+n,-3*I*arccos(a*x))/a^3/((-I*arccos(a*x))^n)+1/8*3^(-1
-n)*arccos(a*x)^n*GAMMA(1+n,3*I*arccos(a*x))/a^3/((I*arccos(a*x))^n)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4732, 4491, 3389, 2212}

$$\int x^2 \arccos(ax)^n \, dx = \frac{\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-i\arccos(ax))}{8a^3} + \frac{3^{-n-1}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-3i\arccos(ax))}{8a^3} + \frac{(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(n+1,i\arccos(ax))}{8a^3} + \frac{3^{-n-1}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(n+1,3i\arccos(ax))}{8a^3}$$

```
[In] Int[x^2*ArcCos[a*x]^n,x]
```

```
[Out] (ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/(8*a^3*((-I)*ArcCos[a*x])^n)
 + (ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(8*a^3*(I*ArcCos[a*x])^n) +
(3^(-1 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-3*I)*ArcCos[a*x]])/(8*a^3*((-I)*Ar
cCos[a*x])^n) + (3^(-1 - n)*ArcCos[a*x]^n*Gamma[1 + n, (3*I)*ArcCos[a*x]])/
(8*a^3*(I*ArcCos[a*x])^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^n\cos^2(x)\sin(x)\,dx,x,\arccos(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int \left(\frac{1}{4}x^n\sin(x)+\frac{1}{4}x^n\sin(3x)\right)\,dx,x,\arccos(ax)\right)}{a^3}\\
&= -\frac{\text{Subst}\left(\int x^n\sin(x)\,dx,x,\arccos(ax)\right)}{4a^3}-\frac{\text{Subst}\left(\int x^n\sin(3x)\,dx,x,\arccos(ax)\right)}{4a^3}\\
&= -\frac{i\text{Subst}\left(\int e^{-ix}x^n\,dx,x,\arccos(ax)\right)}{8a^3}+\frac{i\text{Subst}\left(\int e^{ix}x^n\,dx,x,\arccos(ax)\right)}{8a^3}\\
&\quad -\frac{i\text{Subst}\left(\int e^{-3ix}x^n\,dx,x,\arccos(ax)\right)}{8a^3}+\frac{i\text{Subst}\left(\int e^{3ix}x^n\,dx,x,\arccos(ax)\right)}{8a^3}
\end{aligned}$$

$$= \frac{(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-i\arccos(ax))}{8a^3} + \frac{(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,i\arccos(ax))}{8a^3} + \frac{3^{-1-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-3i\arccos(ax))}{8a^3} + \frac{3^{-1-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,3i\arccos(ax))}{8a^3}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.93

$$\int x^2\arccos(ax)^n\,dx = \frac{\frac{1}{4}\left(\frac{1}{2}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-i\arccos(ax))+\frac{1}{2}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,i\arccos\right.}{}$$

```
[In] Integrate[x^2*ArcCos[a*x]^n,x]
```

```
[Out] (((ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/(2*((-I)*ArcCos[a*x])^n) +
 (ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(2*(I*ArcCos[a*x])^n))/4 + (3^
(-1 - n)*ArcCos[a*x]^n*((I*ArcCos[a*x])^n*Gamma[1 + n, (-3*I)*ArcCos[a*x]]
+ ((-I)*ArcCos[a*x])^n*Gamma[1 + n, (3*I)*ArcCos[a*x]]))/(8*(ArcCos[a*x]^2)
^n))/a^3
```

Maple [F]

$$\int x^2\arccos{(ax)}^n\,dx$$

```
[In] int(x^2*arccos(a*x)^n,x)
```

```
[Out] int(x^2*arccos(a*x)^n,x)
```

Fricas [F]

$$\int x^2\arccos(ax)^n\,dx = \int x^2\arccos{(ax)}^n\,dx$$

```
[In] integrate(x^2*arccos(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x^2*arccos(a*x)^n, x)
```

Sympy [F]

$$\int x^2 \arccos(ax)^n \, dx = \int x^2 \operatorname{acos}^n(ax) \, dx$$

```
[In] integrate(x**2*acos(a*x)**n,x)
[Out] Integral(x**2*acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^2 \arccos(ax)^n \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x^2*arccos(a*x)^n,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x^2 \arccos(ax)^n \, dx = \int x^2 \arccos(ax)^n \, dx$$

```
[In] integrate(x^2*arccos(a*x)^n,x, algorithm="giac")
[Out] integrate(x^2*arccos(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2 \arccos(ax)^n \, dx = \int x^2 \operatorname{acos}(a\,x)^n \, dx$$

```
[In] int(x^2*acos(a*x)^n,x)
[Out] int(x^2*acos(a*x)^n, x)
```

3.132 $\int x \arccos(ax)^n \, dx$

Optimal result

Integrand size = 8, antiderivative size = 83

$$\int x \arccos(ax)^n \, dx = \frac{2^{-3-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-2i\arccos(ax))}{a^2} + \frac{2^{-3-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,2i\arccos(ax))}{a^2}$$

```
[Out] 2^(-3-n)*arccos(a*x)^n*GAMMA(1+n,-2*I*arccos(a*x))/a^2/((-I*arccos(a*x))^n)
+2^(-3-n)*arccos(a*x)^n*GAMMA(1+n,2*I*arccos(a*x))/a^2/((I*arccos(a*x))^n)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4732, 4491, 12, 3389, 2212}

$$\int x \arccos(ax)^n \, dx = \frac{2^{-n-3}\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-2i\arccos(ax))}{a^2} + \frac{2^{-n-3}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(n+1,2i\arccos(ax))}{a^2}$$

```
[In] Int[x*ArcCos[a*x]^n,x]
```

```
[Out] (2^(-3 - n)*ArcCos[a*x]^n*Gamma[1 + n, (-2*I)*ArcCos[a*x]])/(a^2*((-I)*ArcC
os[a*x])^n) + (2^(-3 - n)*ArcCos[a*x]^n*Gamma[1 + n, (2*I)*ArcCos[a*x]])/(a
^2*(I*ArcCos[a*x])^n)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^n \cos(x)\sin(x)\,dx, x, \arccos(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{2}x^n \sin(2x)\,dx, x, \arccos(ax)\right)}{a^2}\\
&= -\frac{\text{Subst}\left(\int x^n \sin(2x)\,dx, x, \arccos(ax)\right)}{2a^2}\\
&= -\frac{i\text{Subst}\left(\int e^{-2ix}x^n\,dx, x, \arccos(ax)\right)}{4a^2} + \frac{i\text{Subst}\left(\int e^{2ix}x^n\,dx, x, \arccos(ax)\right)}{4a^2}\\
&= \frac{2^{-3-n}(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-2i\arccos(ax))}{a^2}\\
&\quad + \frac{2^{-3-n}(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,2i\arccos(ax))}{a^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89

$$\int x \arccos(ax)^n \, dx$$
$$= \frac{2^{-3-n} \arccos(ax)^n \left(\arccos(ax)^2\right)^{-n} \left((i \arccos(ax))^n \Gamma(1+n, -2i \arccos(ax)) + (-i \arccos(ax))^n \Gamma(1+n, 2i \cdots\right.}{a^2}$$

```
[In] Integrate[x*ArcCos[a*x]^n,x]
```

```
[Out] (2^(-3 - n)*ArcCos[a*x]^n*((I*ArcCos[a*x])^n*Gamma[1 + n, (-2*I)*ArcCos[a*x
]] + ((-I)*ArcCos[a*x])^n*Gamma[1 + n, (2*I)*ArcCos[a*x]]))/(a^2*(ArcCos[a*
x]^2)^n)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.10 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\sqrt{\pi}\left(\frac{2 \arccos(ax)^{1+n} \sin(2 \arccos(ax))}{\sqrt{\pi}\,(2+n)} - \frac{2^{\frac{1}{2}-n} \sqrt{\arccos(ax)}\ \mathrm{LommelS1}\left(n+\frac{3}{2}, \frac{3}{2}, 2 \arccos(ax)\right) \sin(2 \arccos(ax))}{\sqrt{\pi}\,(2+n)} - \frac{3\, 2^{-\frac{3}{2}-n} \left(\frac{4}{3}+\frac{2n}{3}\right)(2 \arccos(ax \cdots}{\cdots}\right.}{4a^2}$

```
[In] int(x*arccos(a*x)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*Pi^(1/2)/a^2*(2/Pi^(1/2)/(2+n)*arccos(a*x)^(1+n)*sin(2*arccos(a*x))-2^
(1/2-n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*LommelS1(n+3/2,3/2,2*arccos(a*x))*
sin(2*arccos(a*x))-3*2^(-3/2-n)/Pi^(1/2)/(2+n)/arccos(a*x)^(1/2)*(4/3+2/3*n
)*(2*arccos(a*x)*cos(2*arccos(a*x))-sin(2*arccos(a*x)))*LommelS1(n+1/2,1/2,
2*arccos(a*x)))
```

Fricas [F]

$$\int x \arccos(ax)^n \, dx = \int x \arccos{(ax)}^n \, dx$$

```
[In] integrate(x*arccos(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(x*arccos(a*x)^n, x)
```

Sympy [F]

$$\int x \arccos(ax)^n \, dx = \int x \operatorname{acos}^n(ax) \, dx$$

```
[In] integrate(x*acos(a*x)**n,x)
[Out] Integral(x*acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x \arccos(ax)^n \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(x*arccos(a*x)^n,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int x \arccos(ax)^n \, dx = \int x \arccos(ax)^n \, dx$$

```
[In] integrate(x*arccos(a*x)^n,x, algorithm="giac")
[Out] integrate(x*arccos(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int x \arccos(ax)^n \, dx = \int x \operatorname{acos}(a\,x)^n \, dx$$

```
[In] int(x*acos(a*x)^n,x)
[Out] int(x*acos(a*x)^n, x)
```

3.133 $\int \arccos(ax)^n\,dx$

Optimal result

Integrand size = 6, antiderivative size = 75

$$\int \arccos(ax)^n\,dx = \frac{(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,-i\arccos(ax))}{2a} + \frac{(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n,i\arccos(ax))}{2a}$$

```
[Out] 1/2*arccos(a*x)^n*GAMMA(1+n,-I*arccos(a*x))/a/((-I*arccos(a*x))^n)+1/2*arcc
os(a*x)^n*GAMMA(1+n,I*arccos(a*x))/a/((I*arccos(a*x))^n)
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4720, 3389, 2212}

$$\int \arccos(ax)^n\,dx = \frac{\arccos(ax)^n(-i\arccos(ax))^{-n}\Gamma(n+1,-i\arccos(ax))}{2a} + \frac{(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(n+1,i\arccos(ax))}{2a}$$

```
[In] Int[ArcCos[a*x]^n,x]
```

```
[Out] (ArcCos[a*x]^n*Gamma[1 + n, (-I)*ArcCos[a*x]])/(2*a*((-I)*ArcCos[a*x])^n) +
 (ArcCos[a*x]^n*Gamma[1 + n, I*ArcCos[a*x]])/(2*a*(I*ArcCos[a*x])^n)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
```

```
((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{\text{Subst}\left(\int x^n \sin(x)\,dx, x, \arccos(ax)\right)}{a} \\
&= -\frac{i\text{Subst}\left(\int e^{-ix} x^n\,dx, x, \arccos(ax)\right)}{2a} + \frac{i\text{Subst}\left(\int e^{ix} x^n\,dx, x, \arccos(ax)\right)}{2a} \\
&= \frac{(-i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n, -i\arccos(ax))}{2a} \\
&\quad + \frac{(i\arccos(ax))^{-n}\arccos(ax)^n\Gamma(1+n, i\arccos(ax))}{2a}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93

$$
\int \arccos(ax)^n\,dx
$$

$$
= \frac{\arccos(ax)^n\left(\arccos(ax)^2\right)^{-n}\left((i\arccos(ax))^n\Gamma(1+n, -i\arccos(ax)) + (-i\arccos(ax))^n\Gamma(1+n, i\arccos(\right.}{2a}
$$

```
[In] Integrate[ArcCos[a*x]^n,x]
```

```
[Out] (ArcCos[a*x]^n*((I*ArcCos[a*x])^n*Gamma[1 + n, (-I)*ArcCos[a*x]] + ((-I)*Ar
cCos[a*x])^n*Gamma[1 + n, I*ArcCos[a*x]]))/(2*a*(ArcCos[a*x]^2)^n)
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.87 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.97

method	result
default	$-\frac{2^{n}\sqrt{\pi}\left(\frac{\arccos(ax)^{1+n}2^{-n}\sqrt{-a^{2}x^{2}+1}}{\sqrt{\pi}\,(2+n)}-\frac{2^{-n}\sqrt{\arccos(ax)}\,\mathrm{LommelS1}\left(n+\frac{3}{2},\frac{3}{2},\arccos(ax)\right)\sqrt{-a^{2}x^{2}+1}}{\sqrt{\pi}\,(2+n)}-\frac{3\,2^{-1-n}\left(\frac{4}{3}+\frac{2n}{3}\right)\left(\arccos(ax)ax-\sqrt{-a\right.}{\sqrt{\pi}\,(2+n)}}{a}$

```
[In] int(arccos(a*x)^n,x,method=_RETURNVERBOSE)
```

```
[Out] -2^n*Pi^(1/2)/a*(1/Pi^(1/2)/(2+n)*arccos(a*x)^(1+n)*2^(-n)*(-a^2*x^2+1)^(1/
2)-2^(-n)/Pi^(1/2)/(2+n)*arccos(a*x)^(1/2)*LommelS1(n+3/2,3/2,arccos(a*x))*
(-a^2*x^2+1)^(1/2)-3*2^(-1-n)/Pi^(1/2)/(2+n)/arccos(a*x)^(1/2)*(4/3+2/3*n)*
(arccos(a*x)*a*x-(-a^2*x^2+1)^(1/2))*LommelS1(n+1/2,1/2,arccos(a*x)))
```

Fricas [F]

$$\int \arccos(ax)^n\,dx=\int \arccos\left(ax\right)^n\,dx$$

```
[In] integrate(arccos(a*x)^n,x, algorithm="fricas")
```

```
[Out] integral(arccos(a*x)^n, x)
```

Sympy [F]

$$\int \arccos(ax)^n\,dx=\int \operatorname{acos}^{n}\left(ax\right)dx$$

```
[In] integrate(acos(a*x)**n,x)
```

```
[Out] Integral(acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \arccos(ax)^n\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^n,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [F]

$$\int \arccos(ax)^n\,dx = \int \arccos\left(ax\right)^n\,dx$$

```
[In] integrate(arccos(a*x)^n,x, algorithm="giac")
[Out] integrate(arccos(a*x)^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int \arccos(ax)^n\,dx = \int \mathrm{acos}(a\,x)^n\,dx$$

```
[In] int(acos(a*x)^n,x)
[Out] int(acos(a*x)^n, x)
```

3.134 $\int \frac{\arccos(ax)^n}{x} \, dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\arccos(ax)^n}{x} \, dx = \text{Int}\left(\frac{\arccos(ax)^n}{x}, x\right)$$

```
[Out] Unintegrable(arccos(a*x)^n/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arccos(ax)^n}{x} \, dx = \int \frac{\arccos(ax)^n}{x} \, dx$$

```
[In] Int[ArcCos[a*x]^n/x,x]
```

```
[Out] Defer[Int][ArcCos[a*x]^n/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arccos(ax)^n}{x} \, dx$$

Mathematica [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x} \, dx = \int \frac{\arccos(ax)^n}{x} \, dx$$

```
[In] Integrate[ArcCos[a*x]^n/x,x]
[Out] Integrate[ArcCos[a*x]^n/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.99 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\arccos (ax)^n}{x} dx$$

```
[In] int(arccos(a*x)^n/x,x)
[Out] int(arccos(a*x)^n/x,x)
```

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x} \, dx = \int \frac{\arccos (ax)^n}{x} \, dx$$

```
[In] integrate(arccos(a*x)^n/x,x, algorithm="fricas")
[Out] integral(arccos(a*x)^n/x, x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\arccos(ax)^n}{x} \, dx = \int \frac{\operatorname{acos}^n (ax)}{x} \, dx$$

```
[In] integrate(acos(a*x)**n/x,x)
[Out] Integral(acos(a*x)**n/x, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^n}{x}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^n/x,x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x}\,dx = \int \frac{\arccos{(ax)}^n}{x}\,dx$$

```
[In] integrate(arccos(a*x)^n/x,x, algorithm="giac")
[Out] integrate(arccos(a*x)^n/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x}\,dx = \int \frac{\mathrm{acos}(a\,x)^n}{x}\,dx$$

```
[In] int(acos(a*x)^n/x,x)
[Out] int(acos(a*x)^n/x, x)
```

3.135 $\int \frac{\arccos(ax)^n}{x^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\arccos(ax)^n}{x^2} dx = \text{Int}\left(\frac{\arccos(ax)^n}{x^2}, x\right)$$

```
[Out] Unintegrable(arccos(a*x)^n/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arccos(ax)^n}{x^2} dx = \int \frac{\arccos(ax)^n}{x^2} dx$$

```
[In] Int[ArcCos[a*x]^n/x^2,x]
```

```
[Out] Defer[Int][ArcCos[a*x]^n/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\arccos(ax)^n}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x^2} \, dx = \int \frac{\arccos(ax)^n}{x^2} \, dx$$

```
[In] Integrate[ArcCos[a*x]^n/x^2,x]
[Out] Integrate[ArcCos[a*x]^n/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.77 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{x^2} dx$$

```
[In] int(arccos(a*x)^n/x^2,x)
[Out] int(arccos(a*x)^n/x^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x^2} \, dx = \int \frac{\arccos(ax)^n}{x^2} \, dx$$

```
[In] integrate(arccos(a*x)^n/x^2,x, algorithm="fricas")
[Out] integral(arccos(a*x)^n/x^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.56 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{x^2} \, dx = \int \frac{\operatorname{acos}^n(ax)}{x^2} \, dx$$

```
[In] integrate(acos(a*x)**n/x**2,x)
[Out] Integral(acos(a*x)**n/x**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^n}{x^2}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^n/x^2,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x^2}\,dx = \int \frac{\arccos{(ax)}^n}{x^2}\,dx$$

```
[In] integrate(arccos(a*x)^n/x^2,x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^n/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\arccos(ax)^n}{x^2}\,dx = \int \frac{\mathrm{acos}(a\,x)^n}{x^2}\,dx$$

```
[In] int(acos(a*x)^n/x^2,x)
```

```
[Out] int(acos(a*x)^n/x^2, x)
```

3.136 $\int (bx)^{3/2} \arccos(ax)^n \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \operatorname{Int}\left((bx)^{3/2} \arccos(ax)^n, x\right)$$

```
[Out] Unintegrable((b*x)^(3/2)*arccos(a*x)^n,x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \int (bx)^{3/2} \arccos(ax)^n \, dx$$

```
[In] Int[(b*x)^(3/2)*ArcCos[a*x]^n,x]
```

```
[Out] Defer[Int][(b*x)^(3/2)*ArcCos[a*x]^n, x]
```

Rubi steps

$$\text{integral} = \int (bx)^{3/2} \arccos(ax)^n \, dx$$

Mathematica [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \int (bx)^{3/2} \arccos(ax)^n \, dx$$

```
[In] Integrate[(b*x)^(3/2)*ArcCos[a*x]^n,x]
[Out] Integrate[(b*x)^(3/2)*ArcCos[a*x]^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int (bx)^{\frac{3}{2}} \arccos(ax)^n \, dx$$

```
[In] int((b*x)^(3/2)*arccos(a*x)^n,x)
[Out] int((b*x)^(3/2)*arccos(a*x)^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \int (bx)^{\frac{3}{2}} \arccos(ax)^n \, dx$$

```
[In] integrate((b*x)^(3/2)*arccos(a*x)^n,x, algorithm="fricas")
[Out] integral(sqrt(b*x)*b*x*arccos(a*x)^n, x)
```

Sympy [N/A]

Not integrable

Time = 162.72 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \int (bx)^{\frac{3}{2}} \operatorname{acos}^n(ax) \, dx$$

```
[In] integrate((b*x)**(3/2)*acos(a*x)**n,x)
[Out] Integral((b*x)**(3/2)*acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((b*x)^(3/2)*arccos(a*x)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \int (bx)^{\frac{3}{2}} \arccos(ax)^n \, dx$$

```
[In] integrate((b*x)^(3/2)*arccos(a*x)^n,x, algorithm="giac")
```

```
[Out] integrate((b*x)^(3/2)*arccos(a*x)^n, x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (bx)^{3/2} \arccos(ax)^n \, dx = \int \mathrm{acos}(a\,x)^n\,(b\,x)^{3/2} \, dx$$

```
[In] int(acos(a*x)^n*(b*x)^(3/2),x)
```

```
[Out] int(acos(a*x)^n*(b*x)^(3/2), x)
```

3.137 $\int \sqrt{bx} \arccos(ax)^n \, dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \text{Int}\left(\sqrt{bx} \arccos(ax)^n, x\right)$$

```
[Out] Unintegrable(arccos(a*x)^n*(b*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \int \sqrt{bx} \arccos(ax)^n \, dx$$

```
[In] Int[Sqrt[b*x]*ArcCos[a*x]^n,x]
[Out] Defer[Int][Sqrt[b*x]*ArcCos[a*x]^n, x]
```

Rubi steps

$$\text{integral} = \int \sqrt{bx} \arccos(ax)^n \, dx$$

Mathematica [N/A]

Not integrable

Time = 3.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \int \sqrt{bx} \arccos(ax)^n \, dx$$

```
[In] Integrate[Sqrt[b*x]*ArcCos[a*x]^n,x]
[Out] Integrate[Sqrt[b*x]*ArcCos[a*x]^n, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.56 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \arccos\left(ax\right)^{n} \sqrt{bx}dx$$

```
[In] int(arccos(a*x)^n*(b*x)^(1/2),x)
[Out] int(arccos(a*x)^n*(b*x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \int \sqrt{bx} \arccos\left(ax\right)^{n} \, dx$$

```
[In] integrate(arccos(a*x)^n*(b*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x)*arccos(a*x)^n, x)
```

Sympy [N/A]

Not integrable

Time = 3.96 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \int \sqrt{bx} \operatorname{acos}^{n}\left(ax\right) dx$$

```
[In] integrate(acos(a*x)**n*(b*x)**(1/2),x)
[Out] Integral(sqrt(b*x)*acos(a*x)**n, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^n*(b*x)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \int \sqrt{bx} \arccos(ax)^n \, dx$$

```
[In] integrate(arccos(a*x)^n*(b*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*x)*arccos(a*x)^n, x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \sqrt{bx} \arccos(ax)^n \, dx = \int \mathrm{acos}(a\,x)^n\,\sqrt{b\,x} \, dx$$

```
[In] int(acos(a*x)^n*(b*x)^(1/2),x)
```

```
[Out] int(acos(a*x)^n*(b*x)^(1/2), x)
```

3.138 $\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \mathrm{Int}\left(\frac{\arccos(ax)^n}{\sqrt{bx}},x\right)$$

```
[Out] Unintegrable(arccos(a*x)^n/(b*x)^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx$$

```
[In] Int[ArcCos[a*x]^n/Sqrt[b*x],x]
[Out] Defer[Int][ArcCos[a*x]^n/Sqrt[b*x], x]
```

Rubi steps

$$\text{integral} = \int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx$$

```
[In] Integrate[ArcCos[a*x]^n/Sqrt[b*x],x]
[Out] Integrate[ArcCos[a*x]^n/Sqrt[b*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.89 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\arccos\left(ax\right)^{n}}{\sqrt{bx}}dx$$

```
[In] int(arccos(a*x)^n/(b*x)^(1/2),x)
[Out] int(arccos(a*x)^n/(b*x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\arccos\left(ax\right)^{n}}{\sqrt{bx}}\,dx$$

```
[In] integrate(arccos(a*x)^n/(b*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x)*arccos(a*x)^n/(b*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\operatorname{acos}^{n}\left(ax\right)}{\sqrt{bx}}\,dx$$

```
[In] integrate(acos(a*x)**n/(b*x)**(1/2),x)
[Out] Integral(acos(a*x)**n/sqrt(b*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^n/(b*x)^(1/2),x, algorithm="maxima")
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\arccos{(ax)}^n}{\sqrt{bx}}\,dx$$

```
[In] integrate(arccos(a*x)^n/(b*x)^(1/2),x, algorithm="giac")
[Out] integrate(arccos(a*x)^n/sqrt(b*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{\sqrt{bx}}\,dx = \int \frac{\mathrm{acos}(a\,x)^n}{\sqrt{b\,x}}\,dx$$

```
[In] int(acos(a*x)^n/(b*x)^(1/2),x)
[Out] int(acos(a*x)^n/(b*x)^(1/2), x)
```

3.139 $\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx = \text{Int}\left(\frac{\arccos(ax)^n}{(bx)^{3/2}},x\right)$$

```
[Out] Unintegrable(arccos(a*x)^n/(b*x)^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx = \int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx$$

```
[In] Int[ArcCos[a*x]^n/(b*x)^(3/2),x]
[Out] Defer[Int][ArcCos[a*x]^n/(b*x)^(3/2), x]
```

Rubi steps

$$\text{integral} = \int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx=\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx$$

```
[In] Integrate[ArcCos[a*x]^n/(b*x)^(3/2),x]
[Out] Integrate[ArcCos[a*x]^n/(b*x)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.94 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\arccos\left(ax\right)^{n}}{\left(bx\right)^{\frac{3}{2}}}dx$$

```
[In] int(arccos(a*x)^n/(b*x)^(3/2),x)
[Out] int(arccos(a*x)^n/(b*x)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx=\int \frac{\arccos\left(ax\right)^{n}}{\left(bx\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arccos(a*x)^n/(b*x)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(b*x)*arccos(a*x)^n/(b^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 13.71 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx=\int \frac{\operatorname{acos}^{n}\left(ax\right)}{\left(bx\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(acos(a*x)**n/(b*x)**(3/2),x)
[Out] Integral(acos(a*x)**n/(b*x)**(3/2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(arccos(a*x)^n/(b*x)^(3/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: expt: undefined: 0 to a negativ
e exponent.
```

Giac [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx = \int \frac{\arccos{(ax)}^n}{(bx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(arccos(a*x)^n/(b*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(arccos(a*x)^n/(b*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\arccos(ax)^n}{(bx)^{3/2}}\,dx = \int \frac{\mathrm{acos}(a\,x)^n}{(b\,x)^{3/2}}\,dx$$

```
[In] int(acos(a*x)^n/(b*x)^(3/2),x)
```

```
[Out] int(acos(a*x)^n/(b*x)^(3/2), x)
```

3.140 $\int x^3(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int x^3(a+b\arccos(cx))\,dx=-\frac{3bx\sqrt{1-c^2x^2}}{32c^3}-\frac{bx^3\sqrt{1-c^2x^2}}{16c}+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{3b\arcsin(cx)}{32c^4}$$

```
[Out] 1/4*x^4*(a+b*arccos(c*x))+3/32*b*arcsin(c*x)/c^4-3/32*b*x*(-c^2*x^2+1)^(1/2
)/c^3-1/16*b*x^3*(-c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4724, 327, 222}

$$\int x^3(a+b\arccos(cx))\,dx=\frac{1}{4}x^4(a+b\arccos(cx))+\frac{3b\arcsin(cx)}{32c^4}-\frac{bx^3\sqrt{1-c^2x^2}}{16c}-\frac{3bx\sqrt{1-c^2x^2}}{32c^3}$$

```
[In] Int[x^3*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-3*b*x*Sqrt[1 - c^2*x^2])/(32*c^3) - (b*x^3*Sqrt[1 - c^2*x^2])/(16*c) + (x
^4*(a + b*ArcCos[c*x]))/4 + (3*b*ArcSin[c*x])/(32*c^4)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{4}x^4(a+b\arccos(cx))+\frac{1}{4}(bc)\int\frac{x^4}{\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{bx^3\sqrt{1-c^2x^2}}{16c}+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{(3b)\int\frac{x^2}{\sqrt{1-c^2x^2}}\,dx}{16c}\\
&= -\frac{3bx\sqrt{1-c^2x^2}}{32c^3}-\frac{bx^3\sqrt{1-c^2x^2}}{16c}+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{(3b)\int\frac{1}{\sqrt{1-c^2x^2}}\,dx}{32c^3}\\
&= -\frac{3bx\sqrt{1-c^2x^2}}{32c^3}-\frac{bx^3\sqrt{1-c^2x^2}}{16c}+\frac{1}{4}x^4(a+b\arccos(cx))+\frac{3b\arcsin(cx)}{32c^4}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$
\int x^3(a+b\arccos(cx))\,dx=\frac{ax^4}{4}+b\sqrt{1-c^2x^2}\left(-\frac{3x}{32c^3}-\frac{x^3}{16c}\right)+\frac{1}{4}bx^4\arccos(cx)+\frac{3b\arcsin(cx)}{32c^4}
$$

```
[In] Integrate[x^3*(a + b*ArcCos[c*x]),x]
[Out] (a*x^4)/4 + b*Sqrt[1 - c^2*x^2]*((-3*x)/(32*c^3) - x^3/(16*c)) + (b*x^4*Arc
Cos[c*x])/4 + (3*b*ArcSin[c*x])/(32*c^4)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

method	result	size
parts	$\frac{a\,x^{4}}{4}+\frac{b\left(\frac{c^{4}x^{4}\arccos\left(cx\right)}{4}-\frac{c^{3}x^{3}\sqrt{-c^{2}x^{2}+1}}{16}-\frac{3cx\sqrt{-c^{2}x^{2}+1}}{32}+\frac{3\arcsin\left(cx\right)}{32}\right)}{c^{4}}$	68
derivativedivides	$\frac{\frac{a\,c^{4}x^{4}}{4}+b\left(\frac{c^{4}x^{4}\arccos\left(cx\right)}{4}-\frac{c^{3}x^{3}\sqrt{-c^{2}x^{2}+1}}{16}-\frac{3cx\sqrt{-c^{2}x^{2}+1}}{32}+\frac{3\arcsin\left(cx\right)}{32}\right)}{c^{4}}$	72
default	$\frac{\frac{a\,c^{4}x^{4}}{4}+b\left(\frac{c^{4}x^{4}\arccos\left(cx\right)}{4}-\frac{c^{3}x^{3}\sqrt{-c^{2}x^{2}+1}}{16}-\frac{3cx\sqrt{-c^{2}x^{2}+1}}{32}+\frac{3\arcsin\left(cx\right)}{32}\right)}{c^{4}}$	72

```
[In] int(x^3*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/4*a*x^4+b/c^4*(1/4*c^4*x^4*arccos(c*x)-1/16*c^3*x^3*(-c^2*x^2+1)^(1/2)-3/
32*c*x*(-c^2*x^2+1)^(1/2)+3/32*arcsin(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.82

$$\int x^3(a+b\arccos(cx))\,dx = \frac{8\,ac^4x^4+(8\,bc^4x^4-3\,b)\arccos{(cx)}-(2\,bc^3x^3+3\,bcx)\sqrt{-c^2x^2+1}}{32\,c^4}$$

```
[In] integrate(x^3*(a+b*arccos(c*x)),x, algorithm="fricas")
[Out] 1/32*(8*a*c^4*x^4 + (8*b*c^4*x^4 - 3*b)*arccos(c*x) - (2*b*c^3*x^3 + 3*b*c*
x)*sqrt(-c^2*x^2 + 1))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.12

$$\int x^3(a+b\arccos(cx))\,dx = \begin{cases} \frac{ax^4}{4}+\frac{bx^4\operatorname{acos}{(cx)}}{4}-\frac{bx^3\sqrt{-c^2x^2+1}}{16c}-\frac{3bx\sqrt{-c^2x^2+1}}{32c^3}-\frac{3b\operatorname{acos}{(cx)}}{32c^4} & \text{for}\ c\neq 0\\ \frac{x^4\left(a+\frac{\pi b}{2}\right)}{4} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**3*(a+b*acos(c*x)),x)
[Out] Piecewise((a*x**4/4 + b*x**4*acos(c*x)/4 - b*x**3*sqrt(-c**2*x**2 + 1)/(16*
c) - 3*b*x*sqrt(-c**2*x**2 + 1)/(32*c**3) - 3*b*acos(c*x)/(32*c**4), Ne(c,
0)), (x**4*(a + pi*b/2)/4, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.93

$$\int x^3(a+b\arccos(cx))\,dx = \frac{1}{4}\,ax^4 + \frac{1}{32}\left(8\,x^4\arccos\left(cx\right) - \left(\frac{2\,\sqrt{-c^2x^2+1}x^3}{c^2} + \frac{3\,\sqrt{-c^2x^2+1}x}{c^4} - \frac{3\,\arcsin\left(cx\right)}{c^5}\right)c\right)b$$

```
[In] integrate(x^3*(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] 1/4*a*x^4 + 1/32*(8*x^4*arccos(c*x) - (2*sqrt(-c^2*x^2 + 1)*x^3/c^2 + 3*sqr
t(-c^2*x^2 + 1)*x/c^4 - 3*arcsin(c*x)/c^5)*c)*b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.88

$$\int x^3(a+b\arccos(cx))\,dx = \frac{1}{4}\,bx^4\arccos\left(cx\right) + \frac{1}{4}\,ax^4 - \frac{\sqrt{-c^2x^2+1}bx^3}{16\,c} - \frac{3\,\sqrt{-c^2x^2+1}bx}{32\,c^3} - \frac{3\,b\arccos\left(cx\right)}{32\,c^4}$$

```
[In] integrate(x^3*(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] 1/4*b*x^4*arccos(c*x) + 1/4*a*x^4 - 1/16*sqrt(-c^2*x^2 + 1)*b*x^3/c - 3/32*
sqrt(-c^2*x^2 + 1)*b*x/c^3 - 3/32*b*arccos(c*x)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+b\arccos(cx))\,dx = \int x^3\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)\,dx$$

```
[In] int(x^3*(a + b*acos(c*x)),x)
```

```
[Out] int(x^3*(a + b*acos(c*x)), x)
```

3.141 $\int x^2(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int x^2(a+b\arccos(cx))\,dx = -\frac{b\sqrt{1-c^2x^2}}{3c^3}+\frac{b(1-c^2x^2)^{3/2}}{9c^3}+\frac{1}{3}x^3(a+b\arccos(cx))$$

```
[Out] 1/9*b*(-c^2*x^2+1)^(3/2)/c^3+1/3*x^3*(a+b*arccos(c*x))-1/3*b*(-c^2*x^2+1)^(
1/2)/c^3
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4724, 272, 45}

$$\int x^2(a+b\arccos(cx))\,dx = \frac{1}{3}x^3(a+b\arccos(cx))+\frac{b(1-c^2x^2)^{3/2}}{9c^3}-\frac{b\sqrt{1-c^2x^2}}{3c^3}$$

```
[In] Int[x^2*(a + b*ArcCos[c*x]),x]
```

```
[Out] -1/3*(b*Sqrt[1 - c^2*x^2])/c^3 + (b*(1 - c^2*x^2)^(3/2))/(9*c^3) + (x^3*(a
+ b*ArcCos[c*x]))/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{3}(bc)\int\frac{x^3}{\sqrt{1-c^2x^2}}\,dx\\
&= \frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}(bc)\text{Subst}\left(\int\frac{x}{\sqrt{1-c^2x}}\,dx,x,x^2\right)\\
&= \frac{1}{3}x^3(a+b\arccos(cx))+\frac{1}{6}(bc)\text{Subst}\left(\int\left(\frac{1}{c^2\sqrt{1-c^2x}}-\frac{\sqrt{1-c^2x}}{c^2}\right)\,dx,x,x^2\right)\\
&= -\frac{b\sqrt{1-c^2x^2}}{3c^3}+\frac{b(1-c^2x^2)^{3/2}}{9c^3}+\frac{1}{3}x^3(a+b\arccos(cx))
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int x^2(a+b\arccos(cx))\,dx=\frac{ax^3}{3}+b\left(-\frac{2}{9c^3}-\frac{x^2}{9c}\right)\sqrt{1-c^2x^2}+\frac{1}{3}bx^3\arccos(cx)$$

```
[In] Integrate[x^2*(a + b*ArcCos[c*x]),x]
[Out] (a*x^3)/3 + b*(-2/(9*c^3) - x^2/(9*c))*Sqrt[1 - c^2*x^2] + (b*x^3*ArcCos[c*
x])/3
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

method	result	size
parts	$\frac{x^{3}a}{3}+\frac{b\left(\frac{c^{3}x^{3}\arccos(cx)}{3}-\frac{c^{2}x^{2}\sqrt{-c^{2}x^{2}+1}}{9}-\frac{2\sqrt{-c^{2}x^{2}+1}}{9}\right)}{c^{3}}$	60
derivativedivides	$\frac{\frac{a\,c^{3}x^{3}}{3}+b\left(\frac{c^{3}x^{3}\arccos(cx)}{3}-\frac{c^{2}x^{2}\sqrt{-c^{2}x^{2}+1}}{9}-\frac{2\sqrt{-c^{2}x^{2}+1}}{9}\right)}{c^{3}}$	64
default	$\frac{\frac{a\,c^{3}x^{3}}{3}+b\left(\frac{c^{3}x^{3}\arccos(cx)}{3}-\frac{c^{2}x^{2}\sqrt{-c^{2}x^{2}+1}}{9}-\frac{2\sqrt{-c^{2}x^{2}+1}}{9}\right)}{c^{3}}$	64

```
[In] int(x^2*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/3*x^3*a+b/c^3*(1/3*c^3*x^3*arccos(c*x)-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/9
*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int x^2(a+b\arccos(cx))\,dx=\frac{3\,bc^3x^3\arccos\left(cx\right)+3\,ac^3x^3-\left(bc^2x^2+2\,b\right)\sqrt{-c^2x^2+1}}{9\,c^3}$$

```
[In] integrate(x^2*(a+b*arccos(c*x)),x, algorithm="fricas")
[Out] 1/9*(3*b*c^3*x^3*arccos(c*x) + 3*a*c^3*x^3 - (b*c^2*x^2 + 2*b)*sqrt(-c^2*x^
2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.17

$$\int x^2(a+b\arccos(cx))\,dx=\begin{cases}\frac{ax^3}{3}+\frac{bx^3\operatorname{acos}\left(cx\right)}{3}-\frac{bx^2\sqrt{-c^2x^2+1}}{9c}-\frac{2b\sqrt{-c^2x^2+1}}{9c^3} & \text{for}\ c\neq 0\\ \frac{x^3\left(a+\frac{\pi b}{2}\right)}{3} & \text{otherwise}\end{cases}$$

```
[In] integrate(x**2*(a+b*acos(c*x)),x)
[Out] Piecewise((a*x**3/3 + b*x**3*acos(c*x)/3 - b*x**2*sqrt(-c**2*x**2 + 1)/(9*c
) - 2*b*sqrt(-c**2*x**2 + 1)/(9*c**3), Ne(c, 0)), (x**3*(a + pi*b/2)/3, Tru
e))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int x^2(a+b\arccos(cx))\,dx = \frac{1}{3}\,ax^3 + \frac{1}{9}\left(3\,x^3\arccos\left(cx\right) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\sqrt{-c^2x^2+1}}{c^4}\right)\right)b$$

```
[In] integrate(x^2*(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] 1/3*a*x^3 + 1/9*(3*x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt
(-c^2*x^2 + 1)/c^4))*b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int x^2(a+b\arccos(cx))\,dx = \frac{1}{3}\,bx^3\arccos\left(cx\right) + \frac{1}{3}\,ax^3 - \frac{\sqrt{-c^2x^2+1}bx^2}{9\,c} - \frac{2\sqrt{-c^2x^2+1}b}{9\,c^3}$$

```
[In] integrate(x^2*(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] 1/3*b*x^3*arccos(c*x) + 1/3*a*x^3 - 1/9*sqrt(-c^2*x^2 + 1)*b*x^2/c - 2/9*sq
rt(-c^2*x^2 + 1)*b/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arccos(cx))\,dx = \begin{cases} \frac{a\,x^3}{3} - b\left(\frac{\sqrt{\frac{1}{c^2}-x^2}\,\left(\frac{2}{c^2}+x^2\right)}{9} - \frac{x^3\,\mathrm{acos}(c\,x)}{3}\right) & \text{if } 0<c \\ \int x^2\,\left(a+b\,\mathrm{acos}(c\,x)\right)\,dx & \text{if } \neg 0<c \end{cases}$$

```
[In] int(x^2*(a + b*acos(c*x)),x)
```

```
[Out] piecewise(0 < c, - b*(((1/c^2 - x^2)^(1/2)*(2/c^2 + x^2))/9 - (x^3*acos(c*x
))/3) + (a*x^3)/3, ~0 < c, int(x^2*(a + b*acos(c*x)), x))
```

3.142 $\int x(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 51

$$\int x(a+b\arccos(cx))\,dx=-\frac{bx\sqrt{1-c^2x^2}}{4c}+\frac{1}{2}x^2(a+b\arccos(cx))+\frac{b\arcsin(cx)}{4c^2}$$

```
[Out] 1/2*x^2*(a+b*arccos(c*x))+1/4*b*arcsin(c*x)/c^2-1/4*b*x*(-c^2*x^2+1)^(1/2)/
c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4724, 327, 222}

$$\int x(a+b\arccos(cx))\,dx=\frac{1}{2}x^2(a+b\arccos(cx))+\frac{b\arcsin(cx)}{4c^2}-\frac{bx\sqrt{1-c^2x^2}}{4c}$$

```
[In] Int[x*(a + b*ArcCos[c*x]),x]
```

```
[Out] -1/4*(b*x*Sqrt[1 - c^2*x^2])/c + (x^2*(a + b*ArcCos[c*x]))/2 + (b*ArcSin[c*
x])/(4*c^2)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
```

```
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2(a+b\arccos(cx))+\frac{1}{2}(bc)\int\frac{x^2}{\sqrt{1-c^2x^2}}\,dx\\ &= -\frac{bx\sqrt{1-c^2x^2}}{4c}+\frac{1}{2}x^2(a+b\arccos(cx))+\frac{b\int\frac{1}{\sqrt{1-c^2x^2}}\,dx}{4c}\\ &= -\frac{bx\sqrt{1-c^2x^2}}{4c}+\frac{1}{2}x^2(a+b\arccos(cx))+\frac{b\arcsin(cx)}{4c^2}\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int x(a+b\arccos(cx))\,dx=\frac{ax^2}{2}-\frac{bx\sqrt{1-c^2x^2}}{4c}+\frac{1}{2}bx^2\arccos(cx)+\frac{b\arcsin(cx)}{4c^2}$$

```
[In] Integrate[x*(a + b*ArcCos[c*x]),x]
[Out] (a*x^2)/2 - (b*x*Sqrt[1 - c^2*x^2])/(4*c) + (b*x^2*ArcCos[c*x])/2 + (b*ArcS
in[c*x])/(4*c^2)
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.94

method	result	size
parts	$\frac{a\,x^{2}}{2}+\frac{b\left(\frac{c^{2} x^{2} \arccos\left(c x\right)}{2}-\frac{c x\sqrt{-c^{2} x^{2}+1}}{4}+\frac{\arcsin\left(c x\right)}{4}\right)}{c^{2}}$	48
derivativedivides	$\frac{\frac{c^{2} x^{2} a}{2}+b\left(\frac{c^{2} x^{2} \arccos\left(c x\right)}{2}-\frac{c x\sqrt{-c^{2} x^{2}+1}}{4}+\frac{\arcsin\left(c x\right)}{4}\right)}{c^{2}}$	52
default	$\frac{\frac{c^{2} x^{2} a}{2}+b\left(\frac{c^{2} x^{2} \arccos\left(c x\right)}{2}-\frac{c x\sqrt{-c^{2} x^{2}+1}}{4}+\frac{\arcsin\left(c x\right)}{4}\right)}{c^{2}}$	52

```
[In] int(x*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/2*a*x^2+b/c^2*(1/2*c^2*x^2*arccos(c*x)-1/4*c*x*(-c^2*x^2+1)^(1/2)+1/4*arc
sin(c*x))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int x(a+b\arccos(cx))\,dx=\frac{2\,ac^{2}x^{2}-\sqrt{-c^{2}x^{2}+1}bcx+\left(2\,bc^{2}x^{2}-b\right)\arccos\left(cx\right)}{4\,c^{2}}$$

```
[In] integrate(x*(a+b*arccos(c*x)),x, algorithm="fricas")
[Out] 1/4*(2*a*c^2*x^2 - sqrt(-c^2*x^2 + 1)*b*c*x + (2*b*c^2*x^2 - b)*arccos(c*x)
)/c^2
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int x(a+b\arccos(cx))\,dx=\begin{cases}\frac{ax^{2}}{2}+\frac{bx^{2}\operatorname{acos}\left(cx\right)}{2}-\frac{bx\sqrt{-c^{2}x^{2}+1}}{4c}-\frac{b\operatorname{acos}\left(cx\right)}{4c^{2}} & \text{for}\: c\neq 0\\ \frac{x^{2}\left(a+\frac{\pi b}{2}\right)}{2} & \text{otherwise}\end{cases}$$

```
[In] integrate(x*(a+b*acos(c*x)),x)
[Out] Piecewise((a*x**2/2 + b*x**2*acos(c*x)/2 - b*x*sqrt(-c**2*x**2 + 1)/(4*c) -
 b*acos(c*x)/(4*c**2), Ne(c, 0)), (x**2*(a + pi*b/2)/2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int x(a+b\arccos(cx))\,dx=\frac{1}{2}\,ax^2+\frac{1}{4}\left(2\,x^2\arccos\left(cx\right)-c\left(\frac{\sqrt{-c^2x^2+1}x}{c^2}-\frac{\arcsin\left(cx\right)}{c^3}\right)\right)b$$

```
[In] integrate(x*(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] 1/2*a*x^2 + 1/4*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c
*x)/c^3))*b
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

$$\int x(a+b\arccos(cx))\,dx=\frac{1}{2}\,bx^2\arccos\left(cx\right)+\frac{1}{2}\,ax^2-\frac{\sqrt{-c^2x^2+1}bx}{4\,c}-\frac{b\arccos\left(cx\right)}{4\,c^2}$$

```
[In] integrate(x*(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] 1/2*b*x^2*arccos(c*x) + 1/2*a*x^2 - 1/4*sqrt(-c^2*x^2 + 1)*b*x/c - 1/4*b*ar
ccos(c*x)/c^2
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.88

$$\int x(a+b\arccos(cx))\,dx=\frac{a\,x^2}{2}+\frac{b\left(\frac{\mathrm{acos}(c\,x)\,\left(2\,c^2\,x^2-1\right)}{4}-\frac{c\,x\,\sqrt{1-c^2\,x^2}}{4}\right)}{c^2}$$

```
[In] int(x*(a + b*acos(c*x)),x)
```

```
[Out] (a*x^2)/2 + (b*((acos(c*x)*(2*c^2*x^2 - 1))/4 - (c*x*(1 - c^2*x^2)^(1/2))/4
))/c^2
```

3.143 $\int (a+b \arccos(cx)) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 31

$$\int (a+b \arccos(cx)) \, dx = ax - \frac{b\sqrt{1-c^2x^2}}{c} + bx \arccos(cx)$$

```
[Out] a*x+b*x*arccos(c*x)-b*(-c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4716, 267}

$$\int (a+b \arccos(cx)) \, dx = ax + bx \arccos(cx) - \frac{b\sqrt{1-c^2x^2}}{c}$$

```
[In] Int[a + b*ArcCos[c*x],x]
```

```
[Out] a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= ax + b\int \arccos(cx)\,dx \\
&= ax + bx\arccos(cx) + (bc)\int \frac{x}{\sqrt{1-c^2x^2}}\,dx \\
&= ax - \frac{b\sqrt{1-c^2x^2}}{c} + bx\arccos(cx)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a + b\arccos(cx))\,dx = ax - \frac{b\sqrt{1-c^2x^2}}{c} + bx\arccos(cx)$$

```
[In] Integrate[a + b*ArcCos[c*x],x]
[Out] a*x - (b*Sqrt[1 - c^2*x^2])/c + b*x*ArcCos[c*x]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

method	result	size
default	$ax + \frac{b\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	32
parts	$ax + \frac{b\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	32
derivativedivides	$\frac{cxa+b\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	34

```
[In] int(a+b*arccos(c*x),x,method=_RETURNVERBOSE)
[Out] a*x+b/c*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.03

$$\int (a+b\arccos(cx))\,dx=\frac{bcx\arccos(cx)+acx-\sqrt{-c^2x^2+1}b}{c}$$

```
[In] integrate(a+b*arccos(c*x),x, algorithm="fricas")
[Out] (b*c*x*arccos(c*x) + a*c*x - sqrt(-c^2*x^2 + 1)*b)/c
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (a+b\arccos(cx))\,dx=ax+b\left(\begin{cases} x\operatorname{acos}(cx)-\frac{\sqrt{-c^2x^2+1}}{c} & \text{for } c\neq 0 \\ \frac{\pi x}{2} & \text{otherwise} \end{cases}\right)$$

```
[In] integrate(a+b*acos(c*x),x)
[Out] a*x + b*Piecewise((x*acos(c*x) - sqrt(-c**2*x**2 + 1)/c, Ne(c, 0)), (pi*x/2
, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a+b\arccos(cx))\,dx=ax+\frac{\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)b}{c}$$

```
[In] integrate(a+b*arccos(c*x),x, algorithm="maxima")
[Out] a*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b/c
```

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int (a + b \arccos(cx)) \, dx = ax + \frac{\left(cx \arccos\left(cx\right) - \sqrt{-c^2x^2 + 1}\right)b}{c}$$

```
[In] integrate(a+b*arccos(c*x),x, algorithm="giac")
[Out] a*x + (c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*b/c
```

Mupad [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.94

$$\int (a + b \arccos(cx)) \, dx = a\,x - \frac{b\,\sqrt{1 - c^2\,x^2}}{c} + b\,x\,\mathrm{acos}(c\,x)$$

```
[In] int(a + b*acos(c*x),x)
[Out] a*x - (b*(1 - c^2*x^2)^(1/2))/c + b*x*acos(c*x)
```

3.144 $\int \frac{a+b\arccos(cx)}{x}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 63

$$\int \frac{a+b\arccos(cx)}{x}\,dx = -\frac{i(a+b\arccos(cx))^2}{2b} + (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right) - \frac{1}{2}ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)$$

```
[Out] -1/2*I*(a+b*arccos(c*x))^2/b+(a+b*arccos(c*x))*ln(1+(c*x+I*(-c^2*x^2+1)^(1/
2))^2)-1/2*I*b*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {4722, 3800, 2221, 2317, 2438}

$$\int \frac{a+b\arccos(cx)}{x}\,dx = -\frac{i(a+b\arccos(cx))^2}{2b} + \log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx)) - \frac{1}{2}ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)$$

```
[In] Int[(a + b*ArcCos[c*x])/x,x]
```

```
[Out] ((-1/2*I)*(a + b*ArcCos[c*x])^2)/b + (a + b*ArcCos[c*x])*Log[1 + E^((2*I)*A
rcCos[c*x])] - (I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
```

```
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int (a+bx)\tan(x)\,dx, x, \arccos(cx)\right)\\
&= -\frac{i(a+b\arccos(cx))^2}{2b} + 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)}{1+e^{2ix}}\,dx, x, \arccos(cx)\right)\\
&= -\frac{i(a+b\arccos(cx))^2}{2b} + (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)\\
&\quad - b\text{Subst}\left(\int \log\left(1+e^{2ix}\right)\,dx, x, \arccos(cx)\right)\\
&= -\frac{i(a+b\arccos(cx))^2}{2b} + (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)\\
&\quad + \frac{1}{2}(ib)\text{Subst}\left(\int \frac{\log(1+x)}{x}\,dx, x, e^{2i\arccos(cx)}\right)\\
&= -\frac{i(a+b\arccos(cx))^2}{2b} + (a+b\arccos(cx))\log\left(1+e^{2i\arccos(cx)}\right)\\
&\quad - \frac{1}{2}ib\,\text{PolyLog}\left(2, -e^{2i\arccos(cx)}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{a+b\arccos(cx)}{x}\,dx = -\frac{1}{2}ib\arccos(cx)^2 + b\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right) + a\log(x) - \frac{1}{2}ib\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)$$

```
[In] Integrate[(a + b*ArcCos[c*x])/x,x]
[Out] (-1/2*I)*b*ArcCos[c*x]^2 + b*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] + a
*Log[x] - (I/2)*b*PolyLog[2, -E^((2*I)*ArcCos[c*x])]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.19

method	result
parts	$a\ln(x)+b\left(-\frac{i\arccos(cx)^2}{2}+\arccos(cx)\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)-\frac{i\,\text{polylog}\left(2,-\left(cx+i\right.\right.}{2}\right.$
derivativedivides	$a\ln(cx)+b\left(-\frac{i\arccos(cx)^2}{2}+\arccos(cx)\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)-\frac{i\,\text{polylog}\left(2,-\left(cx+\right.\right.}{2}\right.$
default	$a\ln(cx)+b\left(-\frac{i\arccos(cx)^2}{2}+\arccos(cx)\ln\left(1+\left(cx+i\sqrt{-c^2x^2+1}\right)^2\right)-\frac{i\,\text{polylog}\left(2,-\left(cx+\right.\right.}{2}\right.$

```
[In] int((a+b*arccos(c*x))/x,x,method=_RETURNVERBOSE)
[Out] a*ln(x)+b*(-1/2*I*arccos(c*x)^2+arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))
^2)-1/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))
```

Fricas [F]

$$\int \frac{a+b\arccos(cx)}{x}\,dx = \int \frac{b\arccos(cx)+a}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))/x,x, algorithm="fricas")
[Out] integral((b*arccos(c*x) + a)/x, x)
```

Sympy [F]

$$\int \frac{a+b\arccos(cx)}{x}\,dx=\int \frac{a+b\operatorname{acos}{(cx)}}{x}\,dx$$

```
[In] integrate((a+b*acos(c*x))/x,x)
[Out] Integral((a + b*acos(c*x))/x, x)
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{x}\,dx=\int \frac{b\arccos{(cx)}+a}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))/x,x, algorithm="maxima")
[Out] b*integrate(arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)/x, x) + a*log(x)
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{x}\,dx=\int \frac{b\arccos{(cx)}+a}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))/x,x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x}\,dx=\int \frac{a+b\operatorname{acos}(c\,x)}{x}\,dx$$

```
[In] int((a + b*acos(c*x))/x,x)
[Out] int((a + b*acos(c*x))/x, x)
```

3.145 $\int \frac{a+b\arccos(cx)}{x^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int \frac{a+b\arccos(cx)}{x^2}\,dx=-\frac{a+b\arccos(cx)}{x}+b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)$$

```
[Out] (-a-b*arccos(c*x))/x+b*c*arctanh((-c^2*x^2+1)^(1/2))
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4724, 272, 65, 214}

$$\int \frac{a+b\arccos(cx)}{x^2}\,dx=b\text{carctanh}\left(\sqrt{1-c^2x^2}\right)-\frac{a+b\arccos(cx)}{x}$$

```
[In] Int[(a + b*ArcCos[c*x])/x^2,x]
```

```
[Out] -((a + b*ArcCos[c*x])/x) + b*c*ArcTanh[Sqrt[1 - c^2*x^2]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arccos(cx)}{x} - (bc)\int \frac{1}{x\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{a+b\arccos(cx)}{x} - \frac{1}{2}(bc)\text{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx, x, x^2\right) \\
&= -\frac{a+b\arccos(cx)}{x} + \frac{b\text{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx, x, \sqrt{1-c^2x^2}\right)}{c} \\
&= -\frac{a+b\arccos(cx)}{x} + b c\,\text{arctanh}\left(\sqrt{1-c^2x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.34

$$\int \frac{a+b\arccos(cx)}{x^2}\,dx = -\frac{a}{x} - \frac{b\arccos(cx)}{x} - bc\log(x) + bc\log\left(1+\sqrt{1-c^2x^2}\right)$$

```
[In] Integrate[(a + b*ArcCos[c*x])/x^2,x]
[Out] -(a/x) - (b*ArcCos[c*x])/x - b*c*Log[x] + b*c*Log[1 + Sqrt[1 - c^2*x^2]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.16

method	result	size
parts	$-\frac{a}{x}+bc\left(-\frac{\arccos(cx)}{cx}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)\right)$	37
derivativedivides	$c\left(-\frac{a}{cx}+b\left(-\frac{\arccos(cx)}{cx}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)\right)\right)$	41
default	$c\left(-\frac{a}{cx}+b\left(-\frac{\arccos(cx)}{cx}+\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)\right)\right)$	41

```
[In] int((a+b*arccos(c*x))/x^2,x,method=_RETURNVERBOSE)
[Out] -a/x+b*c*(-1/c/x*arccos(c*x)+arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(30) = 60.

Time = 0.28 (sec) , antiderivative size = 92, normalized size of antiderivative = 2.88

$$\int \frac{a+b\arccos(cx)}{x^2}\,dx = \frac{bcx\log\left(\sqrt{-c^2x^2+1}+1\right)-bcx\log\left(\sqrt{-c^2x^2+1}-1\right)-2\,bx\arctan\left(\frac{\sqrt{-c^2x^2+1}cx}{c^2x^2-1}\right)+2\,(bx-b)\arccos\left(cx\right)}{2\,x}$$

```
[In] integrate((a+b*arccos(c*x))/x^2,x, algorithm="fricas")
[Out] 1/2*(b*c*x*log(sqrt(-c^2*x^2 + 1) + 1) - b*c*x*log(sqrt(-c^2*x^2 + 1) - 1)
- 2*b*x*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 2*(b*x - b)*arccos(c
*x) - 2*a)/x
```

Sympy [A] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.28

$$\int \frac{a+b\arccos(cx)}{x^2}\,dx = -\frac{a}{x}-bc\left(\begin{cases}-\operatorname{acosh}\left(\frac{1}{cx}\right) & \text{for}\ \frac{1}{|c^2x^2|}>1\\ i\operatorname{asin}\left(\frac{1}{cx}\right) & \text{otherwise}\end{cases}\right)-\frac{b\operatorname{acos}\left(cx\right)}{x}$$

```
[In] integrate((a+b*acos(c*x))/x**2,x)
[Out] -a/x - b*c*Piecewise((-acosh(1/(c*x)), 1/Abs(c**2*x**2) > 1), (I*asin(1/(c*
x)), True)) - b*acos(c*x)/x
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.47

$$\int \frac{a+b\arccos(cx)}{x^2}\,dx = \left(c\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|}+\frac{2}{|x|}\right)-\frac{\arccos(cx)}{x}\right)b-\frac{a}{x}$$

```
[In] integrate((a+b*arccos(c*x))/x^2,x, algorithm="maxima")
[Out] (c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*b - a/x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(30) = 60.

Time = 0.36 (sec) , antiderivative size = 347, normalized size of antiderivative = 10.84

$$\begin{aligned}\int \frac{a+b\arccos(cx)}{x^2}\,dx = &-\frac{bc\arccos(cx)}{\frac{c^2x^2-1}{(cx+1)^2}+1}+\frac{bc\log\left(\left|cx+\sqrt{-c^2x^2+1}+1\right|\right)}{\frac{c^2x^2-1}{(cx+1)^2}+1} \\ &-\frac{bc\log\left(\left|-cx+\sqrt{-c^2x^2+1}-1\right|\right)}{\frac{c^2x^2-1}{(cx+1)^2}+1} \\ &-\frac{ac}{\frac{c^2x^2-1}{(cx+1)^2}+1}+\frac{(c^2x^2-1)bc\arccos(cx)}{(cx+1)^2\left(\frac{c^2x^2-1}{(cx+1)^2}+1\right)} \\ &+\frac{(c^2x^2-1)bc\log\left(\left|cx+\sqrt{-c^2x^2+1}+1\right|\right)}{(cx+1)^2\left(\frac{c^2x^2-1}{(cx+1)^2}+1\right)} \\ &-\frac{(c^2x^2-1)bc\log\left(\left|-cx+\sqrt{-c^2x^2+1}-1\right|\right)}{(cx+1)^2\left(\frac{c^2x^2-1}{(cx+1)^2}+1\right)} \\ &+\frac{(c^2x^2-1)ac}{(cx+1)^2\left(\frac{c^2x^2-1}{(cx+1)^2}+1\right)}\end{aligned}$$

```
[In] integrate((a+b*arccos(c*x))/x^2,x, algorithm="giac")
[Out] -b*c*arccos(c*x)/((c^2*x^2 - 1)/(c*x + 1)^2 + 1) + b*c*log(abs(c*x + sqrt(-
c^2*x^2 + 1) + 1))/((c^2*x^2 - 1)/(c*x + 1)^2 + 1) - b*c*log(abs(-c*x + sqr
t(-c^2*x^2 + 1) - 1))/((c^2*x^2 - 1)/(c*x + 1)^2 + 1) - a*c/((c^2*x^2 - 1)/
(c*x + 1)^2 + 1) + (c^2*x^2 - 1)*b*c*arccos(c*x)/((c*x + 1)^2*((c^2*x^2 - 1
)/(c*x + 1)^2 + 1)) + (c^2*x^2 - 1)*b*c*log(abs(c*x + sqrt(-c^2*x^2 + 1) +
1))/((c*x + 1)^2*((c^2*x^2 - 1)/(c*x + 1)^2 + 1)) - (c^2*x^2 - 1)*b*c*log(a
bs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c*x + 1)^2*((c^2*x^2 - 1)/(c*x + 1)^2
+ 1)) + (c^2*x^2 - 1)*a*c/((c*x + 1)^2*((c^2*x^2 - 1)/(c*x + 1)^2 + 1))
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{a+b\arccos(cx)}{x^2}\,dx = b\,c\,\mathrm{atanh}\left(\frac{1}{\sqrt{1-c^2\,x^2}}\right)-\frac{b\,\mathrm{acos}(c\,x)}{x}-\frac{a}{x}$$

```
[In] int((a + b*acos(c*x))/x^2,x)
[Out] b*c*atanh(1/(1 - c^2*x^2)^(1/2)) - (b*acos(c*x))/x - a/x
```

3.146 $\int \frac{a+b\arccos(cx)}{x^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 39

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx=\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\arccos(cx)}{2x^2}$$

```
[Out] 1/2*(-a-b*arccos(c*x))/x^2+1/2*b*c*(-c^2*x^2+1)^(1/2)/x
```

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {4724, 270}

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx=\frac{bc\sqrt{1-c^2x^2}}{2x}-\frac{a+b\arccos(cx)}{2x^2}$$

```
[In] Int[(a + b*ArcCos[c*x])/x^3,x]
```

```
[Out] (b*c*Sqrt[1 - c^2*x^2])/(2*x) - (a + b*ArcCos[c*x])/(2*x^2)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
```

```
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{a+b\arccos(cx)}{2x^2} - \frac{1}{2}(bc)\int \frac{1}{x^2\sqrt{1-c^2x^2}}\,dx$$
$$= \frac{bc\sqrt{1-c^2x^2}}{2x} - \frac{a+b\arccos(cx)}{2x^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx = -\frac{a}{2x^2} + \frac{bc\sqrt{1-c^2x^2}}{2x} - \frac{b\arccos(cx)}{2x^2}$$

```
[In] Integrate[(a + b*ArcCos[c*x])/x^3,x]
[Out] -1/2*a/x^2 + (b*c*Sqrt[1 - c^2*x^2])/(2*x) - (b*ArcCos[c*x])/(2*x^2)
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

method	result	size
parts	$-\frac{a}{2x^2}+b\,c^2\left(-\frac{\arccos(cx)}{2c^2x^2}+\frac{\sqrt{-c^2x^2+1}}{2cx}\right)$	46
derivativedivides	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\arccos(cx)}{2c^2x^2}+\frac{\sqrt{-c^2x^2+1}}{2cx}\right)\right)$	50
default	$c^2\left(-\frac{a}{2c^2x^2}+b\left(-\frac{\arccos(cx)}{2c^2x^2}+\frac{\sqrt{-c^2x^2+1}}{2cx}\right)\right)$	50

```
[In] int((a+b*arccos(c*x))/x^3,x,method=_RETURNVERBOSE)
[Out] -1/2*a/x^2+b*c^2*(-1/2/c^2/x^2*arccos(c*x)+1/2/c/x*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx=\frac{\sqrt{-c^2x^2+1}bcx+ax^2-b\arccos{(cx)}-a}{2\,x^2}$$

```
[In] integrate((a+b*arccos(c*x))/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(-c^2*x^2 + 1)*b*c*x + a*x^2 - b*arccos(c*x) - a)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.62

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx=-\frac{a}{2x^2}-\frac{bc\left(\begin{cases}-\frac{i\sqrt{c^2x^2-1}}{x} & \text{for } |c^2x^2|>1\\ -\frac{\sqrt{-c^2x^2+1}}{x} & \text{otherwise}\end{cases}\right)}{2}-\frac{b\,\mathrm{acos}\,(cx)}{2x^2}$$

```
[In] integrate((a+b*acos(c*x))/x**3,x)
```

```
[Out] -a/(2*x**2) - b*c*Piecewise((-I*sqrt(c**2*x**2 - 1)/x, Abs(c**2*x**2) > 1),
 (-sqrt(-c**2*x**2 + 1)/x, True))/2 - b*acos(c*x)/(2*x**2)
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx=\frac{1}{2}\,b\left(\frac{\sqrt{-c^2x^2+1}c}{x}-\frac{\arccos{(cx)}}{x^2}\right)-\frac{a}{2\,x^2}$$

```
[In] integrate((a+b*arccos(c*x))/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*b*(sqrt(-c^2*x^2 + 1)*c/x - arccos(c*x)/x^2) - 1/2*a/x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 492 vs. $2(33) = 66$.

Time = 0.29 (sec) , antiderivative size = 492, normalized size of antiderivative = 12.62

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx = -\frac{bc^2\arccos(cx)}{2\left(\frac{2\,(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} - \frac{ac^2}{2\left(\frac{2\,(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} + \frac{(c^2x^2-1)bc^2\arccos(cx)}{(cx+1)^2\left(\frac{2\,(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} + \frac{\sqrt{-c^2x^2+1}bc^2}{(cx+1)\left(\frac{2\,(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} + \frac{(c^2x^2-1)ac^2}{(cx+1)^2\left(\frac{2\,(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} - \frac{(c^2x^2-1)^2bc^2\arccos(cx)}{2\,(cx+1)^4\left(\frac{2\,(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} - \frac{(-c^2x^2+1)^{\frac{3}{2}}bc^2}{(cx+1)^3\left(\frac{2\,(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)} - \frac{(c^2x^2-1)^2ac^2}{2\,(cx+1)^4\left(\frac{2\,(c^2x^2-1)}{(cx+1)^2}+\frac{(c^2x^2-1)^2}{(cx+1)^4}+1\right)}$$

```
[In] integrate((a+b*arccos(c*x))/x^3,x, algorithm="giac")
```

```
[Out] -1/2*b*c^2*arccos(c*x)/(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x
+ 1)^4 + 1) - 1/2*a*c^2/(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x
 + 1)^4 + 1) + (c^2*x^2 - 1)*b*c^2*arccos(c*x)/((c*x + 1)^2*(2*(c^2*x^2 - 1
)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^4 + 1)) + sqrt(-c^2*x^2 + 1)*b*c^
2/((c*x + 1)*(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^4 + 1
)) + (c^2*x^2 - 1)*a*c^2/((c*x + 1)^2*(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x
^2 - 1)^2/(c*x + 1)^4 + 1)) - 1/2*(c^2*x^2 - 1)^2*b*c^2*arccos(c*x)/((c*x +
 1)^4*(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^4 + 1)) - (-
c^2*x^2 + 1)^(3/2)*b*c^2/((c*x + 1)^3*(2*(c^2*x^2 - 1)/(c*x + 1)^2 + (c^2*x
^2 - 1)^2/(c*x + 1)^4 + 1)) - 1/2*(c^2*x^2 - 1)^2*a*c^2/((c*x + 1)^4*(2*(c^
2*x^2 - 1)/(c*x + 1)^2 + (c^2*x^2 - 1)^2/(c*x + 1)^4 + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{x^3}\,dx=\int \frac{a+b\,\mathrm{acos}(c\,x)}{x^3}\,dx$$

```
[In] int((a + b*acos(c*x))/x^3,x)
```

```
[Out] int((a + b*acos(c*x))/x^3, x)
```

3.147 $\int \frac{a+b\arccos(cx)}{x^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 62

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx=\frac{bc\sqrt{1-c^2x^2}}{6x^2}-\frac{a+b\arccos(cx)}{3x^3}+\frac{1}{6}bc^3\text{arctanh}\left(\sqrt{1-c^2x^2}\right)$$

```
[Out] 1/3*(-a-b*arccos(c*x))/x^3+1/6*b*c^3*arctanh((-c^2*x^2+1)^(1/2))+1/6*b*c*(-
c^2*x^2+1)^(1/2)/x^2
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4724, 272, 44, 65, 214}

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx=-\frac{a+b\arccos(cx)}{3x^3}+\frac{1}{6}bc^3\text{arctanh}\left(\sqrt{1-c^2x^2}\right)+\frac{bc\sqrt{1-c^2x^2}}{6x^2}$$

```
[In] Int[(a + b*ArcCos[c*x])/x^4,x]
```

```
[Out] (b*c*Sqrt[1 - c^2*x^2])/(6*x^2) - (a + b*ArcCos[c*x])/(3*x^3) + (b*c^3*ArcT
anh[Sqrt[1 - c^2*x^2]])/6
```

Rule 44

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && ILtQ[m, -1] && !Int
egerQ[n] && LtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{a+b\arccos(cx)}{3x^3}-\frac{1}{3}(bc)\int\frac{1}{x^3\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{a+b\arccos(cx)}{3x^3}-\frac{1}{6}(bc)\text{Subst}\left(\int\frac{1}{x^2\sqrt{1-c^2x}}\,dx,x,x^2\right)\\
&= \frac{bc\sqrt{1-c^2x^2}}{6x^2}-\frac{a+b\arccos(cx)}{3x^3}-\frac{1}{12}\left(bc^3\right)\text{Subst}\left(\int\frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)\\
&= \frac{bc\sqrt{1-c^2x^2}}{6x^2}-\frac{a+b\arccos(cx)}{3x^3}+\frac{1}{6}(bc)\text{Subst}\left(\int\frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)\\
&= \frac{bc\sqrt{1-c^2x^2}}{6x^2}-\frac{a+b\arccos(cx)}{3x^3}+\frac{1}{6}bc^3\text{arctanh}\left(\sqrt{1-c^2x^2}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx = -\frac{a}{3x^3}+\frac{bc\sqrt{1-c^2x^2}}{6x^2}-\frac{b\arccos(cx)}{3x^3} - \frac{1}{6}bc^3\log(x)+\frac{1}{6}bc^3\log\left(1+\sqrt{1-c^2x^2}\right)$$

```
[In] Integrate[(a + b*ArcCos[c*x])/x^4,x]
```

```
[Out] -1/3*a/x^3 + (b*c*Sqrt[1 - c^2*x^2])/(6*x^2) - (b*ArcCos[c*x])/(3*x^3) - (b
*c^3*Log[x])/6 + (b*c^3*Log[1 + Sqrt[1 - c^2*x^2]])/6
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.98

method	result	size
parts	$-\frac{a}{3x^3}+b\,c^3\left(-\frac{\arccos(cx)}{3c^3x^3}+\frac{\sqrt{-c^2x^2+1}}{6c^2x^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{6}\right)$	61
derivativedivides	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\arccos(cx)}{3c^3x^3}+\frac{\sqrt{-c^2x^2+1}}{6c^2x^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{6}\right)\right)$	65
default	$c^3\left(-\frac{a}{3c^3x^3}+b\left(-\frac{\arccos(cx)}{3c^3x^3}+\frac{\sqrt{-c^2x^2+1}}{6c^2x^2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{-c^2x^2+1}}\right)}{6}\right)\right)$	65

```
[In] int((a+b*arccos(c*x))/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*a/x^3+b*c^3*(-1/3/c^3/x^3*arccos(c*x)+1/6/c^2/x^2*(-c^2*x^2+1)^(1/2)+1
/6*arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(52) = 104$.

Time = 0.26 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx = \frac{bc^3x^3\log\left(\sqrt{-c^2x^2+1}+1\right)-bc^3x^3\log\left(\sqrt{-c^2x^2+1}-1\right)-4\,bx^3\arctan\left(\frac{\sqrt{-c^2x^2+1}cx}{c^2x^2-1}\right)+2\,\sqrt{-c^2x^2+1}bcx}{12\,x^3}$$

```
[In] integrate((a+b*arccos(c*x))/x^4,x, algorithm="fricas")
```

```
[Out] 1/12*(b*c^3*x^3*log(sqrt(-c^2*x^2 + 1) + 1) - b*c^3*x^3*log(sqrt(-c^2*x^2 +
 1) - 1) - 4*b*x^3*arctan(sqrt(-c^2*x^2 + 1)*c*x/(c^2*x^2 - 1)) + 2*sqrt(-c
^2*x^2 + 1)*b*c*x + 4*(b*x^3 - b)*arccos(c*x) - 4*a)/x^3
```

Sympy [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.92

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx = -\frac{a}{3x^3} - \frac{bc\left(\begin{cases} -\frac{c^2\operatorname{acosh}\left(\frac{1}{cx}\right)}{2} + \frac{c}{2x\sqrt{-1+\frac{1}{c^2x^2}}} - \frac{1}{2cx^3\sqrt{-1+\frac{1}{c^2x^2}}} & \text{for}\ \frac{1}{|c^2x^2|} > 1 \\ \frac{ic^2\operatorname{asin}\left(\frac{1}{cx}\right)}{2} - \frac{ic\sqrt{1-\frac{1}{c^2x^2}}}{2x} & \text{otherwise} \end{cases}\right)}{3} - \frac{b\operatorname{acos}(cx)}{3x^3}$$

```
[In] integrate((a+b*acos(c*x))/x**4,x)
```

```
[Out] -a/(3*x**3) - b*c*Piecewise((-c**2*acosh(1/(c*x))/2 + c/(2*x*sqrt(-1 + 1/(c
**2*x**2))) - 1/(2*c*x**3*sqrt(-1 + 1/(c**2*x**2))), 1/Abs(c**2*x**2) > 1),
 (I*c**2*asin(1/(c*x))/2 - I*c*sqrt(1 - 1/(c**2*x**2))/(2*x), True))/3 - b*
acos(c*x)/(3*x**3)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11

$$\int \frac{a+b\arccos(cx)}{x^4}\,dx = \frac{1}{6}\left(\left(c^2\log\left(\frac{2\sqrt{-c^2x^2+1}}{|x|} + \frac{2}{|x|}\right) + \frac{\sqrt{-c^2x^2+1}}{x^2}\right)c - \frac{2\arccos(cx)}{x^3}\right)b - \frac{a}{3x^3}$$

```
[In] integrate((a+b*arccos(c*x))/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*((c^2*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) + sqrt(-c^2*x^2 + 1)/
x^2)*c - 2*arccos(c*x)/x^3)*b - 1/3*a/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1634 vs. $2(52) = 104$.

Time = 0.59 (sec) , antiderivative size = 1634, normalized size of antiderivative = 26.35

$$\int \frac{a + b \arccos(cx)}{x^4} \, dx = \text{Too large to display}$$

```
[In] integrate((a+b*arccos(c*x))/x^4,x, algorithm="giac")
```

```
[Out] -1/3*b*c^3*arccos(c*x)/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*
x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) + 1/6*b*c^3*log(abs(c*x + sqrt(
-c^2*x^2 + 1) + 1))/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x +
 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) - 1/6*b*c^3*log(abs(-c*x + sqrt(-c
^2*x^2 + 1) - 1))/(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1
)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) - 1/3*a*c^3/(3*(c^2*x^2 - 1)/(c*x +
1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1) + (
c^2*x^2 - 1)*b*c^3*arccos(c*x)/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2 +
3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/2*(c^
2*x^2 - 1)*b*c^3*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c*x + 1)^2*(3*(c^
2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c
*x + 1)^6 + 1)) - 1/2*(c^2*x^2 - 1)*b*c^3*log(abs(-c*x + sqrt(-c^2*x^2 + 1)
 - 1))/((c*x + 1)^2*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x +
 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/3*sqrt(-c^2*x^2 + 1)*b*c^3/((
c*x + 1)*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^
2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + (c^2*x^2 - 1)*a*c^3/((c*x + 1)^2*(3*(c^2*x
^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x
+ 1)^6 + 1)) - (c^2*x^2 - 1)^2*b*c^3*arccos(c*x)/((c*x + 1)^4*(3*(c^2*x^2 -
 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)
^6 + 1)) + 1/2*(c^2*x^2 - 1)^2*b*c^3*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))
/((c*x + 1)^4*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4
+ (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) - 1/2*(c^2*x^2 - 1)^2*b*c^3*log(abs(-c*
x + sqrt(-c^2*x^2 + 1) - 1))/((c*x + 1)^4*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*
(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) - (c^2*x^2
- 1)^2*a*c^3/((c*x + 1)^4*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/
(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/3*(c^2*x^2 - 1)^3*b*c^3
*arccos(c*x)/((c*x + 1)^6*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/
(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/6*(c^2*x^2 - 1)^3*b*c^3
*log(abs(c*x + sqrt(-c^2*x^2 + 1) + 1))/((c*x + 1)^6*(3*(c^2*x^2 - 1)/(c*x
+ 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1))
- 1/6*(c^2*x^2 - 1)^3*b*c^3*log(abs(-c*x + sqrt(-c^2*x^2 + 1) - 1))/((c*x +
 1)^6*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x
^2 - 1)^3/(c*x + 1)^6 + 1)) - 1/3*(c^2*x^2 - 1)^2*sqrt(-c^2*x^2 + 1)*b*c^3/
((c*x + 1)^5*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 +
 (c^2*x^2 - 1)^3/(c*x + 1)^6 + 1)) + 1/3*(c^2*x^2 - 1)^3*a*c^3/((c*x + 1)^6
```

```
*(3*(c^2*x^2 - 1)/(c*x + 1)^2 + 3*(c^2*x^2 - 1)^2/(c*x + 1)^4 + (c^2*x^2 -
1)^3/(c*x + 1)^6 + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arccos(cx)}{x^4} \, dx = \int \frac{a + b \operatorname{acos}(c\,x)}{x^4} \, dx$$

```
[In] int((a + b*acos(c*x))/x^4,x)
[Out] int((a + b*acos(c*x))/x^4, x)
```

3.148 $\int x^2(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int x^2(a+b\arccos(cx))^2\,dx=-\frac{4b^2x}{9c^2}-\frac{2b^2x^3}{27}-\frac{4b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3}-\frac{2bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c}+\frac{1}{3}x^3(a+b\arccos(cx))^2$$

```
[Out] -4/9*b^2*x/c^2-2/27*b^2*x^3+1/3*x^3*(a+b*arccos(c*x))^2-4/9*b*(a+b*arccos(c
*x))*(-c^2*x^2+1)^(1/2)/c^3-2/9*b*x^2*(a+b*arccos(c*x))*(-c^2*x^2+1)^(1/2)/
c
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4724, 4796, 4768, 8, 30}

$$\int x^2(a+b\arccos(cx))^2\,dx=-\frac{2bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c}-\frac{4b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3}+\frac{1}{3}x^3(a+b\arccos(cx))^2-\frac{4b^2x}{9c^2}-\frac{2}{27}b^2x^3$$

```
[In] Int[x^2*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (-4*b^2*x)/(9*c^2) - (2*b^2*x^3)/27 - (4*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[
c*x]))/(9*c^3) - (2*b*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(9*c) + (x
^3*(a + b*ArcCos[c*x])^2)/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{3}x^3(a+b\arccos(cx))^2+\frac{1}{3}(2bc)\int\frac{x^3(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx\\ &= -\frac{2bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c}+\frac{1}{3}x^3(a+b\arccos(cx))^2\\ &\quad-\frac{1}{9}\left(2b^2\right)\int x^2\,dx+\frac{(4b)\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx}{9c}\end{aligned}$$

$$= -\frac{2}{27}b^2x^3 - \frac{4b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3} - \frac{2bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c} + \frac{1}{3}x^3(a+b\arccos(cx))^2 - \frac{(4b^2)\int 1\,dx}{9c^2}$$

$$= -\frac{4b^2x}{9c^2} - \frac{2b^2x^3}{27} - \frac{4b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c^3} - \frac{2bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))}{9c} + \frac{1}{3}x^3(a+b\arccos(cx))^2$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.19

$$\int x^2(a+b\arccos(cx))^2\,dx$$
$$= \frac{9a^2c^3x^3 - 6ab\sqrt{1-c^2x^2}(2+c^2x^2) - 2b^2cx(6+c^2x^2) - 6b\left(-3ac^3x^3 + b\sqrt{1-c^2x^2}(2+c^2x^2)\right)\arccos(cx) +}{27c^3}$$

```
[In] Integrate[x^2*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (9*a^2*c^3*x^3 - 6*a*b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2) - 2*b^2*c*x*(6 + c^2
*x^2) - 6*b*(-3*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2))*ArcCos[c*x]
+ 9*b^2*c^3*x^3*ArcCos[c*x]^2)/(27*c^3)
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.23

method	result
parts	$\frac{a^2x^3}{3} + \frac{b^2\left(\frac{\arccos(cx)^2c^3x^3}{3} - \frac{2\arccos(cx)\left(c^2x^2+2\right)\sqrt{-c^2x^2+1}}{9} - \frac{2c^3x^3}{27} - \frac{4cx}{9}\right)}{c^3} + \frac{2ab\left(\frac{c^3x^3\arccos(cx)}{3} - \frac{c^2x^2\sqrt{-c^2x^2+1}}{9} -\right.}{c^3}$
derivativedivides	$\frac{\frac{a^2c^3x^3}{3} + b^2\left(\frac{\arccos(cx)^2c^3x^3}{3} - \frac{2\arccos(cx)\left(c^2x^2+2\right)\sqrt{-c^2x^2+1}}{9} - \frac{2c^3x^3}{27} - \frac{4cx}{9}\right) + 2ab\left(\frac{c^3x^3\arccos(cx)}{3} - \frac{c^2x^2\sqrt{-c^2x^2+1}}{9} - 2\sqrt{}\right.}{c^3}$
default	$\frac{\frac{a^2c^3x^3}{3} + b^2\left(\frac{\arccos(cx)^2c^3x^3}{3} - \frac{2\arccos(cx)\left(c^2x^2+2\right)\sqrt{-c^2x^2+1}}{9} - \frac{2c^3x^3}{27} - \frac{4cx}{9}\right) + 2ab\left(\frac{c^3x^3\arccos(cx)}{3} - \frac{c^2x^2\sqrt{-c^2x^2+1}}{9} - 2\sqrt{}\right.}{c^3}$

```
[In] int(x^2*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^2*x^3+b^2/c^3*(1/3*arccos(c*x)^2*c^3*x^3-2/9*arccos(c*x)*(c^2*x^2+2)*
(-c^2*x^2+1)^(1/2)-2/27*c^3*x^3-4/9*c*x)+2*a*b/c^3*(1/3*c^3*x^3*arccos(c*x)
-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-2/9*(-c^2*x^2+1)^(1/2))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.09

$$\int x^2(a+b\arccos(cx))^2\,dx$$
$$=\frac{9\,b^2c^3x^3\arccos{(cx)}^2+18\,abc^3x^3\arccos{(cx)}+(9\,a^2-2\,b^2)c^3x^3-12\,b^2cx-6\,(abc^2x^2+2\,ab+(b^2c^2x^2+2}{27\,c^3}$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] 1/27*(9*b^2*c^3*x^3*arccos(c*x)^2 + 18*a*b*c^3*x^3*arccos(c*x) + (9*a^2 - 2
*b^2)*c^3*x^3 - 12*b^2*c*x - 6*(a*b*c^2*x^2 + 2*a*b + (b^2*c^2*x^2 + 2*b^2)
*arccos(c*x))*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.72

$$\int x^2(a+b\arccos(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2x^3}{3}+\frac{2abx^3\operatorname{acos}{(cx)}}{3}-\frac{2abx^2\sqrt{-c^2x^2+1}}{9c}-\frac{4ab\sqrt{-c^2x^2+1}}{9c^3}+\frac{b^2x^3\operatorname{acos}^2{(cx)}}{3}-\frac{2b^2x^3}{27}-\frac{2b^2x^2\sqrt{-c^2x^2+1}\operatorname{acos}{(cx)}}{9c}-\frac{4b^2x}{9c^2}-\\ \frac{x^3\left(a+\frac{\pi b}{2}\right)^2}{3}\end{cases}$$

```
[In] integrate(x**2*(a+b*acos(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**3/3 + 2*a*b*x**3*acos(c*x)/3 - 2*a*b*x**2*sqrt(-c**2*x**
2 + 1)/(9*c) - 4*a*b*sqrt(-c**2*x**2 + 1)/(9*c**3) + b**2*x**3*acos(c*x)**2
/3 - 2*b**2*x**3/27 - 2*b**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(9*c) - 4*
b**2*x/(9*c**2) - 4*b**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(9*c**3), Ne(c, 0))
, (x**3*(a + pi*b/2)**2/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.39

$$\int x^2(a + b\arccos(cx))^2 \, dx$$
$$= \frac{1}{3}\, b^2x^3 \arccos\left(cx\right)^2 + \frac{1}{3}\, a^2x^3$$
$$+ \frac{2}{9}\left(3\,x^3 \arccos\left(cx\right) - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\right)ab$$
$$- \frac{2}{27}\left(3\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\arccos\left(cx\right) + \frac{c^2x^3+6\,x}{c^2}\right)b^2$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*x^3*arccos(c*x)^2 + 1/3*a^2*x^3 + 2/9*(3*x^3*arccos(c*x) - c*(sqrt(
-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4))*a*b - 2/27*(3*c*(sqrt(-c
^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x) + (c^2*x^3 + 6*
x)/c^2)*b^2
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.40

$$\int x^2(a + b\arccos(cx))^2 \, dx = \frac{1}{3}\, b^2x^3 \arccos\left(cx\right)^2 + \frac{2}{3}\, abx^3 \arccos\left(cx\right) + \frac{1}{3}\, a^2x^3 - \frac{2}{27}\, b^2x^3 - \frac{2\,\sqrt{-c^2x^2+1}b^2x^2 \arccos\left(cx\right)}{9\,c} - \frac{2\,\sqrt{-c^2x^2+1}abx^2}{9\,c} - \frac{4\,b^2x}{9\,c^2} - \frac{4\,\sqrt{-c^2x^2+1}b^2 \arccos\left(cx\right)}{9\,c^3} - \frac{4\,\sqrt{-c^2x^2+1}ab}{9\,c^3}$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] 1/3*b^2*x^3*arccos(c*x)^2 + 2/3*a*b*x^3*arccos(c*x) + 1/3*a^2*x^3 - 2/27*b^
2*x^3 - 2/9*sqrt(-c^2*x^2 + 1)*b^2*x^2*arccos(c*x)/c - 2/9*sqrt(-c^2*x^2 +
1)*a*b*x^2/c - 4/9*b^2*x/c^2 - 4/9*sqrt(-c^2*x^2 + 1)*b^2*arccos(c*x)/c^3 -
 4/9*sqrt(-c^2*x^2 + 1)*a*b/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arccos(cx))^2\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^2\,dx$$

```
[In] int(x^2*(a + b*acos(c*x))^2,x)
[Out] int(x^2*(a + b*acos(c*x))^2, x)
```

3.149 $\int x(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int x(a+b\arccos(cx))^2\,dx=-\frac{1}{4}b^2x^2-\frac{bx\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c}-\frac{(a+b\arccos(cx))^2}{4c^2}+\frac{1}{2}x^2(a+b\arccos(cx))^2$$

```
[Out] -1/4*b^2*x^2-1/4*(a+b*arccos(c*x))^2/c^2+1/2*x^2*(a+b*arccos(c*x))^2-1/2*b*
x*(a+b*arccos(c*x))*(-c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4724, 4796, 4738, 30}

$$\int x(a+b\arccos(cx))^2\,dx=-\frac{bx\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c}-\frac{(a+b\arccos(cx))^2}{4c^2}+\frac{1}{2}x^2(a+b\arccos(cx))^2-\frac{1}{4}b^2x^2$$

```
[In] Int[x*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] -1/4*(b^2*x^2) - (b*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/(2*c) - (a + b
*ArcCos[c*x])^2/(4*c^2) + (x^2*(a + b*ArcCos[c*x])^2)/2
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{1}{2}x^2(a+b\arccos(cx))^2+(bc)\int\frac{x^2(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx\\ &= -\frac{bx\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c}+\frac{1}{2}x^2(a+b\arccos(cx))^2-\frac{1}{2}b^2\int x\,dx+\frac{b\int\frac{a+b\arccos(cx)}{\sqrt{1-c^2x^2}}\,dx}{2c}\\ &= -\frac{1}{4}b^2x^2-\frac{bx\sqrt{1-c^2x^2}(a+b\arccos(cx))}{2c}-\frac{(a+b\arccos(cx))^2}{4c^2}+\frac{1}{2}x^2(a+b\arccos(cx))^2\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.37

$$\int x(a+b\arccos(cx))^2\,dx$$
$$=\frac{cx\left(2a^2cx-b^2cx-2ab\sqrt{1-c^2x^2}\right)+2bcx\left(2acx-b\sqrt{1-c^2x^2}\right)\arccos(cx)+b^2(-1+2c^2x^2)\arccos(cx)^2}{4c^2}$$

```
[In] Integrate[x*(a + b*ArcCos[c*x])^2,x]
```

[Out] (c*x*(2*a^2*c*x - b^2*c*x - 2*a*b*Sqrt[1 - c^2*x^2]) + 2*b*c*x*(2*a*c*x - b*Sqrt[1 - c^2*x^2])*ArcCos[c*x] + b^2*(-1 + 2*c^2*x^2)*ArcCos[c*x]^2 + 2*a*b*ArcSin[c*x])/(4*c^2)

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.54

method	result
parts	$\frac{a^2x^2}{2}+\frac{b^2\left(\frac{c^2x^2\arccos(cx)^2}{2}-\frac{\arccos(cx)\left(cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{2}+\frac{\arccos(cx)^2}{4}-\frac{c^2x^2}{4}+\frac{1}{4}\right)}{c^2}+\frac{2ab\left(\frac{c^2x^2\arccos(cx)}{2}-cx\ldots\right.}{c^2}$
derivativedivides	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2\arccos(cx)^2}{2}-\frac{\arccos(cx)\left(cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{2}+\frac{\arccos(cx)^2}{4}-\frac{c^2x^2}{4}+\frac{1}{4}\right)+2ab\left(\frac{c^2x^2\arccos(cx)}{2}-\frac{cx\sqrt{-c\ldots}}{}\right.}{c^2}$
default	$\frac{\frac{c^2x^2a^2}{2}+b^2\left(\frac{c^2x^2\arccos(cx)^2}{2}-\frac{\arccos(cx)\left(cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{2}+\frac{\arccos(cx)^2}{4}-\frac{c^2x^2}{4}+\frac{1}{4}\right)+2ab\left(\frac{c^2x^2\arccos(cx)}{2}-\frac{cx\sqrt{-c\ldots}}{}\right.}{c^2}$

[In] int(x*(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)

[Out] 1/2*a^2*x^2+b^2/c^2*(1/2*c^2*x^2*arccos(c*x)^2-1/2*arccos(c*x)*(c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x))+1/4*arccos(c*x)^2-1/4*c^2*x^2+1/4)+2*a*b/c^2*(1/2*c^2*x^2*arccos(c*x)-1/4*c*x*(-c^2*x^2+1)^(1/2)+1/4*arcsin(c*x))

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.30

$$\int x(a+b\arccos(cx))^2\,dx$$
$$=\frac{(2a^2-b^2)c^2x^2+(2b^2c^2x^2-b^2)\arccos(cx)^2+2(2abc^2x^2-ab)\arccos(cx)-2(b^2cx\arccos(cx)+abcx)\sqrt{\ldots}}{4c^2}$$

[In] integrate(x*(a+b*arccos(c*x))^2,x, algorithm="fricas")

[Out] 1/4*((2*a^2 - b^2)*c^2*x^2 + (2*b^2*c^2*x^2 - b^2)*arccos(c*x)^2 + 2*(2*a*b*c^2*x^2 - a*b)*arccos(c*x) - 2*(b^2*c*x*arccos(c*x) + a*b*c*x)*sqrt(-c^2*x^2 + 1))/c^2

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(65) = 130.

Time = 0.28 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.72

$$\int x(a+b\arccos(cx))^2\,dx$$
$$=\begin{cases}\frac{a^2x^2}{2}+abx^2\operatorname{acos}(cx)-\frac{abx\sqrt{-c^2x^2+1}}{2c}-\frac{ab\operatorname{acos}(cx)}{2c^2}+\frac{b^2x^2\operatorname{acos}^2(cx)}{2}-\frac{b^2x^2}{4}-\frac{b^2x\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{2c}-\frac{b^2\operatorname{acos}^2(cx)}{4c^2}\\ \frac{x^2\left(a+\frac{\pi b}{2}\right)^2}{2}\end{cases}$$

```
[In] integrate(x*(a+b*acos(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x**2/2 + a*b*x**2*acos(c*x) - a*b*x*sqrt(-c**2*x**2 + 1)/(2
*c) - a*b*acos(c*x)/(2*c**2) + b**2*x**2*acos(c*x)**2/2 - b**2*x**2/4 - b**
2*x*sqrt(-c**2*x**2 + 1)*acos(c*x)/(2*c) - b**2*acos(c*x)**2/(4*c**2), Ne(c
, 0)), (x**2*(a + pi*b/2)**2/2, True))
```

Maxima [F]

$$\int x(a+b\arccos(cx))^2\,dx=\int(b\arccos(cx)+a)^2x\,dx$$

```
[In] integrate(x*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*x^2 + 1/2*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin
(c*x)/c^3))*a*b + 1/2*(x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 - 2
*c*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x)/(c^2*x^2 - 1), x))*b^2
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.57

$$\int x(a+b\arccos(cx))^2\,dx=\frac{1}{2}b^2x^2\arccos(cx)^2+abx^2\arccos(cx)+\frac{1}{2}a^2x^2-\frac{1}{4}b^2x^2-\frac{\sqrt{-c^2x^2+1}b^2x\arccos(cx)}{2c}-\frac{\sqrt{-c^2x^2+1}abx}{2c}-\frac{b^2\arccos(cx)^2}{4c^2}-\frac{ab\arccos(cx)}{2c^2}+\frac{b^2}{8c^2}$$

```
[In] integrate(x*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

[Out] 1/2*b^2*x^2*arccos(c*x)^2 + a*b*x^2*arccos(c*x) + 1/2*a^2*x^2 - 1/4*b^2*x^2 - 1/2*sqrt(-c^2*x^2 + 1)*b^2*x*arccos(c*x)/c - 1/2*sqrt(-c^2*x^2 + 1)*a*b*x/c - 1/4*b^2*arccos(c*x)^2/c^2 - 1/2*a*b*arccos(c*x)/c^2 + 1/8*b^2/c^2

Mupad [F(-1)]

Timed out.

$$\int x(a+b\arccos(cx))^2\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))^2\,dx$$

[In] int(x*(a + b*acos(c*x))^2,x)

[Out] int(x*(a + b*acos(c*x))^2, x)

3.150 $\int (a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int (a+b\arccos(cx))^2\,dx = -2b^2x - \frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c} + x(a+b\arccos(cx))^2$$

```
[Out] -2*b^2*x+x*(a+b*arccos(c*x))^2-2*b*(a+b*arccos(c*x))*(-c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {4716, 4768, 8}

$$\int (a+b\arccos(cx))^2\,dx = -\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c} + x(a+b\arccos(cx))^2 - 2b^2x$$

```
[In] Int[(a + b*ArcCos[c*x])^2,x]
```

```
[Out] -2*b^2*x - (2*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x]))/c + x*(a + b*ArcCos[
c*x])^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x(a+b\arccos(cx))^2+(2bc)\int\frac{x(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}+x(a+b\arccos(cx))^2-\left(2b^2\right)\int 1\,dx\\
&= -2b^2x-\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))}{c}+x(a+b\arccos(cx))^2
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\begin{aligned}
\int(a+b\arccos(cx))^2\,dx &= \left(a^2-2b^2\right)x-\frac{2ab\sqrt{1-c^2x^2}}{c}\\
&\quad+\frac{2b\left(acx-b\sqrt{1-c^2x^2}\right)\arccos(cx)}{c}+b^2x\arccos(cx)^2
\end{aligned}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^2,x]
[Out] (a^2 - 2*b^2)*x - (2*a*b*Sqrt[1 - c^2*x^2])/c + (2*b*(a*c*x - b*Sqrt[1 - c^
2*x^2])*ArcCos[c*x])/c + b^2*x*ArcCos[c*x]^2
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{cx\,a^2+b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	74
default	$\frac{cx\,a^2+b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)+2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	74
parts	$a^2x+\frac{b^2\left(\arccos(cx)^2cx-2cx-2\arccos(cx)\sqrt{-c^2x^2+1}\right)}{c}+\frac{2ab\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}$	75

```
[In] int((a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*x*a^2+b^2*(arccos(c*x)^2*c*x-2*c*x-2*arccos(c*x)*(-c^2*x^2+1)^(1/2))
+2*a*b*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.38

$$\int (a+b\arccos(cx))^2\,dx = \frac{b^2cx\arccos{(cx)}^2+2\,abcx\arccos{(cx)}+(a^2-2\,b^2)cx-2\,\sqrt{-c^2x^2+1}(b^2\arccos{(cx)}+ab)}{c}$$

```
[In] integrate((a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] (b^2*c*x*arccos(c*x)^2 + 2*a*b*c*x*arccos(c*x) + (a^2 - 2*b^2)*c*x - 2*sqrt
(-c^2*x^2 + 1)*(b^2*arccos(c*x) + a*b))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(42) = 84.

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.85

$$\int (a+b\arccos(cx))^2\,dx = \begin{cases} a^2x+2abx\operatorname{acos}{(cx)}-\frac{2ab\sqrt{-c^2x^2+1}}{c}+b^2x\operatorname{acos}^2{(cx)}-2b^2x-\frac{2b^2\sqrt{-c^2x^2+1}\operatorname{acos}{(cx)}}{c} & \text{for } c\neq 0 \\ x\left(a+\frac{\pi b}{2}\right)^2 & \text{otherwise} \end{cases}$$

```
[In] integrate((a+b*acos(c*x))**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*x*acos(c*x) - 2*a*b*sqrt(-c**2*x**2 + 1)/c + b**2
*x*acos(c*x)**2 - 2*b**2*x - 2*b**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/c, Ne(c,
 0)), (x*(a + pi*b/2)**2, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int (a+b\arccos(cx))^2\,dx = b^2x\arccos(cx)^2 - 2\,b^2\left(x + \frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right) + a^2x + \frac{2\left(cx\arccos(cx) - \sqrt{-c^2x^2+1}\right)ab}{c}$$

```
[In] integrate((a+b*arccos(c*x))^2,x, algorithm="maxima")
[Out] b^2*x*arccos(c*x)^2 - 2*b^2*(x + sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^2*x
+ 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))*a*b/c
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

$$\int (a+b\arccos(cx))^2\,dx = b^2x\arccos(cx)^2 + 2\,abx\arccos(cx) + a^2x - 2\,b^2x - \frac{2\sqrt{-c^2x^2+1}b^2\arccos(cx)}{c} - \frac{2\sqrt{-c^2x^2+1}ab}{c}$$

```
[In] integrate((a+b*arccos(c*x))^2,x, algorithm="giac")
[Out] b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) + a^2*x - 2*b^2*x - 2*sqrt(-c^2*x
^2 + 1)*b^2*arccos(c*x)/c - 2*sqrt(-c^2*x^2 + 1)*a*b/c
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.04

$$\int (a+b\arccos(cx))^2\,dx = \begin{cases} x\left(a^2+\pi\,a\,b+\frac{\pi^2\,b^2}{4}\right) & \text{if } c=0 \\ a^2\,x+b^2\,x\left(\mathrm{acos}(c\,x)^2-2\right)-\frac{2\,b^2\,\mathrm{acos}(c\,x)\,\sqrt{1-c^2\,x^2}}{c}-\frac{2\,a\,b\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)}{c} & \text{if } c\neq 0 \end{cases}$$

```
[In] int((a + b*acos(c*x))^2,x)
[Out] piecewise(c == 0, x*(a^2 + (b^2*pi^2)/4 + a*b*pi), c ~= 0, a^2*x + b^2*x*(a
cos(c*x)^2 - 2) - (2*b^2*acos(c*x)*(- c^2*x^2 + 1)^(1/2))/c - (2*a*b*((- c^
2*x^2 + 1)^(1/2) - c*x*acos(c*x)))/c)
```

3.151 $\int \frac{(a+b\arccos(cx))^2}{x} dx$

Optimal result

Integrand size = 14, antiderivative size = 92

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x} dx = &-\frac{i(a+b\arccos(cx))^3}{3b} + (a+b\arccos(cx))^2 \log\left(1+e^{2i\arccos(cx)}\right) \\ &- ib(a+b\arccos(cx))\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) \\ &+ \frac{1}{2}b^2 \operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)\end{aligned}$$

```
[Out] -1/3*I*(a+b*arccos(c*x))^3/b+(a+b*arccos(c*x))^2*ln(1+(c*x+I*(-c^2*x^2+1)^(
1/2))^2)-I*b*(a+b*arccos(c*x))*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2
*b^2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4722, 3800, 2221, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^2}{x} dx = &-ib \operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx)) \\ &- \frac{i(a+b\arccos(cx))^3}{3b} + \log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2 \\ &+ \frac{1}{2}b^2 \operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)\end{aligned}$$

```
[In] Int[(a + b*ArcCos[c*x])^2/x,x]
```

```
[Out] ((-1/3*I)*(a + b*ArcCos[c*x])^3)/b + (a + b*ArcCos[c*x])^2*Log[1 + E^((2*I)
*ArcCos[c*x])] - I*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])]
 + (b^2*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/2
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\text{integral} = -\text{Subst}\left(\int (a+bx)^2 \tan(x)\,dx, x, \arccos(cx)\right)$$

$$
\begin{aligned}
&= -\frac{i(a+b\arccos(cx))^3}{3b} + 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^2}{1+e^{2ix}}\,dx, x, \arccos(cx)\right) \\
&= -\frac{i(a+b\arccos(cx))^3}{3b} + (a+b\arccos(cx))^2\log\left(1+e^{2i\arccos(cx)}\right) \\
&\quad - (2b)\text{Subst}\left(\int (a+bx)\log\left(1+e^{2ix}\right)\,dx, x, \arccos(cx)\right) \\
&= -\frac{i(a+b\arccos(cx))^3}{3b} + (a+b\arccos(cx))^2\log\left(1+e^{2i\arccos(cx)}\right) \\
&\quad - ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) \\
&\quad + \left(ib^2\right)\text{Subst}\left(\int \text{PolyLog}\left(2,-e^{2ix}\right)\,dx, x, \arccos(cx)\right) \\
&= -\frac{i(a+b\arccos(cx))^3}{3b} + (a+b\arccos(cx))^2\log\left(1+e^{2i\arccos(cx)}\right) \\
&\quad - ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) \\
&\quad + \frac{1}{2}b^2\text{Subst}\left(\int \frac{\text{PolyLog}(2,-x)}{x}\,dx, x, e^{2i\arccos(cx)}\right) \\
&= -\frac{i(a+b\arccos(cx))^3}{3b} + (a+b\arccos(cx))^2\log\left(1+e^{2i\arccos(cx)}\right) \\
&\quad - ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) + \frac{1}{2}b^2\,\text{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.39

$$
\begin{aligned}
\int \frac{(a+b\arccos(cx))^2}{x}\,dx &= -iab\arccos(cx)^2 - \frac{1}{3}ib^2\arccos(cx)^3 \\
&\quad + 2ab\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right) \\
&\quad + b^2\arccos(cx)^2\log\left(1+e^{2i\arccos(cx)}\right) + a^2\log(cx) \\
&\quad - ib(a+b\arccos(cx))\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) \\
&\quad + \frac{1}{2}b^2\,\text{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)
\end{aligned}
$$

```
[In] Integrate[(a + b*ArcCos[c*x])^2/x,x]
```

```
[Out] (-I)*a*b*ArcCos[c*x]^2 - (I/3)*b^2*ArcCos[c*x]^3 + 2*a*b*ArcCos[c*x]*Log[1
+ E^((2*I)*ArcCos[c*x])] + b^2*ArcCos[c*x]^2*Log[1 + E^((2*I)*ArcCos[c*x])]
 + a^2*Log[c*x] - I*b*(a + b*ArcCos[c*x])*PolyLog[2, -E^((2*I)*ArcCos[c*x])
] + (b^2*PolyLog[3, -E^((2*I)*ArcCos[c*x])])/2
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.01

method	result
parts	$a^{2} \ln \left(x \right)+b^{2} \left(-\frac{i \arccos \left(c x \right)^{3}}{3}+\arccos \left(c x \right)^{2} \ln \left(1+\left(c x +i \sqrt{-c^{2} x^{2}+1}\right)^{2}\right)-i \arccos \left(c x \right)\right.$
derivativedivides	$a^{2} \ln \left(c x \right)+b^{2} \left(-\frac{i \arccos \left(c x \right)^{3}}{3}+\arccos \left(c x \right)^{2} \ln \left(1+\left(c x +i \sqrt{-c^{2} x^{2}+1}\right)^{2}\right)-i \arccos \left(c x\right)\right.$
default	$a^{2} \ln \left(c x \right)+b^{2} \left(-\frac{i \arccos \left(c x \right)^{3}}{3}+\arccos \left(c x \right)^{2} \ln \left(1+\left(c x +i \sqrt{-c^{2} x^{2}+1}\right)^{2}\right)-i \arccos \left(c x\right)\right.$

```
[In] int((a+b*arccos(c*x))^2/x,x,method=_RETURNVERBOSE)
[Out] a^2*ln(x)+b^2*(-1/3*I*arccos(c*x)^3+arccos(c*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)^
(1/2))^2)-I*arccos(c*x)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*polylo
g(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))-I*a*b*arccos(c*x)^2-I*a*b*polylog(2,-(c
*x+I*(-c^2*x^2+1)^(1/2))^2)+2*a*b*arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2
))^2)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx = \int \frac{\left(b\arccos\left(cx\right)+a\right)^2}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/x,x, algorithm="fricas")
[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/x, x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx = \int \frac{\left(a+b\operatorname{acos}\left(cx\right)\right)^2}{x}\,dx$$

```
[In] integrate((a+b*acos(c*x))**2/x,x)
[Out] Integral((a + b*acos(c*x))**2/x, x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(b\arccos(cx)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/x,x, algorithm="maxima")
```

```
[Out] a^2*log(x) + integrate((b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 +
2*a*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x))/x, x)
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(b\arccos(cx)+a)^2}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^2/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{x}\,dx$$

```
[In] int((a + b*acos(c*x))^2/x,x)
```

```
[Out] int((a + b*acos(c*x))^2/x, x)
```

3.152 $\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx = -\frac{(a+b\arccos(cx))^2}{x} - 4ibc(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right) + 2ib^2c\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right) - 2ib^2c\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)$$

```
[Out] -(a+b*arccos(c*x))^2/x-4*I*b*c*(a+b*arccos(c*x))*arctan(c*x+I*(-c^2*x^2+1)^
(1/2))+2*I*b^2*c*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))-2*I*b^2*c*polylog
(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))
```

Rubi [A] (verified)

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4724, 4804, 4266, 2317, 2438}

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx = -4ibc\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx)) - \frac{(a+b\arccos(cx))^2}{x} + 2ib^2c\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right) - 2ib^2c\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)$$

```
[In] Int[(a + b*ArcCos[c*x])^2/x^2,x]
```

```
[Out] -((a + b*ArcCos[c*x])^2/x) - (4*I)*b*c*(a + b*ArcCos[c*x])*ArcTan[E^(I*ArcC
os[c*x])] + (2*I)*b^2*c*PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - (2*I)*b^2*c*Po
lyLog[2, I*E^(I*ArcCos[c*x])]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
```

```
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\frac{(a+b\arccos(cx))^2}{x} - (2bc)\int \frac{a+b\arccos(cx)}{x\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{(a+b\arccos(cx))^2}{x} + (2bc)\text{Subst}\left(\int (a+bx)\sec(x)\,dx, x, \arccos(cx)\right) \\
&= -\frac{(a+b\arccos(cx))^2}{x} - 4ibc(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right) \\
&\quad - \left(2b^2c\right)\text{Subst}\left(\int \log\left(1-ie^{ix}\right)\,dx, x, \arccos(cx)\right) \\
&\quad + \left(2b^2c\right)\text{Subst}\left(\int \log\left(1+ie^{ix}\right)\,dx, x, \arccos(cx)\right)
\end{aligned}
$$

$$= -\frac{(a+b\arccos(cx))^2}{x} - 4ibc(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right) + \left(2ib^2c\right)\text{Subst}\left(\int\frac{\log(1-ix)}{x}\,dx,x,e^{i\arccos(cx)}\right) - \left(2ib^2c\right)\text{Subst}\left(\int\frac{\log(1+ix)}{x}\,dx,x,e^{i\arccos(cx)}\right)$$

$$= -\frac{(a+b\arccos(cx))^2}{x} - 4ibc(a+b\arccos(cx))\arctan\left(e^{i\arccos(cx)}\right) + 2ib^2c\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right) - 2ib^2c\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.51

$$\int\frac{(a+b\arccos(cx))^2}{x^2}\,dx = -\frac{a^2+2ab\left(\arccos(cx)-cx\text{arctanh}\left(\sqrt{1-c^2x^2}\right)\right)+b^2\left(\arccos(cx)^2-2cx\left(\arccos(cx)\left(\log\left(1-ie^{i\arccos(cx)}\right)\right.\right.\right.}{x}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^2/x^2,x]
```

```
[Out] -((a^2 + 2*a*b*(ArcCos[c*x] - c*x*ArcTanh[Sqrt[1 - c^2*x^2]]) + b^2*(ArcCos
[c*x]^2 - 2*c*x*(ArcCos[c*x]*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^(I
*ArcCos[c*x])]) + I*(PolyLog[2, (-I)*E^(I*ArcCos[c*x])] - PolyLog[2, I*E^(I
*ArcCos[c*x])]))))/x)
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.01

method	result
parts	$-\frac{a^2}{x}+b^2c\Big(-\frac{\arccos(cx)^2}{cx}-2\arccos(cx)\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)+2\arccos(cx)\ln\left(1\right.$
derivativedivides	$c\Big(-\frac{a^2}{cx}+b^2\Big(-\frac{\arccos(cx)^2}{cx}-2\arccos(cx)\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)+2\arccos(cx)\ln$
default	$c\Big(-\frac{a^2}{cx}+b^2\Big(-\frac{\arccos(cx)^2}{cx}-2\arccos(cx)\ln\left(1+i\left(cx+i\sqrt{-c^2x^2+1}\right)\right)+2\arccos(cx)\ln$

```
[In] int((a+b*arccos(c*x))^2/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/x+b^2*c*(-arccos(c*x)^2/c/x-2*arccos(c*x)*ln(1+I*(c*x+I*(-c^2*x^2+1)^(
1/2)))+2*arccos(c*x)*ln(1-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+2*I*dilog(1+I*(c*x+
I*(-c^2*x^2+1)^(1/2)))-2*I*dilog(1-I*(c*x+I*(-c^2*x^2+1)^(1/2))))+2*a*b*c*(
-1/c/x*arccos(c*x)+arctanh(1/(-c^2*x^2+1)^(1/2)))
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/x^2,x, algorithm="fricas")
[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)/x^2, x)
```

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{(a+b\operatorname{acos}{(cx)})^2}{x^2}\,dx$$

```
[In] integrate((a+b*acos(c*x))**2/x**2,x)
[Out] Integral((a + b*acos(c*x))**2/x**2, x)
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/x^2,x, algorithm="maxima")
[Out] 2*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*a*b + (2*
c*x*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x)/(c^2*x^3 - x), x) - arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2
)*b^2/x - a^2/x
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx=\int \frac{(b\arccos{(cx)}+a)^2}{x^2}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/x^2,x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^2/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{x^2}\,dx = \int \frac{\left(a+b\,\mathrm{acos}(c\,x)\right)^2}{x^2}\,dx$$

```
[In] int((a + b*acos(c*x))^2/x^2,x)
[Out] int((a + b*acos(c*x))^2/x^2, x)
```

3.153 $\int x^2(a+b\arccos(cx))^3\,dx$

Optimal result

Integrand size = 14, antiderivative size = 178

$$\int x^2(a+b\arccos(cx))^3\,dx = -\frac{4ab^2x}{3c^2}+\frac{14b^3\sqrt{1-c^2x^2}}{9c^3}-\frac{2b^3(1-c^2x^2)^{3/2}}{27c^3}-\frac{4b^3x\arccos(cx)}{3c^2} -\frac{2}{9}b^2x^3(a+b\arccos(cx))-\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^3} -\frac{bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c}+\frac{1}{3}x^3(a+b\arccos(cx))^3$$

```
[Out] -4/3*a*b^2*x/c^2-2/27*b^3*(-c^2*x^2+1)^(3/2)/c^3-4/3*b^3*x*arccos(c*x)/c^2-
2/9*b^2*x^3*(a+b*arccos(c*x))+1/3*x^3*(a+b*arccos(c*x))^3+14/9*b^3*(-c^2*x^
2+1)^(1/2)/c^3-2/3*b*(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2)/c^3-1/3*b*x^2*(
a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4724, 4796, 4768, 4716, 267, 272, 45}

$$\int x^2(a+b\arccos(cx))^3\,dx = -\frac{2}{9}b^2x^3(a+b\arccos(cx))-\frac{bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c} -\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^3}+\frac{1}{3}x^3(a+b\arccos(cx))^3 -\frac{4ab^2x}{3c^2}-\frac{4b^3x\arccos(cx)}{3c^2}-\frac{2b^3(1-c^2x^2)^{3/2}}{27c^3}+\frac{14b^3\sqrt{1-c^2x^2}}{9c^3}$$

```
[In] Int[x^2*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (-4*a*b^2*x)/(3*c^2) + (14*b^3*Sqrt[1 - c^2*x^2])/(9*c^3) - (2*b^3*(1 - c^2
*x^2)^(3/2))/(27*c^3) - (4*b^3*x*ArcCos[c*x])/(3*c^2) - (2*b^2*x^3*(a + b*A
rcCos[c*x]))/9 - (2*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(3*c^3) - (b
*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(3*c) + (x^3*(a + b*ArcCos[c*
x])^3)/3
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{3}x^3(a+b\arccos(cx))^3+(bc)\int\frac{x^3(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c}+\frac{1}{3}x^3(a+b\arccos(cx))^3\\
&\quad -\frac{1}{3}\left(2b^2\right)\int x^2(a+b\arccos(cx))\,dx+\frac{(2b)\int\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\,dx}{3c}\\
&= -\frac{2}{9}b^2x^3(a+b\arccos(cx))-\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^3}\\
&\quad -\frac{bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c}+\frac{1}{3}x^3(a+b\arccos(cx))^3\\
&\quad -\frac{\left(4b^2\right)\int(a+b\arccos(cx))\,dx}{3c^2}-\frac{1}{9}\left(2b^3c\right)\int\frac{x^3}{\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{4ab^2x}{3c^2}-\frac{2}{9}b^2x^3(a+b\arccos(cx))-\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^3}\\
&\quad -\frac{bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c}+\frac{1}{3}x^3(a+b\arccos(cx))^3\\
&\quad -\frac{\left(4b^3\right)\int\arccos(cx)\,dx}{3c^2}-\frac{1}{9}\left(b^3c\right)\text{Subst}\left(\int\frac{x}{\sqrt{1-c^2x}}\,dx,x,x^2\right)\\
&= -\frac{4ab^2x}{3c^2}-\frac{4b^3x\arccos(cx)}{3c^2}-\frac{2}{9}b^2x^3(a+b\arccos(cx))\\
&\quad -\frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^3}-\frac{bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c}\\
&\quad +\frac{1}{3}x^3(a+b\arccos(cx))^3-\frac{\left(4b^3\right)\int\frac{x}{\sqrt{1-c^2x^2}}\,dx}{3c}\\
&\quad -\frac{1}{9}\left(b^3c\right)\text{Subst}\left(\int\left(\frac{1}{c^2\sqrt{1-c^2x}}-\frac{\sqrt{1-c^2x}}{c^2}\right)\,dx,x,x^2\right)
\end{aligned}$$

$$
\begin{aligned}
&= -\frac{4ab^2x}{3c^2} + \frac{14b^3\sqrt{1-c^2x^2}}{9c^3} - \frac{2b^3(1-c^2x^2)^{3/2}}{27c^3} - \frac{4b^3x\arccos(cx)}{3c^2} \\
&\quad - \frac{2}{9}b^2x^3(a+b\arccos(cx)) - \frac{2b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c^3} \\
&\qquad - \frac{bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{3c} + \frac{1}{3}x^3(a+b\arccos(cx))^3
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.22

$$
\int x^2(a+b\arccos(cx))^3\,dx
$$

$$
= \frac{9a^3c^3x^3 - 9a^2b\sqrt{1-c^2x^2}(2+c^2x^2) - 6ab^2cx(6+c^2x^2) + 2b^3\sqrt{1-c^2x^2}(20+c^2x^2) - 3b(-9a^2c^3x^3 + 6ab\sqrt{}}{}
$$

```
[In] Integrate[x^2*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (9*a^3*c^3*x^3 - 9*a^2*b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2) - 6*a*b^2*c*x*(6 +
 c^2*x^2) + 2*b^3*Sqrt[1 - c^2*x^2]*(20 + c^2*x^2) - 3*b*(-9*a^2*c^3*x^3 +
6*a*b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2) + 2*b^2*c*x*(6 + c^2*x^2))*ArcCos[c*x
] - 9*b^2*(-3*a*c^3*x^3 + b*Sqrt[1 - c^2*x^2]*(2 + c^2*x^2))*ArcCos[c*x]^2
+ 9*b^3*c^3*x^3*ArcCos[c*x]^3)/(27*c^3)
```

Maple [A] (verified)

Time = 2.37 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{\frac{a^3c^3x^3}{3}+b^3\left(\frac{c^3x^3\arccos(cx)^3}{3}-\frac{\arccos(cx)^2\left(c^2x^2+2\right)\sqrt{-c^2x^2+1}}{3}+\frac{4\sqrt{-c^2x^2+1}}{3}-\frac{4cx\arccos(cx)}{3}-\frac{2c^3x^3\arccos(cx)}{9}+\frac{2\left(c^2x^2+2\right)}{27}\right.}{}$
default	$\frac{\frac{a^3c^3x^3}{3}+b^3\left(\frac{c^3x^3\arccos(cx)^3}{3}-\frac{\arccos(cx)^2\left(c^2x^2+2\right)\sqrt{-c^2x^2+1}}{3}+\frac{4\sqrt{-c^2x^2+1}}{3}-\frac{4cx\arccos(cx)}{3}-\frac{2c^3x^3\arccos(cx)}{9}+\frac{2\left(c^2x^2+2\right)}{27}\right.}{}$
parts	$\frac{a^3x^3}{3}+\frac{b^3\left(\frac{c^3x^3\arccos(cx)^3}{3}-\frac{\arccos(cx)^2\left(c^2x^2+2\right)\sqrt{-c^2x^2+1}}{3}+\frac{4\sqrt{-c^2x^2+1}}{3}-\frac{4cx\arccos(cx)}{3}-\frac{2c^3x^3\arccos(cx)}{9}+\frac{2\left(c^2x^2+2\right)}{2}\right.}{c^3}$

```
[In] int(x^2*(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/3*a^3*c^3*x^3+b^3*(1/3*c^3*x^3*arccos(c*x)^3-1/3*arccos(c*x)^2*(c^
2*x^2+2)*(-c^2*x^2+1)^(1/2)+4/3*(-c^2*x^2+1)^(1/2)-4/3*c*x*arccos(c*x)-2/9*
c^3*x^3*arccos(c*x)+2/27*(c^2*x^2+2)*(-c^2*x^2+1)^(1/2))+3*a*b^2*(1/3*arcco
s(c*x)^2*c^3*x^3-2/9*arccos(c*x)*(c^2*x^2+2)*(-c^2*x^2+1)^(1/2)-2/27*c^3*x^
```

```
3-4/9*c*x)+3*a^2*b*(1/3*c^3*x^3*arccos(c*x)-1/9*c^2*x^2*(-c^2*x^2+1)^(1/2)-
2/9*(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10

$$\int x^2(a+b\arccos(cx))^3\,dx$$
$$= \frac{9\,b^3c^3x^3\arccos{(cx)}^3+27\,ab^2c^3x^3\arccos{(cx)}^2+3\,(3\,a^3-2\,ab^2)c^3x^3-36\,ab^2cx+3\,((9\,a^2b-2\,b^3)c^3x^3-1}{}$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] 1/27*(9*b^3*c^3*x^3*arccos(c*x)^3 + 27*a*b^2*c^3*x^3*arccos(c*x)^2 + 3*(3*a
^3 - 2*a*b^2)*c^3*x^3 - 36*a*b^2*c*x + 3*((9*a^2*b - 2*b^3)*c^3*x^3 - 12*b^
3*c*x)*arccos(c*x) - ((9*a^2*b - 2*b^3)*c^2*x^2 + 18*a^2*b - 40*b^3 + 9*(b^
3*c^2*x^2 + 2*b^3)*arccos(c*x)^2 + 18*(a*b^2*c^2*x^2 + 2*a*b^2)*arccos(c*x)
)*sqrt(-c^2*x^2 + 1))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.87

$$\int x^2(a+b\arccos(cx))^3\,dx$$
$$= \begin{cases} \frac{a^3x^3}{3}+a^2bx^3\operatorname{acos}{(cx)}-\frac{a^2bx^2\sqrt{-c^2x^2+1}}{3c}-\frac{2a^2b\sqrt{-c^2x^2+1}}{3c^3}+ab^2x^3\operatorname{acos}^2{(cx)}-\frac{2ab^2x^3}{9}-\frac{2ab^2x^2\sqrt{-c^2x^2+1}\operatorname{acos}{(cx)}}{3c} \\ \frac{x^3\left(a+\frac{\pi b}{2}\right)^3}{3} \end{cases}$$

```
[In] integrate(x**2*(a+b*acos(c*x))**3,x)
```

```
[Out] Piecewise((a**3*x**3/3 + a**2*b*x**3*acos(c*x) - a**2*b*x**2*sqrt(-c**2*x**
2 + 1)/(3*c) - 2*a**2*b*sqrt(-c**2*x**2 + 1)/(3*c**3) + a*b**2*x**3*acos(c*
x)**2 - 2*a*b**2*x**3/9 - 2*a*b**2*x**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(3*c
) - 4*a*b**2*x/(3*c**2) - 4*a*b**2*sqrt(-c**2*x**2 + 1)*acos(c*x)/(3*c**3)
+ b**3*x**3*acos(c*x)**3/3 - 2*b**3*x**3*acos(c*x)/9 - b**3*x**2*sqrt(-c**2
*x**2 + 1)*acos(c*x)**2/(3*c) + 2*b**3*x**2*sqrt(-c**2*x**2 + 1)/(27*c) - 4
*b**3*x*acos(c*x)/(3*c**2) - 2*b**3*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/(3*c*
*3) + 40*b**3*sqrt(-c**2*x**2 + 1)/(27*c**3), Ne(c, 0)), (x**3*(a + pi*b/2)
**3/3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.53

$$\int x^2(a+b\arccos(cx))^3\,dx = \frac{1}{3}\,b^3x^3\arccos{(cx)}^3 + ab^2x^3\arccos{(cx)}^2 + \frac{1}{3}\,a^3x^3 + \frac{1}{3}\left(3\,x^3\arccos{(cx)} - c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\right)a^2b - \frac{2}{9}\left(3\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\arccos{(cx)} + \frac{c^2x^3+6\,x}{c^2}\right)ab^2 - \frac{1}{27}\left(9\,c\left(\frac{\sqrt{-c^2x^2+1}x^2}{c^2} + \frac{2\,\sqrt{-c^2x^2+1}}{c^4}\right)\arccos{(cx)}^2 - 2\,c\left(\frac{\sqrt{-c^2x^2+1}x^2 + \frac{20\,\sqrt{-c^2x^2+1}}{c^2}}{c^2} - \frac{3\,(c^2x^3+\right.\right.$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/3*b^3*x^3*arccos(c*x)^3 + a*b^2*x^3*arccos(c*x)^2 + 1/3*a^3*x^3 + 1/3*(3*
x^3*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)
)*a^2*b - 2/9*(3*c*(sqrt(-c^2*x^2 + 1)*x^2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*
arccos(c*x) + (c^2*x^3 + 6*x)/c^2)*a*b^2 - 1/27*(9*c*(sqrt(-c^2*x^2 + 1)*x^
2/c^2 + 2*sqrt(-c^2*x^2 + 1)/c^4)*arccos(c*x)^2 - 2*c*((sqrt(-c^2*x^2 + 1)*
x^2 + 20*sqrt(-c^2*x^2 + 1)/c^2)/c^2 - 3*(c^2*x^3 + 6*x)*arccos(c*x)/c^3))*
b^3
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.62

$$\begin{aligned}\int x^2(a+b\arccos(cx))^3\,dx = {} & \frac{1}{3}\,b^3x^3\arccos{(cx)}^3 + ab^2x^3\arccos{(cx)}^2 + a^2bx^3\arccos{(cx)} \\ & - \frac{2}{9}\,b^3x^3\arccos{(cx)} - \frac{\sqrt{-c^2x^2+1}b^3x^2\arccos{(cx)}^2}{3\,c} \\ & + \frac{1}{3}\,a^3x^3 - \frac{2}{9}\,ab^2x^3 - \frac{2\,\sqrt{-c^2x^2+1}ab^2x^2\arccos{(cx)}}{3\,c} \\ & - \frac{\sqrt{-c^2x^2+1}a^2bx^2}{3\,c} + \frac{2\,\sqrt{-c^2x^2+1}b^3x^2}{27\,c} \\ & - \frac{4\,b^3x\arccos{(cx)}}{3\,c^2} - \frac{2\,\sqrt{-c^2x^2+1}b^3\arccos{(cx)}^2}{3\,c^3} \\ & - \frac{4\,ab^2x}{3\,c^2} - \frac{4\,\sqrt{-c^2x^2+1}ab^2\arccos{(cx)}}{3\,c^3} \\ & - \frac{2\,\sqrt{-c^2x^2+1}a^2b}{3\,c^3} + \frac{40\,\sqrt{-c^2x^2+1}b^3}{27\,c^3}\end{aligned}$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] 1/3*b^3*x^3*arccos(c*x)^3 + a*b^2*x^3*arccos(c*x)^2 + a^2*b*x^3*arccos(c*x)
 - 2/9*b^3*x^3*arccos(c*x) - 1/3*sqrt(-c^2*x^2 + 1)*b^3*x^2*arccos(c*x)^2/c
 + 1/3*a^3*x^3 - 2/9*a*b^2*x^3 - 2/3*sqrt(-c^2*x^2 + 1)*a*b^2*x^2*arccos(c*
x)/c - 1/3*sqrt(-c^2*x^2 + 1)*a^2*b*x^2/c + 2/27*sqrt(-c^2*x^2 + 1)*b^3*x^2
/c - 4/3*b^3*x*arccos(c*x)/c^2 - 2/3*sqrt(-c^2*x^2 + 1)*b^3*arccos(c*x)^2/c
^3 - 4/3*a*b^2*x/c^2 - 4/3*sqrt(-c^2*x^2 + 1)*a*b^2*arccos(c*x)/c^3 - 2/3*s
qrt(-c^2*x^2 + 1)*a^2*b/c^3 + 40/27*sqrt(-c^2*x^2 + 1)*b^3/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arccos(cx))^3\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^3\,dx$$

```
[In] int(x^2*(a + b*acos(c*x))^3,x)
```

```
[Out] int(x^2*(a + b*acos(c*x))^3, x)
```

3.154 $\int x(a+b\arccos(cx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 125

$$\int x(a+b\arccos(cx))^3\,dx=\frac{3b^3x\sqrt{1-c^2x^2}}{8c}-\frac{3}{4}b^2x^2(a+b\arccos(cx))-\frac{3bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4c}-\frac{(a+b\arccos(cx))^3}{4c^2}+\frac{1}{2}x^2(a+b\arccos(cx))^3-\frac{3b^3\arcsin(cx)}{8c^2}$$

```
[Out] -3/4*b^2*x^2*(a+b*arccos(c*x))-1/4*(a+b*arccos(c*x))^3/c^2+1/2*x^2*(a+b*arc
cos(c*x))^3-3/8*b^3*arcsin(c*x)/c^2+3/8*b^3*x*(-c^2*x^2+1)^(1/2)/c-3/4*b*x*
(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {4724, 4796, 4738, 327, 222}

$$\int x(a+b\arccos(cx))^3\,dx=-\frac{3}{4}b^2x^2(a+b\arccos(cx))-\frac{3bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4c}-\frac{(a+b\arccos(cx))^3}{4c^2}+\frac{1}{2}x^2(a+b\arccos(cx))^3-\frac{3b^3\arcsin(cx)}{8c^2}+\frac{3b^3x\sqrt{1-c^2x^2}}{8c}$$

```
[In] Int[x*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (3*b^3*x*Sqrt[1 - c^2*x^2])/(8*c) - (3*b^2*x^2*(a + b*ArcCos[c*x]))/4 - (3*
b*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/(4*c) - (a + b*ArcCos[c*x])^3/
(4*c^2) + (x^2*(a + b*ArcCos[c*x])^3)/2 - (3*b^3*ArcSin[c*x])/(8*c^2)
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2(a+b\arccos(cx))^3 + \frac{1}{2}(3bc)\int \frac{x^2(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{3bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4c} + \frac{1}{2}x^2(a+b\arccos(cx))^3 - \frac{1}{2}(3b^2)\int x(a+b\arccos(cx))\,dx + \frac{(3b)\int \frac{(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\,dx}{4c}$$

$$= -\frac{3}{4}b^2x^2(a+b\arccos(cx)) - \frac{3bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4c} - \frac{(a+b\arccos(cx))^3}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^3 - \frac{1}{4}(3b^3c)\int \frac{x^2}{\sqrt{1-c^2x^2}}\,dx$$

$$= \frac{3b^3x\sqrt{1-c^2x^2}}{8c} - \frac{3}{4}b^2x^2(a+b\arccos(cx)) - \frac{3bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4c} - \frac{(a+b\arccos(cx))^3}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^3 - \frac{(3b^3)\int \frac{1}{\sqrt{1-c^2x^2}}\,dx}{8c}$$

$$= \frac{3b^3x\sqrt{1-c^2x^2}}{8c} - \frac{3}{4}b^2x^2(a+b\arccos(cx)) - \frac{3bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{4c} - \frac{(a+b\arccos(cx))^3}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^3 - \frac{3b^3\arcsin(cx)}{8c^2}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.48

$$\int x(a+b\arccos(cx))^3\,dx = \frac{cx\left(4a^3cx-6ab^2cx-6a^2b\sqrt{1-c^2x^2}+3b^3\sqrt{1-c^2x^2}\right)-6bcx\left(-2a^2cx+b^2cx+2ab\sqrt{1-c^2x^2}\right)\arccos(cx)}{8c}$$

```
[In] Integrate[x*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (c*x*(4*a^3*c*x - 6*a*b^2*c*x - 6*a^2*b*Sqrt[1 - c^2*x^2] + 3*b^3*Sqrt[1 -
c^2*x^2]) - 6*b*c*x*(-2*a^2*c*x + b^2*c*x + 2*a*b*Sqrt[1 - c^2*x^2])*ArcCos
[c*x] - 6*b^2*(a - 2*a*c^2*x^2 + b*c*x*Sqrt[1 - c^2*x^2])*ArcCos[c*x]^2 + 2
*b^3*(-1 + 2*c^2*x^2)*ArcCos[c*x]^3 + (6*a^2*b - 3*b^3)*ArcSin[c*x])/(8*c^2
)
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{c^2x^2\arccos(cx)^3}{2}-\frac{3\arccos(cx)^2\left(cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{4}-\frac{3c^2x^2\arccos(cx)}{4}+\frac{3cx\sqrt{-c^2x^2+1}}{8}+\frac{3\arccos(cx)}{8}+\text{ar}\right.}{}$
default	$\frac{\frac{c^2x^2a^3}{2}+b^3\left(\frac{c^2x^2\arccos(cx)^3}{2}-\frac{3\arccos(cx)^2\left(cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{4}-\frac{3c^2x^2\arccos(cx)}{4}+\frac{3cx\sqrt{-c^2x^2+1}}{8}+\frac{3\arccos(cx)}{8}+\text{ar}\right.}{}$
parts	$\frac{a^3x^2}{2}+\frac{b^3\left(\frac{c^2x^2\arccos(cx)^3}{2}-\frac{3\arccos(cx)^2\left(cx\sqrt{-c^2x^2+1}+\arccos(cx)\right)}{4}-\frac{3c^2x^2\arccos(cx)}{4}+\frac{3cx\sqrt{-c^2x^2+1}}{8}+\frac{3\arccos(cx)}{8}+\text{ar}\right.}{c^2}$

```
[In] int(x*(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/2*c^2*x^2*a^3+b^3*(1/2*c^2*x^2*arccos(c*x)^3-3/4*arccos(c*x)^2*(c*
x*(-c^2*x^2+1)^(1/2)+arccos(c*x))-3/4*c^2*x^2*arccos(c*x)+3/8*c*x*(-c^2*x^2
+1)^(1/2)+3/8*arccos(c*x)+1/2*arccos(c*x)^3)+3*a*b^2*(1/2*c^2*x^2*arccos(c*
x)^2-1/2*arccos(c*x)*(c*x*(-c^2*x^2+1)^(1/2)+arccos(c*x))+1/4*arccos(c*x)^2
-1/4*c^2*x^2+1/4)+3*a^2*b*(1/2*c^2*x^2*arccos(c*x)-1/4*c*x*(-c^2*x^2+1)^(1/
2)+1/4*arcsin(c*x)))
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.35

$$\int x(a+b\arccos(cx))^3\,dx$$
$$=\frac{2\,(2\,a^3-3\,ab^2)c^2x^2+2\,(2\,b^3c^2x^2-b^3)\arccos{(cx)}^3+6\,(2\,ab^2c^2x^2-ab^2)\arccos{(cx)}^2+3\,(2\,(2\,a^2b-b^3)c^2}{8\,c^2}$$

```
[In] integrate(x*(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^3 - 3*a*b^2)*c^2*x^2 + 2*(2*b^3*c^2*x^2 - b^3)*arccos(c*x)^3 +
6*(2*a*b^2*c^2*x^2 - a*b^2)*arccos(c*x)^2 + 3*(2*(2*a^2*b - b^3)*c^2*x^2 -
2*a^2*b + b^3)*arccos(c*x) - 3*(2*b^3*c*x*arccos(c*x)^2 + 4*a*b^2*c*x*arcco
s(c*x) + (2*a^2*b - b^3)*c*x)*sqrt(-c^2*x^2 + 1))/c^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(116) = 232.

Time = 0.31 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.15

$$\int x(a+b\arccos(cx))^3\,dx$$
$$=\begin{cases}\frac{a^3x^2}{2}+\frac{3a^2bx^2\,\mathrm{acos}\,(cx)}{2}-\frac{3a^2bx\sqrt{-c^2x^2+1}}{4c}-\frac{3a^2b\,\mathrm{acos}\,(cx)}{4c^2}+\frac{3ab^2x^2\,\mathrm{acos}^2\,(cx)}{2}-\frac{3ab^2x^2}{4}-\frac{3ab^2x\sqrt{-c^2x^2+1}\,\mathrm{acos}\,(cx)}{2c}-\frac{3ab^2\,\mathrm{ac}}{4}\\ \frac{x^2\left(a+\frac{\pi b}{2}\right)^3}{2}\end{cases}$$

```
[In] integrate(x*(a+b*acos(c*x))**3,x)
```

```
[Out] Piecewise((a**3*x**2/2 + 3*a**2*b*x**2*acos(c*x)/2 - 3*a**2*b*x*sqrt(-c**2*
x**2 + 1)/(4*c) - 3*a**2*b*acos(c*x)/(4*c**2) + 3*a*b**2*x**2*acos(c*x)**2/
2 - 3*a*b**2*x**2/4 - 3*a*b**2*x*sqrt(-c**2*x**2 + 1)*acos(c*x)/(2*c) - 3*a
*b**2*acos(c*x)**2/(4*c**2) + b**3*x**2*acos(c*x)**3/2 - 3*b**3*x**2*acos(c
*x)/4 - 3*b**3*x*sqrt(-c**2*x**2 + 1)*acos(c*x)**2/(4*c) + 3*b**3*x*sqrt(-c
**2*x**2 + 1)/(8*c) - b**3*acos(c*x)**3/(4*c**2) + 3*b**3*acos(c*x)/(8*c**2
), Ne(c, 0)), (x**2*(a + pi*b/2)**3/2, True))
```

Maxima [F]

$$\int x(a+b\arccos(cx))^3\,dx=\int(b\arccos{(cx)}+a)^3x\,dx$$

```
[In] integrate(x*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + 1/2*a^3*x^2 + 3/
4*(2*x^2*arccos(c*x) - c*(sqrt(-c^2*x^2 + 1)*x/c^2 - arcsin(c*x)/c^3))*a^2*
b - integrate(3/2*(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^3*c*x^2*arctan2(sqrt(c*x
+ 1)*sqrt(-c*x + 1), c*x)^2 - 2*(a*b^2*c^2*x^3 - a*b^2*x)*arctan2(sqrt(c*x
+ 1)*sqrt(-c*x + 1), c*x)^2)/(c^2*x^2 - 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. 2(109) = 218.

Time = 0.29 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.85

$$\int x(a+b\arccos(cx))^3\,dx = \frac{1}{2}\,b^3x^2\arccos\left(cx\right)^3 + \frac{3}{2}\,ab^2x^2\arccos\left(cx\right)^2 + \frac{3}{2}\,a^2bx^2\arccos\left(cx\right) - \frac{3}{4}\,b^3x^2\arccos\left(cx\right) - \frac{3\,\sqrt{-c^2x^2+1}b^3x\arccos\left(cx\right)^2}{4\,c} + \frac{1}{2}\,a^3x^2 - \frac{3}{4}\,ab^2x^2 - \frac{3\,\sqrt{-c^2x^2+1}ab^2x\arccos\left(cx\right)}{2\,c} - \frac{b^3\arccos\left(cx\right)^3}{4\,c^2} - \frac{3\,\sqrt{-c^2x^2+1}a^2bx}{4\,c} + \frac{3\,\sqrt{-c^2x^2+1}b^3x}{8\,c} - \frac{3\,ab^2\arccos\left(cx\right)^2}{4\,c^2} - \frac{3\,a^2b\arccos\left(cx\right)}{4\,c^2} + \frac{3\,b^3\arccos\left(cx\right)}{8\,c^2} + \frac{3\,ab^2}{8\,c^2}$$

```
[In] integrate(x*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*x^2*arccos(c*x)^3 + 3/2*a*b^2*x^2*arccos(c*x)^2 + 3/2*a^2*b*x^2*arc
cos(c*x) - 3/4*b^3*x^2*arccos(c*x) - 3/4*sqrt(-c^2*x^2 + 1)*b^3*x*arccos(c*
x)^2/c + 1/2*a^3*x^2 - 3/4*a*b^2*x^2 - 3/2*sqrt(-c^2*x^2 + 1)*a*b^2*x*arcco
s(c*x)/c - 1/4*b^3*arccos(c*x)^3/c^2 - 3/4*sqrt(-c^2*x^2 + 1)*a^2*b*x/c + 3
/8*sqrt(-c^2*x^2 + 1)*b^3*x/c - 3/4*a*b^2*arccos(c*x)^2/c^2 - 3/4*a^2*b*arc
cos(c*x)/c^2 + 3/8*b^3*arccos(c*x)/c^2 + 3/8*a*b^2/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\arccos(cx))^3\,dx = \int x\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^3\,dx$$

```
[In] int(x*(a + b*acos(c*x))^3,x)
```

```
[Out] int(x*(a + b*acos(c*x))^3, x)
```

3.155 $\int (a+b\arccos(cx))^3\,dx$

Optimal result

Integrand size = 10, antiderivative size = 82

$$\int (a+b\arccos(cx))^3\,dx = -6ab^2x+\frac{6b^3\sqrt{1-c^2x^2}}{c}-6b^3x\arccos(cx) - \frac{3b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}+x(a+b\arccos(cx))^3$$

```
[Out] -6*a*b^2*x-6*b^3*x*arccos(c*x)+x*(a+b*arccos(c*x))^3+6*b^3*(-c^2*x^2+1)^(1/
2)/c-3*b*(a+b*arccos(c*x))^2*(-c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {4716, 4768, 267}

$$\int (a+b\arccos(cx))^3\,dx = -\frac{3b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}+x(a+b\arccos(cx))^3 - 6ab^2x-6b^3x\arccos(cx)+\frac{6b^3\sqrt{1-c^2x^2}}{c}$$

```
[In] Int[(a + b*ArcCos[c*x])^3,x]
```

```
[Out] -6*a*b^2*x + (6*b^3*Sqrt[1 - c^2*x^2])/c - 6*b^3*x*ArcCos[c*x] - (3*b*Sqrt[
1 - c^2*x^2]*(a + b*ArcCos[c*x])^2)/c + x*(a + b*ArcCos[c*x])^3
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
```

```
NeQ[p, -1]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x(a+b\arccos(cx))^3+(3bc)\int\frac{x(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{3b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}+x(a+b\arccos(cx))^3-\left(6b^2\right)\int(a+b\arccos(cx))\,dx\\
&= -6ab^2x-\frac{3b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}+x(a+b\arccos(cx))^3-\left(6b^3\right)\int\arccos(cx)\,dx\\
&= -6ab^2x-6b^3x\arccos(cx)-\frac{3b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}\\
&\quad+x(a+b\arccos(cx))^3-\left(6b^3c\right)\int\frac{x}{\sqrt{1-c^2x^2}}\,dx\\
&= -6ab^2x+\frac{6b^3\sqrt{1-c^2x^2}}{c}-6b^3x\arccos(cx)\\
&\quad-\frac{3b\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}{c}+x(a+b\arccos(cx))^3
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.56

$$\int(a+b\arccos(cx))^3\,dx$$

$$=\frac{a(a^2-6b^2)\,cx-3b(a^2-2b^2)\sqrt{1-c^2x^2}+3b\left(a^2cx-2b^2cx-2ab\sqrt{1-c^2x^2}\right)\arccos(cx)+3b^2\left(acx-b\sqrt{\right.}}{c}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (a*(a^2 - 6*b^2)*c*x - 3*b*(a^2 - 2*b^2)*Sqrt[1 - c^2*x^2] + 3*b*(a^2*c*x -
 2*b^2*c*x - 2*a*b*Sqrt[1 - c^2*x^2])*ArcCos[c*x] + 3*b^2*(a*c*x - b*Sqrt[1
 - c^2*x^2])*ArcCos[c*x]^2 + b^3*c*x*ArcCos[c*x]^3)/c
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{cx\,a^3+b^3\left(\arccos(cx)^3 cx-3\arccos(cx)^2\sqrt{-c^2x^2+1}+6\sqrt{-c^2x^2+1}-6cx\arccos(cx)\right)+3a\,b^2\left(\arccos(cx)^2 cx-2cx-2\arccos(\right.}{c}$
default	$\frac{cx\,a^3+b^3\left(\arccos(cx)^3 cx-3\arccos(cx)^2\sqrt{-c^2x^2+1}+6\sqrt{-c^2x^2+1}-6cx\arccos(cx)\right)+3a\,b^2\left(\arccos(cx)^2 cx-2cx-2\arccos(\right.}{c}$
parts	$x\,a^3+\frac{b^3\left(\arccos(cx)^3 cx-3\arccos(cx)^2\sqrt{-c^2x^2+1}+6\sqrt{-c^2x^2+1}-6cx\arccos(cx)\right)}{c}+\frac{3a\,b^2\left(\arccos(cx)^2 cx-2cx-2\,\mathrm{ar}\right.}{c}$

```
[In] int((a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(c*x*a^3+b^3*(arccos(c*x)^3*c*x-3*arccos(c*x)^2*(-c^2*x^2+1)^(1/2)+6*(-
c^2*x^2+1)^(1/2)-6*c*x*arccos(c*x))+3*a*b^2*(arccos(c*x)^2*c*x-2*c*x-2*arcc
os(c*x)*(-c^2*x^2+1)^(1/2))+3*a^2*b*(c*x*arccos(c*x)-(-c^2*x^2+1)^(1/2)))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

$$\int (a+b\arccos(cx))^3\,dx$$
$$=\frac{b^3cx\arccos(cx)^3+3\,ab^2cx\arccos(cx)^2+3\,(a^2b-2\,b^3)cx\arccos(cx)+(a^3-6\,ab^2)cx-3\left(b^3\arccos(cx)^2\right.}{c}$$

```
[In] integrate((a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] (b^3*c*x*arccos(c*x)^3 + 3*a*b^2*c*x*arccos(c*x)^2 + 3*(a^2*b - 2*b^3)*c*x*
arccos(c*x) + (a^3 - 6*a*b^2)*c*x - 3*(b^3*arccos(c*x)^2 + 2*a*b^2*arccos(c
*x) + a^2*b - 2*b^3)*sqrt(-c^2*x^2 + 1))/c
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(76) = 152.

Time = 0.16 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.01

$$\int (a+b\arccos(cx))^3\,dx = \begin{cases} a^3x+3a^2bx\operatorname{acos}(cx)-\frac{3a^2b\sqrt{-c^2x^2+1}}{c}+3ab^2x\operatorname{acos}^2(cx)-6ab^2x-\frac{6ab^2\sqrt{-c^2x^2+1}\operatorname{acos}(cx)}{c}+b^3x\operatorname{acos}^3(cx) \\ x\left(a+\frac{\pi b}{2}\right)^3 \end{cases}$$

```
[In] integrate((a+b*acos(c*x))**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x*acos(c*x) - 3*a**2*b*sqrt(-c**2*x**2 + 1)/c
+ 3*a*b**2*x*acos(c*x)**2 - 6*a*b**2*x - 6*a*b**2*sqrt(-c**2*x**2 + 1)*acos
(c*x)/c + b**3*x*acos(c*x)**3 - 6*b**3*x*acos(c*x) - 3*b**3*sqrt(-c**2*x**2
 + 1)*acos(c*x)**2/c + 6*b**3*sqrt(-c**2*x**2 + 1)/c, Ne(c, 0)), (x*(a + pi
*b/2)**3, True))
```

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.76

$$\begin{aligned}\int (a+b\arccos(cx))^3\,dx &= b^3x\arccos(cx)^3+3\,ab^2x\arccos(cx)^2 \\ &\quad -3\left(\frac{\sqrt{-c^2x^2+1}\arccos(cx)^2}{c}+\frac{2\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)}{c}\right)b^3 \\ &\quad -6\,ab^2\left(x+\frac{\sqrt{-c^2x^2+1}\arccos(cx)}{c}\right)+a^3x+\frac{3\left(cx\arccos(cx)-\sqrt{-c^2x^2+1}\right)a^2b}{c}\end{aligned}$$

```
[In] integrate((a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] b^3*x*arccos(c*x)^3 + 3*a*b^2*x*arccos(c*x)^2 - 3*(sqrt(-c^2*x^2 + 1)*arcco
s(c*x)^2/c + 2*(c*x*arccos(c*x) - sqrt(-c^2*x^2 + 1))/c)*b^3 - 6*a*b^2*(x +
 sqrt(-c^2*x^2 + 1)*arccos(c*x)/c) + a^3*x + 3*(c*x*arccos(c*x) - sqrt(-c^2
*x^2 + 1))*a^2*b/c
```

Giac [A] (verification not implemented)

none

Time = 0.32 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.83

$$\int (a+b\arccos(cx))^3\,dx = b^3x\arccos{(cx)}^3+3\,ab^2x\arccos{(cx)}^2+3\,a^2bx\arccos{(cx)} - 6\,b^3x\arccos{(cx)}-\frac{3\,\sqrt{-c^2x^2+1}b^3\arccos{(cx)}^2}{c} + a^3x-6\,ab^2x-\frac{6\,\sqrt{-c^2x^2+1}ab^2\arccos{(cx)}}{c} - \frac{3\,\sqrt{-c^2x^2+1}a^2b}{c}+\frac{6\,\sqrt{-c^2x^2+1}b^3}{c}$$

```
[In] integrate((a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] b^3*x*arccos(c*x)^3 + 3*a*b^2*x*arccos(c*x)^2 + 3*a^2*b*x*arccos(c*x) - 6*b
^3*x*arccos(c*x) - 3*sqrt(-c^2*x^2 + 1)*b^3*arccos(c*x)^2/c + a^3*x - 6*a*b
^2*x - 6*sqrt(-c^2*x^2 + 1)*a*b^2*arccos(c*x)/c - 3*sqrt(-c^2*x^2 + 1)*a^2*
b/c + 6*sqrt(-c^2*x^2 + 1)*b^3/c
```

Mupad [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.00

$$\int (a+b\arccos(cx))^3\,dx = \begin{cases} x\left(a^3+\frac{3\,\pi\,a^2\,b}{2}+\frac{3\,\pi^2\,a\,b^2}{4}+\frac{\pi^3\,b^3}{8}\right) \\ a^3\,x-b^3\,x\left(6\,\mathrm{acos}(c\,x)-\mathrm{acos}(c\,x)^3\right)-\frac{3\,a^2\,b\left(\sqrt{1-c^2\,x^2}-c\,x\,\mathrm{acos}(c\,x)\right)}{c}+3\,a\,b^2\,x\left(\mathrm{acos}(c\,x)^2-2\right)-\frac{b^3\,\sqrt{1-c^2\,x^2}\,(}{} \end{cases}$$

```
[In] int((a + b*acos(c*x))^3,x)
```

```
[Out] piecewise(c == 0, x*(a^3 + (b^3*pi^3)/8 + (3*a*b^2*pi^2)/4 + (3*a^2*b*pi)/2
), c ~= 0, a^3*x - b^3*x*(6*acos(c*x) - acos(c*x)^3) - (3*a^2*b*((- c^2*x^2
 + 1)^(1/2) - c*x*acos(c*x)))/c + 3*a*b^2*x*(acos(c*x)^2 - 2) - (b^3*(- c^2
*x^2 + 1)^(1/2)*(3*acos(c*x)^2 - 6))/c - (6*a*b^2*acos(c*x)*(- c^2*x^2 + 1)
^(1/2))/c)
```

3.156 $\int \frac{(a+b\arccos(cx))^3}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 127

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^3}{x}\,dx = & -\frac{i(a+b\arccos(cx))^4}{4b}+(a+b\arccos(cx))^3\log\left(1+e^{2i\arccos(cx)}\right) \\ & -\frac{3}{2}ib(a+b\arccos(cx))^2\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) \\ & +\frac{3}{2}b^2(a+b\arccos(cx))\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right) \\ & +\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-e^{2i\arccos(cx)}\right)\end{aligned}$$

```
[Out] -1/4*I*(a+b*arccos(c*x))^4/b+(a+b*arccos(c*x))^3*ln(1+(c*x+I*(-c^2*x^2+1)^(
1/2))^2)-3/2*I*b*(a+b*arccos(c*x))^2*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^
2)+3/2*b^2*(a+b*arccos(c*x))*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/4*I
*b^3*polylog(4,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4722, 3800, 2221, 2611, 6744, 2320, 6724}

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^3}{x}\,dx = & \frac{3}{2}b^2\operatorname{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx)) \\ & -\frac{3}{2}ib\operatorname{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^2 \\ & -\frac{i(a+b\arccos(cx))^4}{4b}+\log\left(1+e^{2i\arccos(cx)}\right)(a+b\arccos(cx))^3 \\ & +\frac{3}{4}ib^3\operatorname{PolyLog}\left(4,-e^{2i\arccos(cx)}\right)\end{aligned}$$

```
[In] Int[(a + b*ArcCos[c*x])^3/x,x]
```

```
[Out] ((-1/4*I)*(a + b*ArcCos[c*x])^4)/b + (a + b*ArcCos[c*x])^3*Log[1 + E^((2*I)
*ArcCos[c*x])] - ((3*I)/2)*b*(a + b*ArcCos[c*x])^2*PolyLog[2, -E^((2*I)*Arc
Cos[c*x])] + (3*b^2*(a + b*ArcCos[c*x])*PolyLog[3, -E^((2*I)*ArcCos[c*x])])
/2 + ((3*I)/4)*b^3*PolyLog[4, -E^((2*I)*ArcCos[c*x])]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3800

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
*((c + d*x)^(m + 1)/(d*(m + 1))), x] - Dist[2*I, Int[(c + d*x)^m*(E^(2*I*(e
 + f*x))/(1 + E^(2*I*(e + f*x)))), x], x] /; FreeQ[{c, d, e, f}, x] && IGtQ
[m, 0]
```

Rule 4722

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/(x_), x_Symbol] :> -Subst[Int[
(a + b*x)^n*Tan[x], x], x, ArcCos[c*x]] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0
]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int (a+bx)^3\tan(x)\,dx, x, \arccos(cx)\right)\\
&= -\frac{i(a+b\arccos(cx))^4}{4b} + 2i\text{Subst}\left(\int \frac{e^{2ix}(a+bx)^3}{1+e^{2ix}}\,dx, x, \arccos(cx)\right)\\
&= -\frac{i(a+b\arccos(cx))^4}{4b} + (a+b\arccos(cx))^3\log\left(1+e^{2i\arccos(cx)}\right)\\
&\quad - (3b)\text{Subst}\left(\int (a+bx)^2\log\left(1+e^{2ix}\right)\,dx, x, \arccos(cx)\right)\\
&= -\frac{i(a+b\arccos(cx))^4}{4b} + (a+b\arccos(cx))^3\log\left(1+e^{2i\arccos(cx)}\right)\\
&\quad - \frac{3}{2}ib(a+b\arccos(cx))^2\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\\
&\quad + \left(3ib^2\right)\text{Subst}\left(\int (a+bx)\,\text{PolyLog}\left(2,-e^{2ix}\right)\,dx, x, \arccos(cx)\right)\\
&= -\frac{i(a+b\arccos(cx))^4}{4b} + (a+b\arccos(cx))^3\log\left(1+e^{2i\arccos(cx)}\right)\\
&\quad - \frac{3}{2}ib(a+b\arccos(cx))^2\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\\
&\quad + \frac{3}{2}b^2(a+b\arccos(cx))\,\text{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)\\
&\quad - \frac{1}{2}\left(3b^3\right)\text{Subst}\left(\int \text{PolyLog}\left(3,-e^{2ix}\right)\,dx, x, \arccos(cx)\right)\\
&= -\frac{i(a+b\arccos(cx))^4}{4b} + (a+b\arccos(cx))^3\log\left(1+e^{2i\arccos(cx)}\right)\\
&\quad - \frac{3}{2}ib(a+b\arccos(cx))^2\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right)\\
&\quad + \frac{3}{2}b^2(a+b\arccos(cx))\,\text{PolyLog}\left(3,-e^{2i\arccos(cx)}\right)\\
&\quad + \frac{1}{4}\left(3ib^3\right)\text{Subst}\left(\int \frac{\text{PolyLog}(3,-x)}{x}\,dx, x, e^{2i\arccos(cx)}\right)
\end{aligned}
$$

$$= -\frac{i(a+b\arccos(cx))^4}{4b} + (a+b\arccos(cx))^3\log\left(1+e^{2i\arccos(cx)}\right) - \frac{3}{2}ib(a+b\arccos(cx))^2\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) + \frac{3}{2}b^2(a+b\arccos(cx))\,\text{PolyLog}\left(3,-e^{2i\arccos(cx)}\right) + \frac{3}{4}ib^3\,\text{PolyLog}\left(4,-e^{2i\arccos(cx)}\right)$$

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.61

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx = \frac{1}{4}\left(-6ia^2b\arccos(cx)^2 - 4iab^2\arccos(cx)^3 - ib^3\arccos(cx)^4 + 12a^2b\arccos(cx)\log\left(1+e^{2i\arccos(cx)}\right) + 12ab^2\arccos(cx)^2\log\left(1+e^{2i\arccos(cx)}\right) + 4b^3\arccos(cx)^3\log\left(1+e^{2i\arccos(cx)}\right) + 4a^3\log(cx) - 6ib(a+b\arccos(cx))^2\,\text{PolyLog}\left(2,-e^{2i\arccos(cx)}\right) + 6b^2(a+b\arccos(cx))\,\text{PolyLog}\left(3,-e^{2i\arccos(cx)}\right) + 3ib^3\,\text{PolyLog}\left(4,-e^{2i\arccos(cx)}\right)\right)$$

```
[In] Integrate[(a + b*ArcCos[c*x])^3/x,x]
```

```
[Out] ((-6*I)*a^2*b*ArcCos[c*x]^2 - (4*I)*a*b^2*ArcCos[c*x]^3 - I*b^3*ArcCos[c*x]
^4 + 12*a^2*b*ArcCos[c*x]*Log[1 + E^((2*I)*ArcCos[c*x])] + 12*a*b^2*ArcCos[
c*x]^2*Log[1 + E^((2*I)*ArcCos[c*x])] + 4*b^3*ArcCos[c*x]^3*Log[1 + E^((2*I
)*ArcCos[c*x])] + 4*a^3*Log[c*x] - (6*I)*b*(a + b*ArcCos[c*x])^2*PolyLog[2,
 -E^((2*I)*ArcCos[c*x])] + 6*b^2*(a + b*ArcCos[c*x])*PolyLog[3, -E^((2*I)*A
rcCos[c*x])] + (3*I)*b^3*PolyLog[4, -E^((2*I)*ArcCos[c*x])])/4
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(160) = 320.

Time = 1.20 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.56

method	result
parts	$a^3 \ln(x) + b^3 \left(-\frac{i \arccos(cx)^4}{4} + \arccos(cx)^3 \ln\left(1 + \left(cx + i\sqrt{-c^2x^2+1}\right)^2\right) - \frac{3i \arccos(cx)^2}{}\right.$
derivativedivides	$a^3 \ln(cx) + b^3 \left(-\frac{i \arccos(cx)^4}{4} + \arccos(cx)^3 \ln\left(1 + \left(cx + i\sqrt{-c^2x^2+1}\right)^2\right) - \frac{3i \arccos(cx)^2}{}\right.$
default	$a^3 \ln(cx) + b^3 \left(-\frac{i \arccos(cx)^4}{4} + \arccos(cx)^3 \ln\left(1 + \left(cx + i\sqrt{-c^2x^2+1}\right)^2\right) - \frac{3i \arccos(cx)^2}{}\right.$

[In] int((a+b*arccos(c*x))^3/x,x,method=_RETURNVERBOSE)

[Out] a^3*ln(x)+b^3*(-1/4*I*arccos(c*x)^4+arccos(c*x)^3*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-3/2*I*arccos(c*x)^2*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/2*arccos(c*x)*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+3/4*I*polylog(4,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))+3*a*b^2*(-1/3*I*arccos(c*x)^3+arccos(c*x)^2*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-I*arccos(c*x)*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2)+1/2*polylog(3,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))+3*a^2*b*(-1/2*I*arccos(c*x)^2+arccos(c*x)*ln(1+(c*x+I*(-c^2*x^2+1)^(1/2))^2)-1/2*I*polylog(2,-(c*x+I*(-c^2*x^2+1)^(1/2))^2))

Fricas [F]

$$\int \frac{(a + b \arccos(cx))^3}{x} \, dx = \int \frac{(b \arccos(cx) + a)^3}{x} \, dx$$

[In] integrate((a+b*arccos(c*x))^3/x,x, algorithm="fricas")

[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) + a^3)/x, x)

Sympy [F]

$$\int \frac{(a + b \arccos(cx))^3}{x} \, dx = \int \frac{(a + b \operatorname{acos}(cx))^3}{x} \, dx$$

[In] integrate((a+b*acos(c*x))**3/x,x)

[Out] Integral((a + b*acos(c*x))**3/x, x)

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx = \int \frac{(b\arccos(cx)+a)^3}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^3/x,x, algorithm="maxima")
```

```
[Out] a^3*log(x) + integrate((b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 +
3*a*b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 3*a^2*b*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x))/x, x)
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx = \int \frac{(b\arccos(cx)+a)^3}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^3/x,x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)^3/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^3}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^3}{x}\,dx$$

```
[In] int((a + b*acos(c*x))^3/x,x)
```

```
[Out] int((a + b*acos(c*x))^3/x, x)
```

3.157 $\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 151

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx = &-\frac{(a+b\arccos(cx))^3}{x}-6ibc(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)\\ &+6ib^2c(a+b\arccos(cx))\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)\\ &-6ib^2c(a+b\arccos(cx))\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\\ &-6b^3c\,\text{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)+6b^3c\,\text{PolyLog}\left(3,ie^{i\arccos(cx)}\right)\end{aligned}$$

```
[Out] -(a+b*arccos(c*x))^3/x-6*I*b*c*(a+b*arccos(c*x))^2*arctan(c*x+I*(-c^2*x^2+1
)^(1/2))+6*I*b^2*c*(a+b*arccos(c*x))*polylog(2,-I*(c*x+I*(-c^2*x^2+1)^(1/2)
))-6*I*b^2*c*(a+b*arccos(c*x))*polylog(2,I*(c*x+I*(-c^2*x^2+1)^(1/2)))-6*b^
3*c*polylog(3,-I*(c*x+I*(-c^2*x^2+1)^(1/2)))+6*b^3*c*polylog(3,I*(c*x+I*(-c
^2*x^2+1)^(1/2)))
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4724, 4804, 4266, 2611, 2320, 6724}

$$\begin{aligned}\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx = &-6ibc\arctan\left(e^{i\arccos(cx)}\right)(a+b\arccos(cx))^2\\ &+6ib^2c\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)(a+b\arccos(cx))\\ &-6ib^2c\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)(a+b\arccos(cx))\\ &-\frac{(a+b\arccos(cx))^3}{x}-6b^3c\,\text{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)\\ &+6b^3c\,\text{PolyLog}\left(3,ie^{i\arccos(cx)}\right)\end{aligned}$$

```
[In] Int[(a + b*ArcCos[c*x])^3/x^2,x]
```

```
[Out] -((a + b*ArcCos[c*x])^3/x) - (6*I)*b*c*(a + b*ArcCos[c*x])^2*ArcTan[E^(I*Ar
cCos[c*x])] + (6*I)*b^2*c*(a + b*ArcCos[c*x])*PolyLog[2, (-I)*E^(I*ArcCos[c
*x])] - (6*I)*b^2*c*(a + b*ArcCos[c*x])*PolyLog[2, I*E^(I*ArcCos[c*x])] - 6
*b^3*c*PolyLog[3, (-I)*E^(I*ArcCos[c*x])] + 6*b^3*c*PolyLog[3, I*E^(I*ArcCo
s[c*x])]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4266

```
Int[csc[(e_.) + Pi*(k_.) + (f_.)*(x_)]*((c_.) + (d_.)*(x_))^(m_.), x_Symbol
] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^(I*k*Pi)*E^(I*(e + f*x))]/f), x] + (-Di
st[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 - E^(I*k*Pi)*E^(I*(e + f*x))], x],
x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Log[1 + E^(I*k*Pi)*E^(I*(e + f*x))
], x], x]) /; FreeQ[{c, d, e, f}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4804

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_))/Sqrt[(d_) + (e_.)*
(x_)^2], x_Symbol] :> Dist[(-(c^(m + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[
d + e*x^2]], Subst[Int[(a + b*x)^n*Cos[x]^m, x], x, ArcCos[c*x]], x] /; Fre
eQ[{a, b, c, d, e}, x] && EqQ[c^2*d + e, 0] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{(a+b\arccos(cx))^3}{x}-(3bc)\int\frac{(a+b\arccos(cx))^2}{x\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{(a+b\arccos(cx))^3}{x}+(3bc)\text{Subst}\left(\int(a+bx)^2\sec(x)\,dx,x,\arccos(cx)\right)\\
&= -\frac{(a+b\arccos(cx))^3}{x}-6ibc(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)\\
&\quad-\left(6b^2c\right)\text{Subst}\left(\int(a+bx)\log\left(1-ie^{ix}\right)\,dx,x,\arccos(cx)\right)\\
&\quad+\left(6b^2c\right)\text{Subst}\left(\int(a+bx)\log\left(1+ie^{ix}\right)\,dx,x,\arccos(cx)\right)\\
&= -\frac{(a+b\arccos(cx))^3}{x}-6ibc(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)\\
&\quad+6ib^2c(a+b\arccos(cx))\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)\\
&\quad-6ib^2c(a+b\arccos(cx))\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\\
&\quad-\left(6ib^3c\right)\text{Subst}\left(\int\text{PolyLog}\left(2,-ie^{ix}\right)\,dx,x,\arccos(cx)\right)\\
&\quad+\left(6ib^3c\right)\text{Subst}\left(\int\text{PolyLog}\left(2,ie^{ix}\right)\,dx,x,\arccos(cx)\right)\\
&= -\frac{(a+b\arccos(cx))^3}{x}-6ibc(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)\\
&\quad+6ib^2c(a+b\arccos(cx))\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)\\
&\quad-6ib^2c(a+b\arccos(cx))\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\\
&\quad-\left(6b^3c\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,-ix)}{x}\,dx,x,e^{i\arccos(cx)}\right)\\
&\quad+\left(6b^3c\right)\text{Subst}\left(\int\frac{\text{PolyLog}(2,ix)}{x}\,dx,x,e^{i\arccos(cx)}\right)\\
&= -\frac{(a+b\arccos(cx))^3}{x}-6ibc(a+b\arccos(cx))^2\arctan\left(e^{i\arccos(cx)}\right)\\
&\quad+6ib^2c(a+b\arccos(cx))\,\text{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)\\
&\quad-6ib^2c(a+b\arccos(cx))\,\text{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\\
&\quad-6b^3c\,\text{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)+6b^3c\,\text{PolyLog}\left(3,ie^{i\arccos(cx)}\right)
\end{aligned}$$

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 308 vs. 2(151) = 302.

Time = 0.22 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.04

$$\begin{aligned}&\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx\\&=-\frac{a^3}{x}-\frac{3a^2b\arccos(cx)}{x}-3a^2bc\log(x)+3a^2bc\log\left(1+\sqrt{1-c^2x^2}\right)\\&\quad+3ab^2c\Bigg(-\frac{\arccos(cx)^2}{cx}+2\big(\arccos(cx)\left(\log\left(1-ie^{i\arccos(cx)}\right)-\log\left(1+ie^{i\arccos(cx)}\right)\right)\\&\qquad+i\big(\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\big)\big)\Bigg)\\&\quad+b^3c\Bigg(-\frac{\arccos(cx)^3}{cx}+3\big(\arccos(cx)^2\left(\log\left(1-ie^{i\arccos(cx)}\right)-\log\left(1+ie^{i\arccos(cx)}\right)\right)\\&\qquad+2i\arccos(cx)\left(\operatorname{PolyLog}\left(2,-ie^{i\arccos(cx)}\right)-\operatorname{PolyLog}\left(2,ie^{i\arccos(cx)}\right)\right)\\&\qquad-2\big(\operatorname{PolyLog}\left(3,-ie^{i\arccos(cx)}\right)-\operatorname{PolyLog}\left(3,ie^{i\arccos(cx)}\right)\big)\big)\Bigg)\end{aligned}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^3/x^2,x]
```

```
[Out] -(a^3/x) - (3*a^2*b*ArcCos[c*x])/x - 3*a^2*b*c*Log[x] + 3*a^2*b*c*Log[1 + S
qrt[1 - c^2*x^2]] + 3*a*b^2*c*(-(ArcCos[c*x]^2/(c*x)) + 2*(ArcCos[c*x]*(Log
[1 - I*E^(I*ArcCos[c*x])] - Log[1 + I*E^(I*ArcCos[c*x])]) + I*(PolyLog[2, (
-I)*E^(I*ArcCos[c*x])] - PolyLog[2, I*E^(I*ArcCos[c*x])]))) + b^3*c*(-(ArcC
os[c*x]^3/(c*x)) + 3*(ArcCos[c*x]^2*(Log[1 - I*E^(I*ArcCos[c*x])] - Log[1 +
 I*E^(I*ArcCos[c*x])]) + (2*I)*ArcCos[c*x]*(PolyLog[2, (-I)*E^(I*ArcCos[c*x
])] - PolyLog[2, I*E^(I*ArcCos[c*x])]) - 2*(PolyLog[3, (-I)*E^(I*ArcCos[c*x
])] - PolyLog[3, I*E^(I*ArcCos[c*x])])))
```

Maple [F]

$$\int \frac{(a+b\arccos(cx))^3}{x^2}dx$$

```
[In] int((a+b*arccos(c*x))^3/x^2,x)
```

```
[Out] int((a+b*arccos(c*x))^3/x^2,x)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx = \int \frac{(b\arccos(cx)+a)^3}{x^2}\,dx$$

[In] integrate((a+b*arccos(c*x))^3/x^2,x, algorithm="fricas")

[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) + a^3)/x^2, x)

Sympy [F]

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^3}{x^2}\,dx$$

[In] integrate((a+b*acos(c*x))**3/x**2,x)

[Out] Integral((a + b*acos(c*x))**3/x**2, x)

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx = \int \frac{(b\arccos(cx)+a)^3}{x^2}\,dx$$

[In] integrate((a+b*arccos(c*x))^3/x^2,x, algorithm="maxima")

[Out] 3*(c*log(2*sqrt(-c^2*x^2 + 1)/abs(x) + 2/abs(x)) - arccos(c*x)/x)*a^2*b - a^3/x - (b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 - x*integrate(3*(sqrt(c*x + 1)*sqrt(-c*x + 1)*b^3*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + (a*b^2*c^2*x^2 - a*b^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2)/(c^2*x^4 - x^2), x))/x

Giac [F]

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx = \int \frac{(b\arccos(cx)+a)^3}{x^2}\,dx$$

[In] integrate((a+b*arccos(c*x))^3/x^2,x, algorithm="giac")

[Out] integrate((b*arccos(c*x) + a)^3/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^3}{x^2}\,dx=\int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^3}{x^2}\,dx$$

```
[In] int((a + b*acos(c*x))^3/x^2,x)
```

```
[Out] int((a + b*acos(c*x))^3/x^2, x)
```

3.158 $\int \frac{x^2}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 121

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx = \frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{4bc^3} + \frac{\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{4bc^3} - \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3} - \frac{\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3}$$

```
[Out] -1/4*cos(a/b)*Si((a+b*arccos(c*x))/b)/b/c^3-1/4*cos(3*a/b)*Si(3*(a+b*arccos
(c*x))/b)/b/c^3+1/4*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c^3+1/4*Ci(3*(a+b*ar
ccos(c*x))/b)*sin(3*a/b)/b/c^3
```

Rubi [A] (verified)

Time = 0.13 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {4732, 4491, 3384, 3380, 3383}

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx = \frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3} + \frac{\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3} - \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3} - \frac{\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3}$$

```
[In] Int[x^2/(a + b*ArcCos[c*x]),x]
```

```
[Out] (CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(4*b*c^3) + (CosIntegral[(3*(
a + b*ArcCos[c*x]))/b]*Sin[(3*a)/b])/(4*b*c^3) - (Cos[a/b]*SinIntegral[(a +
 b*ArcCos[c*x])/b])/(4*b*c^3) - (Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[
c*x]))/b])/(4*b*c^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\cos^2\left(\frac{a}{b}-\frac{x}{b}\right)\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{bc^3}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{\sin\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}+\frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)\,dx, x, a+b\arccos(cx)\right)}{bc^3}$$

$$= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{4bc^3}+\frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{4bc^3}$$

$$
\begin{aligned}
&= -\frac{\cos\left(\frac{a}{b}\right)\operatorname{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{4bc^3} \\
&\quad - \frac{\cos\left(\frac{3a}{b}\right)\operatorname{Subst}\left(\int \frac{\sin\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{4bc^3} \\
&\quad + \frac{\sin\left(\frac{a}{b}\right)\operatorname{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{4bc^3} \\
&\quad + \frac{\sin\left(\frac{3a}{b}\right)\operatorname{Subst}\left(\int \frac{\cos\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{4bc^3} \\
&= \frac{\operatorname{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{4bc^3} + \frac{\operatorname{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{4bc^3} \\
&\quad - \frac{\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4bc^3} - \frac{\cos\left(\frac{3a}{b}\right)\operatorname{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4bc^3}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.75

$$
\int \frac{x^2}{a+b\arccos(cx)}\,dx = -\frac{-\operatorname{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right) - \operatorname{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{3a}{b}\right) + \cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\text{ar}\right.}{4bc^3}
$$

```
[In] Integrate[x^2/(a + b*ArcCos[c*x]),x]
```

```
[Out] -1/4*(-(CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b]) - CosIntegral[3*(a/b + Arc
Cos[c*x])]*Sin[(3*a)/b] + Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]] + Cos[(3*
a)/b]*SinIntegral[3*(a/b + ArcCos[c*x])])/(b*c^3)
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

method	result	siz
derivativedivides	$\frac{-\frac{\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)}{4b}+\frac{\operatorname{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)}{4b}-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{4b}+\frac{\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{4b}}{c^3}$	10
default	$\frac{-\frac{\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)}{4b}+\frac{\operatorname{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)}{4b}-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{4b}+\frac{\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{4b}}{c^3}$	10

```
[In] int(x^2/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(-1/4*Si(3*arccos(c*x)+3*a/b)*cos(3*a/b)/b+1/4*Ci(3*arccos(c*x)+3*a/b
)*sin(3*a/b)/b-1/4*Si(arccos(c*x)+a/b)*cos(a/b)/b+1/4*Ci(arccos(c*x)+a/b)*s
in(a/b)/b)
```

Fricas [F]

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx = \int \frac{x^2}{b\arccos(cx)+a}\,dx$$

```
[In] integrate(x^2/(a+b*arccos(c*x)),x, algorithm="fricas")
[Out] integral(x^2/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx = \int \frac{x^2}{a+b\operatorname{acos}(cx)}\,dx$$

```
[In] integrate(x**2/(a+b*acos(c*x)),x)
[Out] Integral(x**2/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx = \int \frac{x^2}{b\arccos(cx)+a}\,dx$$

```
[In] integrate(x^2/(a+b*arccos(c*x)),x, algorithm="maxima")
[Out] integrate(x^2/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.42

$$\begin{aligned}\int \frac{x^2}{a+b\arccos(cx)}\,dx &= \frac{\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{bc^3} \\ &\quad - \frac{\cos\left(\frac{a}{b}\right)^3\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{bc^3} \\ &\quad - \frac{\operatorname{Ci}\left(\frac{3a}{b}+3\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc^3} + \frac{\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{4bc^3} \\ &\quad + \frac{3\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3a}{b}+3\arccos(cx)\right)}{4bc^3} - \frac{\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{4bc^3}\end{aligned}$$

```
[In] integrate(x^2/(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b*c^3) - cos(a/b)^
3*sin_integral(3*a/b + 3*arccos(c*x))/(b*c^3) - 1/4*cos_integral(3*a/b + 3*
arccos(c*x))*sin(a/b)/(b*c^3) + 1/4*cos_integral(a/b + arccos(c*x))*sin(a/b
)/(b*c^3) + 3/4*cos(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b*c^3) - 1/4*
cos(a/b)*sin_integral(a/b + arccos(c*x))/(b*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a+b\arccos(cx)}\,dx=\int \frac{x^2}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

```
[In] int(x^2/(a + b*acos(c*x)),x)
```

```
[Out] int(x^2/(a + b*acos(c*x)), x)
```

3.159 $\int \frac{x}{a+b \arccos(cx)} dx$

Optimal result

Integrand size = 12, antiderivative size = 63

$$\int \frac{x}{a+b \arccos(cx)} dx = \frac{\text{CosIntegral}\left(\frac{2(a+b \arccos(cx))}{b}\right) \sin\left(\frac{2a}{b}\right)}{2bc^2} - \frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arccos(cx))}{b}\right)}{2bc^2}$$

```
[Out] -1/2*cos(2*a/b)*Si(2*(a+b*arccos(c*x))/b)/b/c^2+1/2*Ci(2*(a+b*arccos(c*x))/
b)*sin(2*a/b)/b/c^2
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4732, 4491, 12, 3384, 3380, 3383}

$$\int \frac{x}{a+b \arccos(cx)} dx = \frac{\sin\left(\frac{2a}{b}\right) \text{CosIntegral}\left(\frac{2(a+b \arccos(cx))}{b}\right)}{2bc^2} - \frac{\cos\left(\frac{2a}{b}\right) \text{Si}\left(\frac{2(a+b \arccos(cx))}{b}\right)}{2bc^2}$$

```
[In] Int[x/(a + b*ArcCos[c*x]),x]
```

```
[Out] (CosIntegral[(2*(a + b*ArcCos[c*x]))/b]*Sin[(2*a)/b])/(2*b*c^2) - (Cos[(2*a
)/b]*SinIntegral[(2*(a + b*ArcCos[c*x]))/b])/(2*b*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4732

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{bc^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\,dx, x, a+b\arccos(cx)\right)}{bc^2}\\
&= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{2bc^2}\\
&= -\frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{2bc^2}\\
&\quad + \frac{\sin\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{2bc^2}
\end{aligned}$$

$$= \frac{\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{2bc^2} - \frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{2bc^2}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{x}{a+b\arccos(cx)}\,dx = -\frac{-\text{CosIntegral}\left(\frac{2a}{b}+2\arccos(cx)\right)\sin\left(\frac{2a}{b}\right)+\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2a}{b}+2\arccos(cx)\right)}{2bc^2}$$

```
[In] Integrate[x/(a + b*ArcCos[c*x]),x]
[Out] -1/2*(-(CosIntegral[(2*a)/b + 2*ArcCos[c*x]]*Sin[(2*a)/b]) + Cos[(2*a)/b]*S
inIntegral[(2*a)/b + 2*ArcCos[c*x]])/(b*c^2)
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{-\frac{\text{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)}{2b}+\frac{\text{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)}{2b}}{c^2}$	58
default	$\frac{-\frac{\text{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)}{2b}+\frac{\text{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)}{2b}}{c^2}$	58

```
[In] int(x/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/c^2*(-1/2*Si(2*arccos(c*x)+2*a/b)*cos(2*a/b)/b+1/2*Ci(2*arccos(c*x)+2*a/b
)*sin(2*a/b)/b)
```

Fricas [F]

$$\int \frac{x}{a+b\arccos(cx)}\,dx = \int \frac{x}{b\arccos(cx)+a}\,dx$$

```
[In] integrate(x/(a+b*arccos(c*x)),x, algorithm="fricas")
[Out] integral(x/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{x}{a+b\arccos(cx)}\,dx=\int \frac{x}{a+b\,\mathrm{acos}\,(cx)}\,dx$$

```
[In] integrate(x/(a+b*acos(c*x)),x)
[Out] Integral(x/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x}{a+b\arccos(cx)}\,dx=\int \frac{x}{b\arccos\,(cx)+a}\,dx$$

```
[In] integrate(x/(a+b*arccos(c*x)),x, algorithm="maxima")
[Out] integrate(x/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.37

$$\int \frac{x}{a+b\arccos(cx)}\,dx=\frac{\cos\left(\frac{a}{b}\right)\mathrm{Ci}\left(\frac{2a}{b}+2\arccos\left(cx\right)\right)\sin\left(\frac{a}{b}\right)}{bc^2}-\frac{\cos\left(\frac{a}{b}\right)^2\mathrm{Si}\left(\frac{2a}{b}+2\arccos\left(cx\right)\right)}{bc^2}+\frac{\mathrm{Si}\left(\frac{2a}{b}+2\arccos\left(cx\right)\right)}{2\,bc^2}$$

```
[In] integrate(x/(a+b*arccos(c*x)),x, algorithm="giac")
[Out] cos(a/b)*cos_integral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b*c^2) - cos(a/b)^2*
sin_integral(2*a/b + 2*arccos(c*x))/(b*c^2) + 1/2*sin_integral(2*a/b + 2*ar
ccos(c*x))/(b*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a + b \arccos(cx)}\, dx = \int \frac{x}{a + b\,\mathrm{acos}\,(c\,x)}\, dx$$

```
[In] int(x/(a + b*acos(c*x)),x)
[Out] int(x/(a + b*acos(c*x)), x)
```

3.160 $\int \frac{1}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 54

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{bc}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

```
[Out] -cos(a/b)*Si((a+b*arccos(c*x))/b)/b/c+Ci((a+b*arccos(c*x))/b)*sin(a/b)/b/c
```

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {4720, 3384, 3380, 3383}

$$\int \frac{1}{a+b\arccos(cx)}\,dx=\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}-\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}$$

```
[In] Int[(a + b*ArcCos[c*x])^(-1),x]
```

```
[Out] (CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(b*c) - (Cos[a/b]*SinIntegral
[(a + b*ArcCos[c*x])/b])/(b*c)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
```

```
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{bc} \\
&= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{bc} \\
&\quad + \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{bc} \\
&= \frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{bc} - \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{bc}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.85

$$\int \frac{1}{a+b\arccos(cx)}\,dx = -\frac{-\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)+\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{bc}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^(-1),x]
[Out] -((-(CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b]) + Cos[a/b]*SinIntegral[a/b +
ArcCos[c*x]])/(b*c))
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91

method	result	size
derivativedivides	$\frac{-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b}+\frac{\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{b}}{c}$	49
default	$\frac{-\frac{\operatorname{Si}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b}+\frac{\operatorname{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)}{b}}{c}$	49

```
[In] int(1/(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
[Out] 1/c*(-Si(arccos(c*x)+a/b)*cos(a/b)/b+Ci(arccos(c*x)+a/b)*sin(a/b)/b)
```

Fricas [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \int \frac{1}{b\arccos(cx)+a}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*arccos(c*x) + a), x)
```

Sympy [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \int \frac{1}{a+b\operatorname{acos}(cx)}\,dx$$

```
[In] integrate(1/(a+b*acos(c*x)),x)
[Out] Integral(1/(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \int \frac{1}{b\arccos(cx)+a}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x)),x, algorithm="maxima")
[Out] integrate(1/(b*arccos(c*x) + a), x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \frac{\operatorname{Ci}\left(\frac{a}{b}+\arccos\left(cx\right)\right)\sin\left(\frac{a}{b}\right)}{bc} - \frac{\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos\left(cx\right)\right)}{bc}$$

```
[In] integrate(1/(a+b*arccos(c*x)),x, algorithm="giac")
[Out] cos_integral(a/b + arccos(c*x))*sin(a/b)/(b*c) - cos(a/b)*sin_integral(a/b
+ arccos(c*x))/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{a+b\arccos(cx)}\,dx = \int \frac{1}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

```
[In] int(1/(a + b*acos(c*x)),x)
[Out] int(1/(a + b*acos(c*x)), x)
```

3.161 $\int \frac{1}{x(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{x(a+b\arccos(cx))},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arccos(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx = \int \frac{1}{x(a+b\arccos(cx))}\,dx$$

```
[In] Int[1/(x*(a + b*ArcCos[c*x])),x]
```

```
[Out] Defer[Int][1/(x*(a + b*ArcCos[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x(a+b\arccos(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx=\int \frac{1}{x(a+b\arccos(cx))}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcCos[c*x])),x]
[Out] Integrate[1/(x*(a + b*ArcCos[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.47 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\arccos\left(cx\right)\right)}dx$$

```
[In] int(1/x/(a+b*arccos(c*x)),x)
[Out] int(1/x/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx=\int \frac{1}{(b\arccos\left(cx\right)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x*arccos(c*x) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx=\int \frac{1}{x\left(a+b\operatorname{acos}\left(cx\right)\right)}\,dx$$

```
[In] integrate(1/x/(a+b*acos(c*x)),x)
[Out] Integral(1/(x*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx = \int \frac{1}{(b\arccos(cx)+a)x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> an error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Not invertible Error: Bad Argument Va
lue
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

```
[In] int(1/(x*(a + b*acos(c*x))),x)
```

```
[Out] int(1/(x*(a + b*acos(c*x))), x)
```

3.162 $\int \frac{1}{x^2(a+b\arccos(cx))} dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x^2(a+b\arccos(cx))} dx = \text{Int}\left(\frac{1}{x^2(a+b\arccos(cx))}, x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*arccos(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2(a+b\arccos(cx))} dx = \int \frac{1}{x^2(a+b\arccos(cx))} dx$$

```
[In] Int[1/(x^2*(a + b*ArcCos[c*x])),x]
[Out] Defer[Int][1/(x^2*(a + b*ArcCos[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2(a+b\arccos(cx))} dx$$

Mathematica [N/A]

Not integrable

Time = 2.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx=\int \frac{1}{x^2(a+b\arccos(cx))}\,dx$$

```
[In] Integrate[1/(x^2*(a + b*ArcCos[c*x])),x]
[Out] Integrate[1/(x^2*(a + b*ArcCos[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.60 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a+b\arccos\left(cx\right)\right)}dx$$

```
[In] int(1/x^2/(a+b*arccos(c*x)),x)
[Out] int(1/x^2/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.36

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx=\int \frac{1}{(b\arccos\left(cx\right)+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x)),x, algorithm="fricas")
[Out] integral(1/(b*x^2*arccos(c*x) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx=\int \frac{1}{x^2\left(a+b\operatorname{acos}\left(cx\right)\right)}\,dx$$

```
[In] integrate(1/x**2/(a+b*acos(c*x)),x)
[Out] Integral(1/(x**2*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx=\int \frac{1}{(b\arccos{(cx)}+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x)),x, algorithm="maxima")
[Out] integrate(1/((b*arccos(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx=\int \frac{1}{(b\arccos{(cx)}+a)x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x)),x, algorithm="giac")
[Out] integrate(1/((b*arccos(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))}\,dx=\int \frac{1}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}\,dx$$

```
[In] int(1/(x^2*(a + b*acos(c*x))),x)
[Out] int(1/(x^2*(a + b*acos(c*x))), x)
```

3.163 $\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 155

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = \frac{x^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3} - \frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3} - \frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3} - \frac{3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3}$$

```
[Out] -1/4*Ci((a+b*arccos(c*x))/b)*cos(a/b)/b^2/c^3-3/4*Ci(3*(a+b*arccos(c*x))/b)
*cos(3*a/b)/b^2/c^3-1/4*Si((a+b*arccos(c*x))/b)*sin(a/b)/b^2/c^3-3/4*Si(3*(
a+b*arccos(c*x))/b)*sin(3*a/b)/b^2/c^3+x^2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arcc
os(c*x))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4728, 3384, 3380, 3383}

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = -\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3} - \frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3} - \frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3} - \frac{3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3} + \frac{x^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

```
[In] Int[x^2/(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (x^2*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) - (Cos[a/b]*CosIntegral[(
a + b*ArcCos[c*x])/b])/(4*b^2*c^3) - (3*Cos[(3*a)/b]*CosIntegral[(3*(a + b*
ArcCos[c*x]))/b])/(4*b^2*c^3) - (Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b
])/(4*b^2*c^3) - (3*Sin[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/(4
*b^2*c^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
```

```
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}+\frac{\text{Subst}\left(\int\left(-\frac{3\cos\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x}-\frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)dx,x,a+b\arccos(cx)\right)}{b^2c^3}\\
&= \frac{x^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\text{Subst}\left(\int\frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{x}dx,x,a+b\arccos(cx)\right)}{4b^2c^3}\\
&\quad -\frac{3\text{Subst}\left(\int\frac{\cos\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}dx,x,a+b\arccos(cx)\right)}{4b^2c^3}\\
&= \frac{x^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int\frac{\cos\left(\frac{x}{b}\right)}{x}dx,x,a+b\arccos(cx)\right)}{4b^2c^3}\\
&\quad -\frac{\left(3\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{3x}{b}\right)}{x}dx,x,a+b\arccos(cx)\right)}{4b^2c^3}\\
&\quad -\frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int\frac{\sin\left(\frac{x}{b}\right)}{x}dx,x,a+b\arccos(cx)\right)}{4b^2c^3}\\
&\quad -\frac{\left(3\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{3x}{b}\right)}{x}dx,x,a+b\arccos(cx)\right)}{4b^2c^3}\\
&= \frac{x^2\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3}\\
&\quad -\frac{3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3}\\
&\quad -\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{4b^2c^3}-\frac{3\sin\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{4b^2c^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = -\frac{-\frac{4bc^2x^2\sqrt{1-c^2x^2}}{a+b\arccos(cx)} + \cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right) + 3\cos\left(\frac{3a}{b}\right)\text{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right) + \text{si}}{4b^2c^3}$$

```
[In] Integrate[x^2/(a + b*ArcCos[c*x])^2,x]
```

```
[Out] -1/4*((-4*b*c^2*x^2*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) + Cos[a/b]*CosIn
tegral[a/b + ArcCos[c*x]] + 3*Cos[(3*a)/b]*CosIntegral[3*(a/b + ArcCos[c*x]
)] + Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]] + 3*Sin[(3*a)/b]*SinIntegral[3
*(a/b + ArcCos[c*x])])/(b^2*c^3)
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95

method	result
derivativedivides	$\frac{\frac{\sin(3\arccos(cx))}{4(a+b\arccos(cx))b}-\frac{3\left(\text{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)+\text{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)\right)}{4b^2}+\frac{\sqrt{-c^2x^2+1}}{4(a+b\arccos(cx))b}-\frac{\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin(}{}}{c^3}$
default	$\frac{\frac{\sin(3\arccos(cx))}{4(a+b\arccos(cx))b}-\frac{3\left(\text{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)\sin\left(\frac{3a}{b}\right)+\text{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)\cos\left(\frac{3a}{b}\right)\right)}{4b^2}+\frac{\sqrt{-c^2x^2+1}}{4(a+b\arccos(cx))b}-\frac{\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin(}{}}{c^3}$

```
[In] int(x^2/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/4*sin(3*arccos(c*x))/(a+b*arccos(c*x))/b-3/4*(Si(3*arccos(c*x)+3*a
/b)*sin(3*a/b)+Ci(3*arccos(c*x)+3*a/b)*cos(3*a/b))/b^2+1/4*(-c^2*x^2+1)^(1/
2)/(a+b*arccos(c*x))/b-1/4*(Si(arccos(c*x)+a/b)*sin(a/b)+Ci(arccos(c*x)+a/b
)*cos(a/b))/b^2)
```

Fricas [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^2}\,dx$$

```
[In] integrate(x^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral(x^2/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(a+b\,\mathrm{acos}\,(cx)\right)^2}\,dx$$

```
[In] integrate(x**2/(a+b*acos(c*x))**2,x)
```

```
[Out] Integral(x**2/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate(x^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] (sqrt(c*x + 1)*sqrt(-c*x + 1)*x^2 - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x) + a*b*c)*integrate((3*c^2*x^3 - 2*x)*sqrt(c*x + 1)*sqrt(-c*x + 1
)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-
c*x + 1), c*x)), x))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*
b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(145) = 290$.

Time = 0.32 (sec) , antiderivative size = 615, normalized size of antiderivative = 3.97

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = -\frac{3\,b\arccos\left(cx\right)\cos\left(\frac{a}{b}\right)^3\operatorname{Ci}\left(\frac{3\,a}{b}+3\,\arccos\left(cx\right)\right)}{b^3c^3\arccos\left(cx\right)+ab^2c^3} - \frac{3\,b\arccos\left(cx\right)\cos\left(\frac{a}{b}\right)^2\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3\,a}{b}+3\,\arccos\left(cx\right)\right)}{b^3c^3\arccos\left(cx\right)+ab^2c^3} + \frac{\sqrt{-c^2x^2+1}bc^2x^2}{b^3c^3\arccos\left(cx\right)+ab^2c^3} - \frac{3\,a\cos\left(\frac{a}{b}\right)^3\operatorname{Ci}\left(\frac{3\,a}{b}+3\,\arccos\left(cx\right)\right)}{b^3c^3\arccos\left(cx\right)+ab^2c^3} - \frac{3\,a\cos\left(\frac{a}{b}\right)^2\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3\,a}{b}+3\,\arccos\left(cx\right)\right)}{b^3c^3\arccos\left(cx\right)+ab^2c^3} + \frac{9\,b\arccos\left(cx\right)\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{3\,a}{b}+3\,\arccos\left(cx\right)\right)}{4\,\left(b^3c^3\arccos\left(cx\right)+ab^2c^3\right)} - \frac{b\arccos\left(cx\right)\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arccos\left(cx\right)\right)}{4\,\left(b^3c^3\arccos\left(cx\right)+ab^2c^3\right)} + \frac{3\,b\arccos\left(cx\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3\,a}{b}+3\,\arccos\left(cx\right)\right)}{4\,\left(b^3c^3\arccos\left(cx\right)+ab^2c^3\right)} - \frac{b\arccos\left(cx\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos\left(cx\right)\right)}{4\,\left(b^3c^3\arccos\left(cx\right)+ab^2c^3\right)} + \frac{9\,a\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{3\,a}{b}+3\,\arccos\left(cx\right)\right)}{4\,\left(b^3c^3\arccos\left(cx\right)+ab^2c^3\right)} - \frac{a\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arccos\left(cx\right)\right)}{4\,\left(b^3c^3\arccos\left(cx\right)+ab^2c^3\right)} + \frac{3\,a\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{3\,a}{b}+3\,\arccos\left(cx\right)\right)}{4\,\left(b^3c^3\arccos\left(cx\right)+ab^2c^3\right)} - \frac{a\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos\left(cx\right)\right)}{4\,\left(b^3c^3\arccos\left(cx\right)+ab^2c^3\right)}$$

```
[In] integrate(x^2/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] -3*b*arccos(c*x)*cos(a/b)^3*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*ar
ccos(c*x) + a*b^2*c^3) - 3*b*arccos(c*x)*cos(a/b)^2*sin(a/b)*sin_integral(3
*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + sqrt(-c^2*x^2 + 1
)*b*c^2*x^2/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 3*a*cos(a/b)^3*cos_integral
(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 3*a*cos(a/b)^2*
sin(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c
^3) + 9/4*b*arccos(c*x)*cos(a/b)*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*c
^3*arccos(c*x) + a*b^2*c^3) - 1/4*b*arccos(c*x)*cos(a/b)*cos_integral(a/b +
 arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 3/4*b*arccos(c*x)*sin(a/b
```

```
)*sin_integral(3*a/b + 3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1
/4*b*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c^3*arccos(c
*x) + a*b^2*c^3) + 9/4*a*cos(a/b)*cos_integral(3*a/b + 3*arccos(c*x))/(b^3*
c^3*arccos(c*x) + a*b^2*c^3) - 1/4*a*cos(a/b)*cos_integral(a/b + arccos(c*x
))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) + 3/4*a*sin(a/b)*sin_integral(3*a/b +
3*arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3) - 1/4*a*sin(a/b)*sin_integ
ral(a/b + arccos(c*x))/(b^3*c^3*arccos(c*x) + a*b^2*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\arccos(cx))^2}\,dx = \int \frac{x^2}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int(x^2/(a + b*acos(c*x))^2,x)
[Out] int(x^2/(a + b*acos(c*x))^2, x)
```

3.164 $\int \frac{x}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 91

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx = \frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{\cos\left(\frac{2a}{b}\right)\operatorname{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^2} - \frac{\sin\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^2}$$

```
[Out] -Ci(2*(a+b*arccos(c*x))/b)*cos(2*a/b)/b^2/c^2-Si(2*(a+b*arccos(c*x))/b)*sin
(2*a/b)/b^2/c^2+x*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {4728, 3384, 3380, 3383}

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx = -\frac{\cos\left(\frac{2a}{b}\right)\operatorname{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^2} - \frac{\sin\left(\frac{2a}{b}\right)\operatorname{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^2} + \frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

```
[In] Int[x/(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (x*Sqrt[1 - c^2*x^2])/(b*c*(a + b*ArcCos[c*x])) - (Cos[(2*a)/b]*CosIntegral
[(2*(a + b*ArcCos[c*x]))/b])/(b^2*c^2) - (Sin[(2*a)/b]*SinIntegral[(2*(a +
b*ArcCos[c*x]))/b])/(b^2*c^2)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{b^2c^2} \\
&= \frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{b^2c^2} \\
&\quad - \frac{\sin\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{b^2c^2} \\
&= \frac{x\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{\cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^2} - \frac{\sin\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^2c^2}
\end{aligned}
$$

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.88

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx = \frac{\frac{bcx\sqrt{1-c^2x^2}}{a+b\arccos(cx)} - \cos\left(\frac{2a}{b}\right)\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right) - \sin\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)}{b^2c^2}$$

```
[In] Integrate[x/(a + b*ArcCos[c*x])^2,x]
[Out] ((b*c*x*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - Cos[(2*a)/b]*CosIntegral[2
*(a/b + ArcCos[c*x])] - Sin[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])])/(b
^2*c^2)
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{\sin(2\arccos(cx))}{2(a+b\arccos(cx))b} - \frac{\text{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)+\text{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)}{b^2}}{c^2}$	78
default	$\frac{\frac{\sin(2\arccos(cx))}{2(a+b\arccos(cx))b} - \frac{\text{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)+\text{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\sin\left(\frac{2a}{b}\right)}{b^2}}{c^2}$	78

```
[In] int(x/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
[Out] 1/c^2*(1/2*sin(2*arccos(c*x))/(a+b*arccos(c*x))/b-(Ci(2*arccos(c*x)+2*a/b)*
cos(2*a/b)+Si(2*arccos(c*x)+2*a/b)*sin(2*a/b))/b^2)
```

Fricas [F]

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx = \int \frac{x}{(b\arccos(cx)+a)^2}\,dx$$

```
[In] integrate(x/(a+b*arccos(c*x))^2,x, algorithm="fricas")
[Out] integral(x/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx = \int \frac{x}{\left(a+b\,\mathrm{acos}\,(cx)\right)^2}\,dx$$

```
[In] integrate(x/(a+b*acos(c*x))**2,x)
```

```
[Out] Integral(x/(a + b*acos(c*x))**2, x)
```

Maxima [F]

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx = \int \frac{x}{\left(b\arccos\left(cx\right)+a\right)^2}\,dx$$

```
[In] integrate(x/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] (sqrt(c*x + 1)*sqrt(-c*x + 1)*x - (b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x +
1), c*x) + a*b*c)*integrate((2*c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(a
*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x
+ 1), c*x)), x))/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(89) = 178$.

Time = 0.32 (sec) , antiderivative size = 323, normalized size of antiderivative = 3.55

$$\begin{aligned}\int \frac{x}{(a+b\arccos(cx))^2}\,dx = &-\frac{2\,b\arccos\left(cx\right)\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos\left(cx\right)\right)}{b^3c^2\arccos\left(cx\right)+ab^2c^2}\\&-\frac{2\,b\arccos\left(cx\right)\cos\left(\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos\left(cx\right)\right)}{b^3c^2\arccos\left(cx\right)+ab^2c^2}\\&-\frac{2\,a\cos\left(\frac{a}{b}\right)^2\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos\left(cx\right)\right)}{b^3c^2\arccos\left(cx\right)+ab^2c^2}\\&-\frac{2\,a\cos\left(\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{2\,a}{b}+2\,\arccos\left(cx\right)\right)}{b^3c^2\arccos\left(cx\right)+ab^2c^2}\\&+\frac{\sqrt{-c^2x^2+1}bcx}{b^3c^2\arccos\left(cx\right)+ab^2c^2}+\frac{b\arccos\left(cx\right)\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos\left(cx\right)\right)}{b^3c^2\arccos\left(cx\right)+ab^2c^2}\\&+\frac{a\operatorname{Ci}\left(\frac{2\,a}{b}+2\,\arccos\left(cx\right)\right)}{b^3c^2\arccos\left(cx\right)+ab^2c^2}\end{aligned}$$

```
[In] integrate(x/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] -2*b*arccos(c*x)*cos(a/b)^2*cos_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*ar
ccos(c*x) + a*b^2*c^2) - 2*b*arccos(c*x)*cos(a/b)*sin(a/b)*sin_integral(2*a
/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 2*a*cos(a/b)^2*cos_
integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2*c^2) - 2*a*cos
(a/b)*sin(a/b)*sin_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a
*b^2*c^2) + sqrt(-c^2*x^2 + 1)*b*c*x/(b^3*c^2*arccos(c*x) + a*b^2*c^2) + b*
arccos(c*x)*cos_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^
2*c^2) + a*cos_integral(2*a/b + 2*arccos(c*x))/(b^3*c^2*arccos(c*x) + a*b^2
*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\arccos(cx))^2}\,dx=\int \frac{x}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int(x/(a + b*acos(c*x))^2,x)
```

```
[Out] int(x/(a + b*acos(c*x))^2, x)
```

3.165 $\int \frac{1}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 86

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx=\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}-\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}$$

```
[Out] -Ci((a+b*arccos(c*x))/b)*cos(a/b)/b^2/c-Si((a+b*arccos(c*x))/b)*sin(a/b)/b^
2/c+(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))
```

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4718, 4810, 3384, 3380, 3383}

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx=-\frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}-\frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}+\frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}$$

```
[In] Int[(a + b*ArcCos[c*x])^(-2),x]
```

```
[Out] Sqrt[1 - c^2*x^2]/(b*c*(a + b*ArcCos[c*x])) - (Cos[a/b]*CosIntegral[(a + b*
ArcCos[c*x])/b])/(b^2*c) - (Sin[a/b]*SinIntegral[(a + b*ArcCos[c*x])/b])/(b
^2*c)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}+\frac{c\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))}\,dx}{b} \\
&= \frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx,x,a+b\arccos(cx)\right)}{b^2c} \\
&= \frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))}-\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\arccos(cx)\right)}{b^2c} \\
&\quad-\frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{x}\,dx,x,a+b\arccos(cx)\right)}{b^2c}
\end{aligned}
$$

$$= \frac{\sqrt{1-c^2x^2}}{bc(a+b\arccos(cx))} - \frac{\cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c} - \frac{\sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^2c}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.84

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \frac{\frac{b\sqrt{1-c^2x^2}}{a+b\arccos(cx)} - \cos\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right) - \sin\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^2c}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^(-2),x]
[Out] ((b*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x]) - Cos[a/b]*CosIntegral[a/b + Arc
Cos[c*x]] - Sin[a/b]*SinIntegral[a/b + ArcCos[c*x]])/(b^2*c)
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\frac{\sqrt{-c^2x^2+1}}{(a+b\arccos(cx))b} - \frac{\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+\text{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b^2}}{c}$	74
default	$\frac{\frac{\sqrt{-c^2x^2+1}}{(a+b\arccos(cx))b} - \frac{\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)\sin\left(\frac{a}{b}\right)+\text{Ci}\left(\arccos(cx)+\frac{a}{b}\right)\cos\left(\frac{a}{b}\right)}{b^2}}{c}$	74

```
[In] int(1/(a+b*arccos(c*x))^2,x,method=_RETURNVERBOSE)
[Out] 1/c*((-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))/b-(Si(arccos(c*x)+a/b)*sin(a/b)+C
i(arccos(c*x)+a/b)*cos(a/b))/b^2)
```

Fricas [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))^2}\,dx$$

```
[In] integrate(1/(a+b*acos(c*x))**2,x)
[Out] Integral((a + b*acos(c*x))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^2,x, algorithm="maxima")
[Out] -((b^2*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c^2)*integrate(
sqrt(c*x + 1)*sqrt(-c*x + 1)*x/(a*b*c^2*x^2 - a*b + (b^2*c^2*x^2 - b^2)*arc
tan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) - sqrt(c*x + 1)*sqrt(-c*x + 1)
)/(b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(84) = 168.

Time = 0.29 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.24

$$\begin{aligned}\int \frac{1}{(a+b\arccos(cx))^2}\,dx = &-\frac{b\arccos(cx)\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c}\\&-\frac{b\arccos(cx)\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c}\\&-\frac{a\cos\left(\frac{a}{b}\right)\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c}\\&-\frac{a\sin\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^3c\arccos(cx)+ab^2c}+\frac{\sqrt{-c^2x^2+1}\,b}{b^3c\arccos(cx)+ab^2c}\end{aligned}$$

```
[In] integrate(1/(a+b*arccos(c*x))^2,x, algorithm="giac")
[Out] -b*arccos(c*x)*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3*c*arccos(c*x)
+ a*b^2*c) - b*arccos(c*x)*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c*
arccos(c*x) + a*b^2*c) - a*cos(a/b)*cos_integral(a/b + arccos(c*x))/(b^3*c*
arccos(c*x) + a*b^2*c) - a*sin(a/b)*sin_integral(a/b + arccos(c*x))/(b^3*c*
arccos(c*x) + a*b^2*c) + sqrt(-c^2*x^2 + 1)*b/(b^3*c*arccos(c*x) + a*b^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(cx))^2}\,dx=\int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2}\,dx$$

```
[In] int(1/(a + b*acos(c*x))^2,x)
[Out] int(1/(a + b*acos(c*x))^2, x)
```

3.166 $\int \frac{1}{x(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{1}{x(a+b\arccos(cx))^2},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arccos(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x(a+b\arccos(cx))^2}\,dx$$

```
[In] Int[1/(x*(a + b*ArcCos[c*x])^2),x]
[Out] Defer[Int][1/(x*(a + b*ArcCos[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x(a+b\arccos(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x(a+b\arccos(cx))^2}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcCos[c*x])^2),x]
[Out] Integrate[1/(x*(a + b*ArcCos[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\arccos\left(cx\right)\right)^{2}}dx$$

```
[In] int(1/x/(a+b*arccos(c*x))^2,x)
[Out] int(1/x/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.14

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^{2}x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x*arccos(c*x)^2 + 2*a*b*x*arccos(c*x) + a^2*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x\left(a+b\operatorname{acos}\left(cx\right)\right)^{2}}\,dx$$

```
[In] integrate(1/x/(a+b*acos(c*x))**2,x)
[Out] Integral(1/(x*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 166, normalized size of antiderivative = 11.86

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] -((b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x)*integrate(
sqrt(c*x + 1)*sqrt(-c*x + 1)/(a*b*c^3*x^4 - a*b*c*x^2 + (b^2*c^3*x^4 - b^2*
c*x^2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) - sqrt(c*x + 1)*sqrt
(-c*x + 1))/(b^2*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x)
```

Giac [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^2x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((b*arccos(c*x) + a)^2*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

```
[In] int(1/(x*(a + b*acos(c*x))^2),x)
```

```
[Out] int(1/(x*(a + b*acos(c*x))^2), x)
```

3.167 $\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \mathrm{Int}\left(\frac{1}{x^2(a+b\arccos(cx))^2},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*arccos(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx$$

```
[In] Int[1/(x^2*(a + b*ArcCos[c*x])^2),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*ArcCos[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 56.93 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx$$

```
[In] Integrate[1/(x^2*(a + b*ArcCos[c*x])^2),x]
[Out] Integrate[1/(x^2*(a + b*ArcCos[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.63 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a+b\arccos\left(cx\right)\right)^2}dx$$

```
[In] int(1/x^2/(a+b*arccos(c*x))^2,x)
[Out] int(1/x^2/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.57

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^2x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^2,x, algorithm="fricas")
[Out] integral(1/(b^2*x^2*arccos(c*x)^2 + 2*a*b*x^2*arccos(c*x) + a^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x^2\left(a+b\operatorname{acos}\left(cx\right)\right)^2}\,dx$$

```
[In] integrate(1/x**2/(a+b*acos(c*x))**2,x)
[Out] Integral(1/(x**2*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 181, normalized size of antiderivative = 12.93

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^2x^2}\,dx$$

[In] integrate(1/x^2/(a+b*arccos(c*x))^2,x, algorithm="maxima")

[Out] ((b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*x^2)*integra
te((c^2*x^2 - 2)*sqrt(c*x + 1)*sqrt(-c*x + 1)/(a*b*c^3*x^5 - a*b*c*x^3 + (b
^2*c^3*x^5 - b^2*c*x^3)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + s
qrt(c*x + 1)*sqrt(-c*x + 1))/(b^2*c*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1
), c*x) + a*b*c*x^2)

Giac [N/A]

Not integrable

Time = 0.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^2x^2}\,dx$$

[In] integrate(1/x^2/(a+b*arccos(c*x))^2,x, algorithm="giac")

[Out] integrate(1/((b*arccos(c*x) + a)^2*x^2), x)

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^2}\,dx = \int \frac{1}{x^2\,{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

[In] int(1/(x^2*(a + b*acos(c*x))^2),x)

[Out] int(1/(x^2*(a + b*acos(c*x))^2), x)

3.168 $\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 197

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = \frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{x}{b^2c^2(a+b\arccos(cx))} + \frac{3x^3}{2b^2(a+b\arccos(cx))} - \frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{8b^3c^3} - \frac{9\,\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{8b^3c^3} + \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^3c^3} + \frac{9\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{8b^3c^3}$$

```
[Out] -x/b^2/c^2/(a+b*arccos(c*x))+3/2*x^3/b^2/(a+b*arccos(c*x))+1/8*cos(a/b)*Si(
(a+b*arccos(c*x))/b)/b^3/c^3+9/8*cos(3*a/b)*Si(3*(a+b*arccos(c*x))/b)/b^3/c
^3-1/8*Ci((a+b*arccos(c*x))/b)*sin(a/b)/b^3/c^3-9/8*Ci(3*(a+b*arccos(c*x))/
b)*sin(3*a/b)/b^3/c^3+1/2*x^2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^2
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used

$= \{4730, 4808, 4732, 4491, 3384, 3380, 3383, 4720\}$

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = -\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^3c^3} - \frac{9\sin\left(\frac{3a}{b}\right)\text{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{8b^3c^3} + \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^3c^3} + \frac{9\cos\left(\frac{3a}{b}\right)\text{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{8b^3c^3} - \frac{x}{b^2c^2(a+b\arccos(cx))} + \frac{3x^3}{2b^2(a+b\arccos(cx))} + \frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

```
[In] Int[x^2/(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (x^2*Sqrt[1 - c^2*x^2])/(2*b*c*(a + b*ArcCos[c*x])^2) - x/(b^2*c^2*(a + b*A
rcCos[c*x])) + (3*x^3)/(2*b^2*(a + b*ArcCos[c*x])) - (CosIntegral[(a + b*Ar
cCos[c*x])/b]*Sin[a/b])/(8*b^3*c^3) - (9*CosIntegral[(3*(a + b*ArcCos[c*x])
)/b]*Sin[(3*a)/b])/(8*b^3*c^3) + (Cos[a/b]*SinIntegral[(a + b*ArcCos[c*x])/
b])/(8*b^3*c^3) + (9*Cos[(3*a)/b]*SinIntegral[(3*(a + b*ArcCos[c*x]))/b])/(
8*b^3*c^3)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
```

```
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx}{bc} + \frac{(3c)\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx}{2b}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{x}{b^2c^2(a+b\arccos(cx))} + \frac{3x^3}{2b^2(a+b\arccos(cx))} - \frac{9\int \frac{x^2}{a+b\arccos(cx)}\,dx}{2b^2} + \frac{\int \frac{1}{a+b\arccos(cx)}\,dx}{b^2c^2}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{x}{b^2c^2(a+b\arccos(cx))} + \frac{3x^3}{2b^2(a+b\arccos(cx))} + \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{b^3c^3} - \frac{9\text{Subst}\left(\int \frac{\cos^2\left(\frac{a}{b}-\frac{x}{b}\right)\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{2b^3c^3}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{x}{b^2c^2(a+b\arccos(cx))} + \frac{3x^3}{2b^2(a+b\arccos(cx))} - \frac{9\text{Subst}\left(\int \left(\frac{\sin\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4x} + \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{4x}\right)dx, x, a+b\arccos(cx)\right)}{2b^3c^3} - \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{b^3c^3} + \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{b^3c^3}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{x}{b^2c^2(a+b\arccos(cx))} + \frac{3x^3}{2b^2(a+b\arccos(cx))} + \frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{b^3c^3} - \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^3c^3} - \frac{9\text{Subst}\left(\int \frac{\sin\left(\frac{3a}{b}-\frac{3x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{8b^3c^3} - \frac{9\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{8b^3c^3}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{x}{b^2c^2(a+b\arccos(cx))} + \frac{3x^3}{2b^2(a+b\arccos(cx))} + \frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{b^3c^3} - \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{b^3c^3} + \frac{\left(9\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{8b^3c^3} + \frac{\left(9\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{8b^3c^3} - \frac{\left(9\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{8b^3c^3} - \frac{\left(9\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{3x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{8b^3c^3}$$

$$= \frac{x^2\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{x}{b^2c^2(a+b\arccos(cx))} + \frac{3x^3}{2b^2(a+b\arccos(cx))} - \frac{\operatorname{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{8b^3c^3} - \frac{9\operatorname{CosIntegral}\left(\frac{3(a+b\arccos(cx))}{b}\right)\sin\left(\frac{3a}{b}\right)}{8b^3c^3} + \frac{\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{8b^3c^3} + \frac{9\cos\left(\frac{3a}{b}\right)\operatorname{Si}\left(\frac{3(a+b\arccos(cx))}{b}\right)}{8b^3c^3}$$

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.86

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = \frac{\frac{4b^2x^2\sqrt{1-c^2x^2}}{c(a+b\arccos(cx))^2} - \frac{8bx}{c^2(a+b\arccos(cx))} + \frac{12bx^3}{a+b\arccos(cx)} - \frac{\operatorname{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{c^3} - \frac{9\operatorname{CosIntegral}\left(3\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{3a}{b}\right.}{c^3}}{8b^3}$$

```
[In] Integrate[x^2/(a + b*ArcCos[c*x])^3,x]
```

```
[Out] ((4*b^2*x^2*Sqrt[1 - c^2*x^2])/(c*(a + b*ArcCos[c*x])^2) - (8*b*x)/(c^2*(a
+ b*ArcCos[c*x])) + (12*b*x^3)/(a + b*ArcCos[c*x]) - (CosIntegral[a/b + Arc
Cos[c*x]]*Sin[a/b])/c^3 - (9*CosIntegral[3*(a/b + ArcCos[c*x])]*Sin[(3*a)/b
])/c^3 + (Cos[a/b]*SinIntegral[a/b + ArcCos[c*x]])/c^3 + (9*Cos[(3*a)/b]*Si
nIntegral[3*(a/b + ArcCos[c*x])])/c^3)/(8*b^3)
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{\sin(3\arccos(cx))}{8(a+b\arccos(cx))^2 b} + \frac{\frac{9\arccos(cx)\cos\left(\frac{3a}{b}\right)\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)b}{8} - \frac{9\arccos(cx)\sin\left(\frac{3a}{b}\right)\operatorname{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)b}{8} + \frac{9\cos\left(\frac{3a}{b}\right)\operatorname{Si}(3\arccos \ldots}{8}}{(a+b\arccos(cx))b^3}$
default	$\frac{\sin(3\arccos(cx))}{8(a+b\arccos(cx))^2 b} + \frac{\frac{9\arccos(cx)\cos\left(\frac{3a}{b}\right)\operatorname{Si}\left(3\arccos(cx)+\frac{3a}{b}\right)b}{8} - \frac{9\arccos(cx)\sin\left(\frac{3a}{b}\right)\operatorname{Ci}\left(3\arccos(cx)+\frac{3a}{b}\right)b}{8} + \frac{9\cos\left(\frac{3a}{b}\right)\operatorname{Si}(3\arccos \ldots}{8}}{(a+b\arccos(cx))b^3}$

```
[In] int(x^2/(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^3*(1/8*sin(3*arccos(c*x))/(a+b*arccos(c*x))^2/b+3/8*(3*arccos(c*x)*cos(
3*a/b)*Si(3*arccos(c*x)+3*a/b)*b-3*arccos(c*x)*sin(3*a/b)*Ci(3*arccos(c*x)+
3*a/b)*b+3*cos(3*a/b)*Si(3*arccos(c*x)+3*a/b)*a-3*sin(3*a/b)*Ci(3*arccos(c*
x)+3*a/b)*a+cos(3*arccos(c*x))*b)/(a+b*arccos(c*x))/b^3+1/8*(-c^2*x^2+1)^(1
/2)/(a+b*arccos(c*x))^2/b+1/8*(arccos(c*x)*cos(a/b)*Si(arccos(c*x)+a/b)*b-a
```

```
rccos(c*x)*sin(a/b)*Ci(arccos(c*x)+a/b)*b+cos(a/b)*Si(arccos(c*x)+a/b)*a-si
n(a/b)*Ci(arccos(c*x)+a/b)*a+x*b*c)/(a+b*arccos(c*x))/b^3)
```

Fricas [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^3}\,dx$$

```
[In] integrate(x^2/(a+b*arccos(c*x))^3,x, algorithm="fricas")
[Out] integral(x^2/(b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*
x) + a^3), x)
```

Sympy [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = \int \frac{x^2}{(a+b\operatorname{acos}(cx))^3}\,dx$$

```
[In] integrate(x**2/(a+b*acos(c*x))**3,x)
[Out] Integral(x**2/(a + b*acos(c*x))**3, x)
```

Maxima [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^3}\,dx$$

```
[In] integrate(x^2/(a+b*arccos(c*x))^3,x, algorithm="maxima")
[Out] 1/2*(3*a*c^2*x^3 + sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x^2 - 2*a*x + (3*b*c^2*
x^3 - 2*b*x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 2*(b^4*c^2*arctan
2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*x + 1)*
sqrt(-c*x + 1), c*x) + a^2*b^2*c^2)*integrate(1/2*(9*c^2*x^2 - 2)/(b^3*c^2*
arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b^2*c^2), x))/(b^4*c^2*arcta
n2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*x + 1)
*sqrt(-c*x + 1), c*x) + a^2*b^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1479 vs. $2(183) = 366$.

Time = 0.39 (sec) , antiderivative size = 1479, normalized size of antiderivative = 7.51

$$\int \frac{x^2}{(a + b\arccos(cx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] 3/2*b^2*c^3*x^3*arccos(c*x)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x
) + a^2*b^3*c^3) + 3/2*a*b*c^3*x^3/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arc
cos(c*x) + a^2*b^3*c^3) - 9/2*b^2*arccos(c*x)^2*cos(a/b)^2*cos_integral(3*a
/b + 3*arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*
x) + a^2*b^3*c^3) + 9/2*b^2*arccos(c*x)^2*cos(a/b)^3*sin_integral(3*a/b + 3
*arccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^
3) - 9*a*b*arccos(c*x)*cos(a/b)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a
/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 9*a*b
*arccos(c*x)*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(c*x))/(b^5*c^3*arccos
(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 1/2*sqrt(-c^2*x^2 + 1)*b
^2*c^2*x^2/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3)
+ 9/8*b^2*arccos(c*x)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b^5*c
^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 9/2*a^2*cos(a/b
)^2*cos_integral(3*a/b + 3*arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2
*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 1/8*b^2*arccos(c*x)^2*cos_integral(
a/b + arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x
) + a^2*b^3*c^3) - 27/8*b^2*arccos(c*x)^2*cos(a/b)*sin_integral(3*a/b + 3*a
rccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3)
 + 9/2*a^2*cos(a/b)^3*sin_integral(3*a/b + 3*arccos(c*x))/(b^5*c^3*arccos(c
*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 1/8*b^2*arccos(c*x)^2*cos(
a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*a
rccos(c*x) + a^2*b^3*c^3) - b^2*c*x*arccos(c*x)/(b^5*c^3*arccos(c*x)^2 + 2*
a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 9/4*a*b*arccos(c*x)*cos_integral(3*a
/b + 3*arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*
x) + a^2*b^3*c^3) - 1/4*a*b*arccos(c*x)*cos_integral(a/b + arccos(c*x))*sin
(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 27/
4*a*b*arccos(c*x)*cos(a/b)*sin_integral(3*a/b + 3*arccos(c*x))/(b^5*c^3*arc
cos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) + 1/4*a*b*arccos(c*x)*c
os(a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^
3*arccos(c*x) + a^2*b^3*c^3) - a*b*c*x/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3
*arccos(c*x) + a^2*b^3*c^3) + 9/8*a^2*cos_integral(3*a/b + 3*arccos(c*x))*s
in(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 1
/8*a^2*cos_integral(a/b + arccos(c*x))*sin(a/b)/(b^5*c^3*arccos(c*x)^2 + 2*
a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3) - 27/8*a^2*cos(a/b)*sin_integral(3*a/b
 + 3*arccos(c*x))/(b^5*c^3*arccos(c*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^
```

```
3*c^3) + 1/8*a^2*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c^3*arccos(c
*x)^2 + 2*a*b^4*c^3*arccos(c*x) + a^2*b^3*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\arccos(cx))^3}\,dx=\int \frac{x^2}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^3}\,dx$$

```
[In] int(x^2/(a + b*acos(c*x))^3,x)
[Out] int(x^2/(a + b*acos(c*x))^3, x)
```

3.169 $\int \frac{x}{(a+b\arccos(cx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 130

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{1}{2b^2c^2(a+b\arccos(cx))} + \frac{x^2}{b^2(a+b\arccos(cx))} - \frac{\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{b^3c^2} + \frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^3c^2}$$

```
[Out] -1/2/b^2/c^2/(a+b*arccos(c*x))+x^2/b^2/(a+b*arccos(c*x))+cos(2*a/b)*Si(2*(a
+b*arccos(c*x))/b)/b^3/c^2-Ci(2*(a+b*arccos(c*x))/b)*sin(2*a/b)/b^3/c^2+1/2
*x*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {4730, 4808, 4732, 4491, 12, 3384, 3380, 3383, 4738}

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = -\frac{\sin\left(\frac{2a}{b}\right)\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^3c^2} + \frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^3c^2} - \frac{1}{2b^2c^2(a+b\arccos(cx))} + \frac{x^2}{b^2(a+b\arccos(cx))} + \frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

```
[In] Int[x/(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (x*Sqrt[1 - c^2*x^2])/(2*b*c*(a + b*ArcCos[c*x])^2) - 1/(2*b^2*c^2*(a + b*A
rcCos[c*x])) + x^2/(b^2*(a + b*ArcCos[c*x])) - (CosIntegral[(2*(a + b*ArcCo
s[c*x]))/b]*Sin[(2*a)/b])/(b^3*c^2) + (Cos[(2*a)/b]*SinIntegral[(2*(a + b*A
rcCos[c*x]))/b])/(b^3*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx}{2bc} + \frac{c\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx}{b} \\
&= \frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{1}{2b^2c^2(a+b\arccos(cx))} + \frac{x^2}{b^2(a+b\arccos(cx))} - \frac{2\int \frac{x}{a+b\arccos(cx)}\,dx}{b^2} \\
&= \frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{1}{2b^2c^2(a+b\arccos(cx))} + \frac{x^2}{b^2(a+b\arccos(cx))} \\
&\quad - \frac{2\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{b^3c^2} \\
&= \frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{1}{2b^2c^2(a+b\arccos(cx))} \\
&\quad + \frac{x^2}{b^2(a+b\arccos(cx))} - \frac{2\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2x}\,dx, x, a+b\arccos(cx)\right)}{b^3c^2} \\
&= \frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{1}{2b^2c^2(a+b\arccos(cx))} \\
&\quad + \frac{x^2}{b^2(a+b\arccos(cx))} - \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}-\frac{2x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{b^3c^2}
\end{aligned}$$

$$= \frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{1}{2b^2c^2(a+b\arccos(cx))} + \frac{x^2}{b^2(a+b\arccos(cx))} + \frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{b^3c^2} - \frac{\sin\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{b^3c^2}$$

$$= \frac{x\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} - \frac{1}{2b^2c^2(a+b\arccos(cx))} + \frac{x^2}{b^2(a+b\arccos(cx))} - \frac{\text{CosIntegral}\left(\frac{2(a+b\arccos(cx))}{b}\right)\sin\left(\frac{2a}{b}\right)}{b^3c^2} + \frac{\cos\left(\frac{2a}{b}\right)\text{Si}\left(\frac{2(a+b\arccos(cx))}{b}\right)}{b^3c^2}$$

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.82

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \frac{\frac{b^2cx\sqrt{1-c^2x^2}}{(a+b\arccos(cx))^2} + \frac{b(-1+2c^2x^2)}{a+b\arccos(cx)} - 2\,\text{CosIntegral}\left(2\left(\frac{a}{b}+\arccos(cx)\right)\right)\sin\left(\frac{2a}{b}\right) + 2\cos\left(\frac{2a}{b}\right)\text{Si}\left(2\left(\frac{a}{b}+\arccos(cx\right.\right.}{2b^3c^2}$$

```
[In] Integrate[x/(a + b*ArcCos[c*x])^3,x]
```

```
[Out] ((b^2*c*x*Sqrt[1 - c^2*x^2])/(a + b*ArcCos[c*x])^2 + (b*(-1 + 2*c^2*x^2))/(
a + b*ArcCos[c*x]) - 2*CosIntegral[2*(a/b + ArcCos[c*x])]*Sin[(2*a)/b] + 2*
Cos[(2*a)/b]*SinIntegral[2*(a/b + ArcCos[c*x])])/(2*b^3*c^2)
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{\sin(2\arccos(cx))}{4(a+b\arccos(cx))^2b} - \frac{2\arccos(cx)\sin\left(\frac{2a}{b}\right)\text{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)b - 2\arccos(cx)\,\text{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)b + 2\sin\left(\frac{2a}{b}\right)\text{Ci}\left(2\,\text{arcc}\right.}{2(a+b\arccos(cx))b^3}}{c^2}$
default	$\frac{\frac{\sin(2\arccos(cx))}{4(a+b\arccos(cx))^2b} - \frac{2\arccos(cx)\sin\left(\frac{2a}{b}\right)\text{Ci}\left(2\arccos(cx)+\frac{2a}{b}\right)b - 2\arccos(cx)\,\text{Si}\left(2\arccos(cx)+\frac{2a}{b}\right)\cos\left(\frac{2a}{b}\right)b + 2\sin\left(\frac{2a}{b}\right)\text{Ci}\left(2\,\text{arcc}\right.}{2(a+b\arccos(cx))b^3}}{c^2}$

```
[In] int(x/(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/4*sin(2*arccos(c*x))/(a+b*arccos(c*x))^2/b-1/2*(2*arccos(c*x)*sin(
2*a/b)*Ci(2*arccos(c*x)+2*a/b)*b-2*arccos(c*x)*Si(2*arccos(c*x)+2*a/b)*cos(
2*a/b)*b+2*sin(2*a/b)*Ci(2*arccos(c*x)+2*a/b)*a-2*Si(2*arccos(c*x)+2*a/b)*c
os(2*a/b)*a-cos(2*arccos(c*x))*b)/(a+b*arccos(c*x))/b^3)
```

Fricas [F]

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \int \frac{x}{(b\arccos(cx)+a)^3}\,dx$$

```
[In] integrate(x/(a+b*arccos(c*x))^3,x, algorithm="fricas")
[Out] integral(x/(b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
 + a^3), x)
```

Sympy [F]

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \int \frac{x}{(a+b\,\mathrm{acos}\,(cx))^3}\,dx$$

```
[In] integrate(x/(a+b*acos(c*x))**3,x)
[Out] Integral(x/(a + b*acos(c*x))**3, x)
```

Maxima [F]

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \int \frac{x}{(b\arccos(cx)+a)^3}\,dx$$

```
[In] integrate(x/(a+b*arccos(c*x))^3,x, algorithm="maxima")
[Out] 1/2*(2*a*c^2*x^2 + sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x + (2*b*c^2*x^2 - b)*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - 4*(b^4*c^2*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x) + a^2*b^2*c^2)*integrate(x/(b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1)
, c*x) + a*b^2), x) - a)/(b^4*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x
)^2 + 2*a*b^3*c^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(124) = 248$.

Time = 0.33 (sec) , antiderivative size = 860, normalized size of antiderivative = 6.62

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx = \text{Too large to display}$$

```
[In] integrate(x/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] b^2*c^2*x^2*arccos(c*x)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) +
a^2*b^3*c^2) - 2*b^2*arccos(c*x)^2*cos(a/b)*cos_integral(2*a/b + 2*arccos(c
*x))*sin(a/b)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^
2) + 2*b^2*arccos(c*x)^2*cos(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^
5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) + a*b*c^2*x^2/
(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) - 4*a*b*arc
cos(c*x)*cos(a/b)*cos_integral(2*a/b + 2*arccos(c*x))*sin(a/b)/(b^5*c^2*arc
cos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) + 4*a*b*arccos(c*x)*cos
(a/b)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^5*c^2*arccos(c*x)^2 + 2*a*b^
4*c^2*arccos(c*x) + a^2*b^3*c^2) - 2*a^2*cos(a/b)*cos_integral(2*a/b + 2*ar
ccos(c*x))*sin(a/b)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*
b^3*c^2) - b^2*arccos(c*x)^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^5*c^2*a
rccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) + 2*a^2*cos(a/b)^2*si
n_integral(2*a/b + 2*arccos(c*x))/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arcc
os(c*x) + a^2*b^3*c^2) + 1/2*sqrt(-c^2*x^2 + 1)*b^2*c*x/(b^5*c^2*arccos(c*x
)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) - 2*a*b*arccos(c*x)*sin_integr
al(2*a/b + 2*arccos(c*x))/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x)
+ a^2*b^3*c^2) - 1/2*b^2*arccos(c*x)/(b^5*c^2*arccos(c*x)^2 + 2*a*b^4*c^2*a
rccos(c*x) + a^2*b^3*c^2) - a^2*sin_integral(2*a/b + 2*arccos(c*x))/(b^5*c^
2*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2) - 1/2*a*b/(b^5*c^2
*arccos(c*x)^2 + 2*a*b^4*c^2*arccos(c*x) + a^2*b^3*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\arccos(cx))^3}\,dx=\int \frac{x}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^3}\,dx$$

```
[In] int(x/(a + b*acos(c*x))^3,x)
```

```
[Out] int(x/(a + b*acos(c*x))^3, x)
```

3.170 $\int \frac{1}{(a+b\arccos(cx))^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 111

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx = \frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}+\frac{x}{2b^2(a+b\arccos(cx))} - \frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{2b^3c}+\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^3c}$$

```
[Out] 1/2*x/b^2/(a+b*arccos(c*x))+1/2*cos(a/b)*Si((a+b*arccos(c*x))/b)/b^3/c-1/2*
Ci((a+b*arccos(c*x))/b)*sin(a/b)/b^3/c+1/2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arcc
os(c*x))^2
```

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {4718, 4808, 4720, 3384, 3380, 3383}

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx = -\frac{\sin\left(\frac{a}{b}\right)\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^3c}+\frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^3c} +\frac{x}{2b^2(a+b\arccos(cx))}+\frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}$$

```
[In] Int[(a + b*ArcCos[c*x])^(-3),x]
```

```
[Out] Sqrt[1 - c^2*x^2]/(2*b*c*(a + b*ArcCos[c*x])^2) + x/(2*b^2*(a + b*ArcCos[c*
x])) - (CosIntegral[(a + b*ArcCos[c*x])/b]*Sin[a/b])/(2*b^3*c) + (Cos[a/b]*
SinIntegral[(a + b*ArcCos[c*x])/b])/(2*b^3*c)
```

Rule 3380

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[SinInte
gral[e + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*e - c*f, 0]
```

Rule 3383

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CosInte
gral[e - Pi/2 + f*x]/d, x] /; FreeQ[{c, d, e, f}, x] && EqQ[d*(e - Pi/2) -
c*f, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}+\frac{c\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^2}\,dx}{2b}\\
&= \frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2}+\frac{x}{2b^2(a+b\arccos(cx))}-\frac{\int \frac{1}{a+b\arccos(cx)}\,dx}{2b^2}
\end{aligned}$$

$$= \frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} + \frac{x}{2b^2(a+b\arccos(cx))} - \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{2b^3c}$$

$$= \frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} + \frac{x}{2b^2(a+b\arccos(cx))} + \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{2b^3c} - \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{x}\,dx, x, a+b\arccos(cx)\right)}{2b^3c}$$

$$= \frac{\sqrt{1-c^2x^2}}{2bc(a+b\arccos(cx))^2} + \frac{x}{2b^2(a+b\arccos(cx))} - \frac{\text{CosIntegral}\left(\frac{a+b\arccos(cx)}{b}\right)\sin\left(\frac{a}{b}\right)}{2b^3c} + \frac{\cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a+b\arccos(cx)}{b}\right)}{2b^3c}$$

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.80

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx = \frac{\frac{b\left(acx+b\sqrt{1-c^2x^2}+bcx\arccos(cx)\right)}{(a+b\arccos(cx))^2} - \text{CosIntegral}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right) + \cos\left(\frac{a}{b}\right)\text{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{2b^3c}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^(-3),x]
```

```
[Out] ((b*(a*c*x + b*Sqrt[1 - c^2*x^2] + b*c*x*ArcCos[c*x]))/(a + b*ArcCos[c*x])^
2 - CosIntegral[a/b + ArcCos[c*x]]*Sin[a/b] + Cos[a/b]*SinIntegral[a/b + Ar
cCos[c*x]])/(2*b^3*c)
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.25

method	result
derivativedivides	$\frac{\frac{\sqrt{-c^2x^2+1}}{2(a+b\arccos(cx))^2 b} + \frac{\arccos(cx)\cos\left(\frac{a}{b}\right)\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)b - \arccos(cx)\sin\left(\frac{a}{b}\right)\text{Ci}\left(\arccos(cx)+\frac{a}{b}\right)b + \cos\left(\frac{a}{b}\right)\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)a - \sin}{2(a+b\arccos(cx))b^3}}{c}$
default	$\frac{\frac{\sqrt{-c^2x^2+1}}{2(a+b\arccos(cx))^2 b} + \frac{\arccos(cx)\cos\left(\frac{a}{b}\right)\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)b - \arccos(cx)\sin\left(\frac{a}{b}\right)\text{Ci}\left(\arccos(cx)+\frac{a}{b}\right)b + \cos\left(\frac{a}{b}\right)\text{Si}\left(\arccos(cx)+\frac{a}{b}\right)a - \sin}{2(a+b\arccos(cx))b^3}}{c}$

```
[In] int(1/(a+b*arccos(c*x))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*(-c^2*x^2+1)^(1/2)/(a+b*arccos(c*x))^2/b+1/2*(arccos(c*x)*cos(a/b)
*Si(arccos(c*x)+a/b)*b-arccos(c*x)*sin(a/b)*Ci(arccos(c*x)+a/b)*b+cos(a/b)*
Si(arccos(c*x)+a/b)*a-sin(a/b)*Ci(arccos(c*x)+a/b)*a+x*b*c)/(a+b*arccos(c*x
))/b^3)
```

Fricas [F]

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx=\int \frac{1}{(b\arccos{(cx)}+a)^3}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^3,x, algorithm="fricas")
```

```
[Out] integral(1/(b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x)
 + a^3), x)
```

Sympy [F]

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx=\int \frac{1}{(a+b\operatorname{acos}{(cx)})^3}\,dx$$

```
[In] integrate(1/(a+b*acos(c*x))**3,x)
```

```
[Out] Integral((a + b*acos(c*x))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx=\int \frac{1}{(b\arccos{(cx)}+a)^3}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(b*c*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*c*x + sqrt(c*x +
1)*sqrt(-c*x + 1)*b - 2*(b^4*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2
 + 2*a*b^3*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c)*integr
ate(1/2/(b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b^2), x))/(b^4*
c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(101) = 202$.

Time = 0.30 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.33

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx = -\frac{b^2\arccos(cx)^2\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} + \frac{b^2\arccos(cx)^2\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} + \frac{b^2cx\arccos(cx)}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} - \frac{ab\arccos(cx)\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c} + \frac{ab\arccos(cx)\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c} + \frac{abcx}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} - \frac{a^2\operatorname{Ci}\left(\frac{a}{b}+\arccos(cx)\right)\sin\left(\frac{a}{b}\right)}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} + \frac{a^2\cos\left(\frac{a}{b}\right)\operatorname{Si}\left(\frac{a}{b}+\arccos(cx)\right)}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)} + \frac{\sqrt{-c^2x^2+1}\,b^2}{2\left(b^5c\arccos(cx)^2+2\,ab^4c\arccos(cx)+a^2b^3c\right)}$$

```
[In] integrate(1/(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] -1/2*b^2*arccos(c*x)^2*cos_integral(a/b + arccos(c*x))*sin(a/b)/(b^5*c*arcc
os(c*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) + 1/2*b^2*arccos(c*x)^2*cos(
a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c*arccos(c*x)^2 + 2*a*b^4*c*arcco
s(c*x) + a^2*b^3*c) + 1/2*b^2*c*x*arccos(c*x)/(b^5*c*arccos(c*x)^2 + 2*a*b^
4*c*arccos(c*x) + a^2*b^3*c) - a*b*arccos(c*x)*cos_integral(a/b + arccos(c*
x))*sin(a/b)/(b^5*c*arccos(c*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) + a*
b*arccos(c*x)*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c*arccos(c*x)^2
 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) + 1/2*a*b*c*x/(b^5*c*arccos(c*x)^2 +
2*a*b^4*c*arccos(c*x) + a^2*b^3*c) - 1/2*a^2*cos_integral(a/b + arccos(c*x)
)*sin(a/b)/(b^5*c*arccos(c*x)^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c) + 1/2*
a^2*cos(a/b)*sin_integral(a/b + arccos(c*x))/(b^5*c*arccos(c*x)^2 + 2*a*b^4
*c*arccos(c*x) + a^2*b^3*c) + 1/2*sqrt(-c^2*x^2 + 1)*b^2/(b^5*c*arccos(c*x)
^2 + 2*a*b^4*c*arccos(c*x) + a^2*b^3*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(cx))^3}\,dx = \int \frac{1}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^3}\,dx$$

```
[In] int(1/(a + b*acos(c*x))^3,x)
[Out] int(1/(a + b*acos(c*x))^3, x)
```

3.171 $\int \frac{1}{x(a+b\arccos(cx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \text{Int}\left(\frac{1}{x(a+b\arccos(cx))^3},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arccos(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \int \frac{1}{x(a+b\arccos(cx))^3}\,dx$$

```
[In] Int[1/(x*(a + b*ArcCos[c*x])^3),x]
[Out] Defer[Int][1/(x*(a + b*ArcCos[c*x])^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x(a+b\arccos(cx))^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.63 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx=\int \frac{1}{x(a+b\arccos(cx))^3}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcCos[c*x])^3),x]
[Out] Integrate[1/(x*(a + b*ArcCos[c*x])^3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.66 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\left(a+b\arccos\left(cx\right)\right)^{3}}dx$$

```
[In] int(1/x/(a+b*arccos(c*x))^3,x)
[Out] int(1/x/(a+b*arccos(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 3.21

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx=\int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^{3}x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^3,x, algorithm="fricas")
[Out] integral(1/(b^3*x*arccos(c*x)^3 + 3*a*b^2*x*arccos(c*x)^2 + 3*a^2*b*x*arcco
s(c*x) + a^3*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \int \frac{1}{x\,(a+b\,\mathrm{acos}\,(cx))^3}\,dx$$

```
[In] integrate(1/x/(a+b*acos(c*x))**3,x)
[Out] Integral(1/(x*(a + b*acos(c*x))**3), x)
```

Maxima [N/A]

Not integrable

Time = 2.40 (sec) , antiderivative size = 251, normalized size of antiderivative = 17.93

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^3x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^3,x, algorithm="maxima")
[Out] 1/2*(sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x + b*arctan2(sqrt(c*x + 1)*sqrt(-c*x
 + 1), c*x) + 2*(b^4*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 +
 2*a*b^3*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c^2*x
^2)*integrate(1/(b^3*c^2*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a
*b^2*c^2*x^3), x) + a)/(b^4*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c
*x)^2 + 2*a*b^3*c^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^
2*c^2*x^2)
```

Giac [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^3x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^3,x, algorithm="giac")
[Out] integrate(1/((b*arccos(c*x) + a)^3*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+b\arccos(cx))^3}\,dx=\int \frac{1}{x\,(a+b\,\mathrm{acos}\,(c\,x))^3}\,dx$$

```
[In] int(1/(x*(a + b*acos(c*x))^3),x)
[Out] int(1/(x*(a + b*acos(c*x))^3), x)
```

3.172 $\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx = \operatorname{Int}\left(\frac{1}{x^2(a+b\arccos(cx))^3},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*arccos(c*x))^3,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx = \int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx$$

```
[In] Int[1/(x^2*(a + b*ArcCos[c*x])^3),x]
```

```
[Out] Defer[Int][1/(x^2*(a + b*ArcCos[c*x])^3), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx$$

Mathematica [N/A]

Not integrable

Time = 24.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx=\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx$$

```
[In] Integrate[1/(x^2*(a + b*ArcCos[c*x])^3),x]
[Out] Integrate[1/(x^2*(a + b*ArcCos[c*x])^3), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.93 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\left(a+b\arccos\left(cx\right)\right)^3}dx$$

```
[In] int(1/x^2/(a+b*arccos(c*x))^3,x)
[Out] int(1/x^2/(a+b*arccos(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 3.79

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx=\int \frac{1}{\left(b\arccos\left(cx\right)+a\right)^3x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^3,x, algorithm="fricas")
[Out] integral(1/(b^3*x^2*arccos(c*x)^3 + 3*a*b^2*x^2*arccos(c*x)^2 + 3*a^2*b*x^2
*arccos(c*x) + a^3*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 1.72 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx = \int \frac{1}{x^2\,(a+b\,\mathrm{acos}\,(cx))^3}\,dx$$

```
[In] integrate(1/x**2/(a+b*acos(c*x))**3,x)
[Out] Integral(1/(x**2*(a + b*acos(c*x))**3), x)
```

Maxima [N/A]

Not integrable

Time = 2.89 (sec) , antiderivative size = 284, normalized size of antiderivative = 20.29

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx = \int \frac{1}{(b\arccos(cx)+a)^3x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^3,x, algorithm="maxima")
[Out] -1/2*(a*c^2*x^2 - sqrt(c*x + 1)*sqrt(-c*x + 1)*b*c*x + (b*c^2*x^2 - 2*b)*ar
ctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + 2*(b^4*c^2*x^3*arctan2(sqrt(c*x
+ 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c
*x + 1), c*x) + a^2*b^2*c^2*x^3)*integrate(1/2*(c^2*x^2 - 6)/(b^3*c^2*x^4*a
rctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b^2*c^2*x^4), x) - 2*a)/(b^4*
c^2*x^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 2*a*b^3*c^2*x^3*arct
an2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a^2*b^2*c^2*x^3)
```

Giac [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx = \int \frac{1}{(b\arccos(cx)+a)^3x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^3,x, algorithm="giac")
[Out] integrate(1/((b*arccos(c*x) + a)^3*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+b\arccos(cx))^3}\,dx = \int \frac{1}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^3}\,dx$$

```
[In] int(1/(x^2*(a + b*acos(c*x))^3),x)
[Out] int(1/(x^2*(a + b*acos(c*x))^3), x)
```

3.173 $\int x^2\sqrt{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 242

$$\int x^2\sqrt{a+b\arccos(cx)}\,dx=\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c^3}-\frac{\sqrt{b}\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{12c^3}-\frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{4c^3}-\frac{\sqrt{b}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{12c^3}$$

```
[Out] -1/72*cos(3*a/b)*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))
*b^(1/2)*6^(1/2)*Pi^(1/2)/c^3-1/72*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*
x))^(1/2)/b^(1/2))*sin(3*a/b)*b^(1/2)*6^(1/2)*Pi^(1/2)/c^3-1/8*cos(a/b)*Fre
snelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*b^(1/2)*2^(1/2)*Pi^
(1/2)/c^3-1/8*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*si
n(a/b)*b^(1/2)*2^(1/2)*Pi^(1/2)/c^3+1/3*x^3*(a+b*arccos(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4726, 4810, 3393, 3387, 3386, 3432, 3385, 3433}

$$\int x^2\sqrt{a+b\arccos(cx)}\,dx=-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c^3}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{b}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{12c^3}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c^3}-\frac{\sqrt{\frac{\pi}{6}}\sqrt{b}\sin\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{12c^3}+\frac{1}{3}x^3\sqrt{a+b\arccos(cx)}$$

```
[In] Int[x^2*Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] (x^3*Sqrt[a + b*ArcCos[c*x]])/3 - (Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sq
rt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(4*c^3) - (Sqrt[b]*Sqrt[Pi/6]*C
os[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(12*c^3
) - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[
b]]*Sin[a/b])/(4*c^3) - (Sqrt[b]*Sqrt[Pi/6]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b
*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(12*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
```

```
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3\sqrt{a+b\arccos(cx)}+\frac{1}{6}(bc)\int\frac{x^3}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}\,dx\\
&= \frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\text{Subst}\left(\int\frac{\cos^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{6c^3}\\
&= \frac{1}{3}x^3\sqrt{a+b\arccos(cx)}-\frac{\text{Subst}\left(\int\left(\frac{\cos\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}}+\frac{3\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx,x,a+b\arccos(cx)\right)}{6c^3}
\end{aligned}
$$

$$= \frac{1}{3}x^3\sqrt{a+b\arccos(cx)} - \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{24c^3} - \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{8c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\arccos(cx)} - \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{8c^3} - \frac{\cos\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{24c^3} - \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{8c^3} - \frac{\sin\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{24c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\arccos(cx)} - \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{4c^3} - \frac{\cos\left(\frac{3a}{b}\right)\text{Subst}\left(\int \cos\left(\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{12c^3} - \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{4c^3} - \frac{\sin\left(\frac{3a}{b}\right)\text{Subst}\left(\int \sin\left(\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{12c^3}$$

$$= \frac{1}{3}x^3\sqrt{a+b\arccos(cx)} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c^3} - \frac{\sqrt{b}\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{12c^3} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{4c^3} - \frac{\sqrt{b}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{12c^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.33 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.95

$$\int x^2\sqrt{a+b\arccos(cx)}\,dx =$$

$$-\frac{ibe^{-\frac{3ia}{b}}\left(-9e^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+9e^{\frac{4ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},\frac{i(a+b\arccos(cx))}{b}\right)+\sqrt{3}\right.}{72c^3\sqrt{a+b\arccos}}$$

```
[In] Integrate[x^2*Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] ((-1/72*I)*b*(-9*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3
/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 9*E^(((4*I)*a)/b)*Sqrt[(I*(a + b*ArcCos
[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(-(Sqrt[((-I)*(a
 + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*ArcCos[c*x]))/b]) + E^(((6*
I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((3*I)*(a + b*ArcCos[c*
x]))/b])))/(c^3*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Maple [A] (verified)

Time = 1.89 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.50

method	result
default	$\frac{9\sin\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,b-\cos\left(\frac{3a}{b}\right)\operatorname{FresnelC}\left(\frac{3\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{3}{b}}\,b}\right)\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{3}{b}}\,\sqrt{a+}}{}$

```
[In] int(x^2*(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/72/c^3/(a+b*arccos(c*x))^(1/2)*(9*sin(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/
b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arcc
os(c*x))^(1/2)*b-cos(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*a
rccos(c*x))^(1/2)/b)*2^(1/2)*Pi^(1/2)*(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*
b-9*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*cos(a
/b)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*b+sin(3*a/b)*Fres
nelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*2^(1/2)*Pi^
(1/2)*(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*b+18*arccos(c*x)*cos(-(a+b*arcco
s(c*x))/b+a/b)*b+18*cos(-(a+b*arccos(c*x))/b+a/b)*a+6*arccos(c*x)*cos(-3*(a
+b*arccos(c*x))/b+3*a/b)*b+6*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*a)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2 \sqrt{a + b \arccos(cx)}\, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2 \sqrt{a + b \arccos(cx)}\, dx = \int x^2 \sqrt{a + b \operatorname{acos}(cx)}\, dx$$

```
[In] integrate(x**2*(a+b*acos(c*x))**(1/2),x)
[Out] Integral(x**2*sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int x^2 \sqrt{a + b \arccos(cx)}\, dx = \int \sqrt{b \arccos(cx) + a} x^2\, dx$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arccos(c*x) + a)*x^2, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.96 (sec) , antiderivative size = 1057, normalized size of antiderivative = 4.37

$$\int x^2 \sqrt{a + b \arccos(cx)}\, dx = \text{Too large to display}$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
[Out] -1/8*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt
(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((
I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c^3) + 1/16*sqrt(2)*sqrt(pi)*b^2*erf(-
```

```
1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arc
cos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b
)))*c^3) + 1/8*I*sqrt(2)*sqrt(pi)*a*b*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x)
+ a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(
-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c^3) + 1/16*sqrt(2)*sqrt(pi
)*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*
sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) +
b*sqrt(abs(b)))*c^3) - 1/4*I*sqrt(pi)*a*sqrt(b)*erf(-1/2*sqrt(6)*sqrt(b*arc
cos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b
))*e^(3*I*a/b)/((sqrt(6)*b + I*sqrt(6)*b^2/abs(b))*c^3) + 1/24*sqrt(pi)*b^(
3/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(
b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*b + I*sqrt(6)*b^2/
abs(b))*c^3) + 1/4*I*sqrt(pi)*a*sqrt(b)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x)
 + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3
*I*a/b)/((sqrt(6)*b - I*sqrt(6)*b^2/abs(b))*c^3) + 1/24*sqrt(pi)*b^(3/2)*er
f(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arcco
s(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*b - I*sqrt(6)*b^2/abs(b)
)*c^3) + 1/4*I*sqrt(pi)*a*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b)
- 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(
6)*sqrt(b) + I*sqrt(6)*b^(3/2)/abs(b))*c^3) + 1/4*I*sqrt(pi)*a*erf(-1/2*I*s
qrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x
) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c^3*(I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*s
qrt(abs(b)))) - 1/4*I*sqrt(pi)*a*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/
sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/
b)/(c^3*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - 1/4*I*sqrt(pi
)*a*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b
*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*sqrt(b) - I*sqrt(6
)*b^(3/2)/abs(b))*c^3) + 1/24*sqrt(b*arccos(c*x) + a)*e^(3*I*arccos(c*x))/c
^3 + 1/8*sqrt(b*arccos(c*x) + a)*e^(I*arccos(c*x))/c^3 + 1/8*sqrt(b*arccos(
c*x) + a)*e^(-I*arccos(c*x))/c^3 + 1/24*sqrt(b*arccos(c*x) + a)*e^(-3*I*arc
cos(c*x))/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{a+b\arccos(cx)}\,dx=\int x^2\,\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

```
[In] int(x^2*(a + b*acos(c*x))^(1/2),x)
[Out] int(x^2*(a + b*acos(c*x))^(1/2), x)
```

3.174 $\int x\sqrt{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 137

$$\int x\sqrt{a+b\arccos(cx)}\,dx=-\frac{\sqrt{a+b\arccos(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}-\frac{\sqrt{b}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8c^2}-\frac{\sqrt{b}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{8c^2}$$

```
[Out] -1/8*cos(2*a/b)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*b^(1/2
)*Pi^(1/2)/c^2-1/8*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin
(2*a/b)*b^(1/2)*Pi^(1/2)/c^2-1/4*(a+b*arccos(c*x))^(1/2)/c^2+1/2*x^2*(a+b*a
rccos(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {4726, 4810, 3393, 3387, 3386, 3432, 3385, 3433}

$$\int x\sqrt{a+b\arccos(cx)}\,dx=-\frac{\sqrt{\pi}\sqrt{b}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8c^2}-\frac{\sqrt{\pi}\sqrt{b}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8c^2}-\frac{\sqrt{a+b\arccos(cx)}}{4c^2}+\frac{1}{2}x^2\sqrt{a+b\arccos(cx)}$$

[In] Int[x*Sqrt[a + b*ArcCos[c*x]],x]

[Out] -1/4*Sqrt[a + b*ArcCos[c*x]]/c^2 + (x^2*Sqrt[a + b*ArcCos[c*x]])/2 - (Sqrt[b]*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/(8*c^2) - (Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/(8*c^2)

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
```

```
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2\sqrt{a+b\arccos(cx)} + \frac{1}{4}(bc)\int \frac{x^2}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}\,dx$$

$$= \frac{1}{2}x^2\sqrt{a+b\arccos(cx)} - \frac{\text{Subst}\left(\int \frac{\cos^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{4c^2}$$

$$= \frac{1}{2}x^2\sqrt{a+b\arccos(cx)} - \frac{\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\cos\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)\,dx, x, a+b\arccos(cx)\right)}{4c^2}$$

$$= -\frac{\sqrt{a+b\arccos(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\arccos(cx)} - \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{8c^2}$$

$$= -\frac{\sqrt{a+b\arccos(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\arccos(cx)} - \frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{8c^2} - \frac{\sin\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{8c^2}$$

$$= -\frac{\sqrt{a+b\arccos(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\arccos(cx)} - \frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \cos\left(\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{4c^2} - \frac{\sin\left(\frac{2a}{b}\right)\text{Subst}\left(\int \sin\left(\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{4c^2}$$

$$= -\frac{\sqrt{a+b\arccos(cx)}}{4c^2} + \frac{1}{2}x^2\sqrt{a+b\arccos(cx)} - \frac{\sqrt{b}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{8c^2} - \frac{\sqrt{b}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{8c^2}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int x\sqrt{a+b\arccos(cx)}\,dx$$
$$= \frac{2\sqrt{a+b\arccos(cx)}\cos(2\arccos(cx))-\sqrt{b}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-\sqrt{b}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{a+}}{}\right.}{8c^2}$$

[In] Integrate[x*Sqrt[a + b*ArcCos[c*x]],x]

[Out] (2*Sqrt[a + b*ArcCos[c*x]]*Cos[2*ArcCos[c*x]] - Sqrt[b]*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])] - Sqrt[b]*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/(8*c^2)

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.36

method	result
default	$\frac{-\sqrt{\pi}\sqrt{-\frac{1}{b}}\sqrt{a+b\arccos(cx)}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{2}\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{-\frac{2}{b}}\,b}\right)b+\sqrt{\pi}\sqrt{-\frac{1}{b}}\sqrt{a+b\arccos(cx)}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{2}\sqrt{a+b\arccos}}{\sqrt{\pi}\sqrt{-\frac{2}{b}}}\right.}{8c^2\sqrt{a+b\arccos(cx)}}$

[In] int(x*(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/8/c^2/(a+b*arccos(c*x))^(1/2)*(-Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b+Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b+2*arccos(c*x)*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*b+2*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*a)

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+b\arccos(cx)}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x*(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

$$\int x\sqrt{a+b\arccos(cx)}\,dx=\int x\sqrt{a+b\operatorname{acos}(cx)}\,dx$$

```
[In] integrate(x*(a+b*acos(c*x))**(1/2),x)
[Out] Integral(x*sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int x\sqrt{a+b\arccos(cx)}\,dx=\int\sqrt{b\arccos(cx)+a}x\,dx$$

```
[In] integrate(x*(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arccos(c*x) + a)*x, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 448, normalized size of antiderivative = 3.27

$$\int x\sqrt{a+b\arccos(cx)}\,dx = -\frac{i\sqrt{\pi}a\sqrt{b}\operatorname{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}-\frac{i\sqrt{b\arccos(cx)+a}\sqrt{b}}{|b|}\right)e^{\left(\frac{2ia}{b}\right)}}{4\left(b+\frac{ib^2}{|b|}\right)c^2}$$
$$+\frac{\sqrt{\pi}b^{\frac{3}{2}}\operatorname{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}-\frac{i\sqrt{b\arccos(cx)+a}\sqrt{b}}{|b|}\right)e^{\left(\frac{2ia}{b}\right)}}{16\left(b+\frac{ib^2}{|b|}\right)c^2}$$
$$+\frac{i\sqrt{\pi}a\sqrt{b}\operatorname{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}+\frac{i\sqrt{b\arccos(cx)+a}\sqrt{b}}{|b|}\right)e^{\left(-\frac{2ia}{b}\right)}}{4\left(b-\frac{ib^2}{|b|}\right)c^2}$$
$$+\frac{\sqrt{\pi}b^{\frac{3}{2}}\operatorname{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}+\frac{i\sqrt{b\arccos(cx)+a}\sqrt{b}}{|b|}\right)e^{\left(-\frac{2ia}{b}\right)}}{16\left(b-\frac{ib^2}{|b|}\right)c^2}$$
$$-\frac{i\sqrt{\pi}a\operatorname{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}+\frac{i\sqrt{b\arccos(cx)+a}\sqrt{b}}{|b|}\right)e^{\left(-\frac{2ia}{b}\right)}}{4c^2\left(\sqrt{b}-\frac{ib^{\frac{3}{2}}}{|b|}\right)}$$
$$+\frac{i\sqrt{\pi}a\operatorname{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}-\frac{i\sqrt{b\arccos(cx)+a}\sqrt{b}}{|b|}\right)e^{\left(\frac{2ia}{b}\right)}}{4\sqrt{b}c^2\left(\frac{ib}{|b|}+1\right)}$$
$$+\frac{\sqrt{b\arccos(cx)+a}e^{(2i\arccos(cx))}}{8c^2}$$
$$+\frac{\sqrt{b\arccos(cx)+a}e^{(-2i\arccos(cx))}}{8c^2}$$

```
[In] integrate(x*(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(pi)*a*sqrt(b)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*a
rccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2) + 1/16
*sqrt(pi)*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos(c*
x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2) + 1/4*I*sqrt(p
i)*a*sqrt(b)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) +
a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b - I*b^2/abs(b))*c^2) + 1/16*sqrt(pi)*b^
(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) + a)*sqrt
(b)/abs(b))*e^(-2*I*a/b)/((b - I*b^2/abs(b))*c^2) - 1/4*I*sqrt(pi)*a*erf(-s
qrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*
e^(-2*I*a/b)/(c^2*(sqrt(b) - I*b^(3/2)/abs(b))) + 1/4*I*sqrt(pi)*a*erf(-sqr
t(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^
(2*I*a/b)/(sqrt(b)*c^2*(I*b/abs(b) + 1)) + 1/8*sqrt(b*arccos(c*x) + a)*e^(2
*I*arccos(c*x))/c^2 + 1/8*sqrt(b*arccos(c*x) + a)*e^(-2*I*arccos(c*x))/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{a+b\arccos(cx)}\,dx=\int x\,\sqrt{a+b\,\mathrm{acos}\,(c\,x)}\,dx$$

```
[In] int(x*(a + b*acos(c*x))^(1/2),x)
[Out] int(x*(a + b*acos(c*x))^(1/2), x)
```

3.175 $\int \sqrt{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 121

$$\int \sqrt{a+b\arccos(cx)}\,dx = x\sqrt{a+b\arccos(cx)} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{c} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{c}$$

```
[Out] -1/2*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*b^
(1/2)*2^(1/2)*Pi^(1/2)/c-1/2*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1
/2)/b^(1/2))*sin(a/b)*b^(1/2)*2^(1/2)*Pi^(1/2)/c+x*(a+b*arccos(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4716, 4810, 3387, 3386, 3432, 3385, 3433}

$$\int \sqrt{a+b\arccos(cx)}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{c} - \frac{\sqrt{\frac{\pi}{2}}\sqrt{b}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{c} + x\sqrt{a+b\arccos(cx)}$$

```
[In] Int[Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] x*Sqrt[a + b*ArcCos[c*x]] - (Sqrt[b]*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/P
i]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/c - (Sqrt[b]*Sqrt[Pi/2]*FresnelS[(Sqr
t[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/c
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
```

```
0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= x\sqrt{a+b\arccos(cx)} + \frac{1}{2}(bc)\int \frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}\,dx \\
&= x\sqrt{a+b\arccos(cx)} - \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{2c} \\
&= x\sqrt{a+b\arccos(cx)} - \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{2c} \\
&\quad - \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{2c} \\
&= x\sqrt{a+b\arccos(cx)} - \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{c} \\
&\quad - \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{c} \\
&= x\sqrt{a+b\arccos(cx)} - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{c} \\
&\quad - \frac{\sqrt{b}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{c}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int \sqrt{a+b\arccos(cx)}\,dx = -\frac{ibe^{-\frac{ia}{b}}\left(-\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},\frac{i(a+b\arccos(cx))}{b}\right)\right)}{2c\sqrt{a+b\arccos(cx)}}$$

```
[In] Integrate[Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] ((-1/2*I)*b*(-(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-I)*(a + b*A
rcCos[c*x]))/b]) + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/
2, (I*(a + b*ArcCos[c*x]))/b]))/(c*E^((I*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Maple [A] (verified)

Time = 1.84 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.54

method	result
default	$\frac{-\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\cos\left(\frac{a}{b}\right)\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+b\arccos(cx)}\,b+\sin\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sqrt{a+}}{2c\sqrt{a+b\arccos(cx)}}$

```
[In] int((a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c/(a+b*arccos(c*x))^(1/2)*(-FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b
*arccos(c*x))^(1/2)/b)*cos(a/b)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c
*x))^(1/2)*b+sin(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*
x))^(1/2)/b)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)*b+2*arcc
os(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*b+2*cos(-(a+b*arccos(c*x))/b+a/b)*a)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{a+b\arccos(cx)}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{a+b\arccos(cx)}\,dx = \int \sqrt{a+b\operatorname{acos}(cx)}\,dx$$

```
[In] integrate((a+b*acos(c*x))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\arccos(cx)}\,dx = \int \sqrt{b\arccos{(cx)}+a}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 531, normalized size of antiderivative = 4.39

$$\begin{aligned}\int \sqrt{a+b\arccos(cx)}\,dx = &-\frac{i\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(\frac{i\,a}{b}\right)}}{2\left(\frac{i\,b^2}{\sqrt{|b|}}+b\sqrt{|b|}\right)c} \\ &+\frac{\sqrt{2}\sqrt{\pi}b^2\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(\frac{i\,a}{b}\right)}}{4\left(\frac{i\,b^2}{\sqrt{|b|}}+b\sqrt{|b|}\right)c} \\ &+\frac{i\sqrt{2}\sqrt{\pi}ab\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(-\frac{i\,a}{b}\right)}}{2\left(-\frac{i\,b^2}{\sqrt{|b|}}+b\sqrt{|b|}\right)c} \\ &+\frac{\sqrt{2}\sqrt{\pi}b^2\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(-\frac{i\,a}{b}\right)}}{4\left(-\frac{i\,b^2}{\sqrt{|b|}}+b\sqrt{|b|}\right)c} \\ &+\frac{i\sqrt{\pi}a\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(\frac{i\,a}{b}\right)}}{c\left(\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)} \\ &-\frac{i\sqrt{\pi}a\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}}-\frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2\,b}\right)e^{\left(-\frac{i\,a}{b}\right)}}{c\left(-\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)} \\ &+\frac{\sqrt{b\arccos{(cx)}+a}e^{(i\arccos(cx))}}{2\,c} \\ &+\frac{\sqrt{b\arccos{(cx)}+a}e^{(-i\arccos(cx))}}{2\,c}\end{aligned}$$

```
[In] integrate((a+b*arccos(c*x))^(1/2),x, algorithm="giac")
[Out] -1/2*I*sqrt(2)*sqrt(pi)*a*b*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt
(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((
```

```
I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) + 1/4*sqrt(2)*sqrt(pi)*b^2*erf(-1/2
*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos
(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))
*c) + 1/2*I*sqrt(2)*sqrt(pi)*a*b*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/
sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/
b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) + 1/4*sqrt(2)*sqrt(pi)*b^2*er
f(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*a
rccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(a
bs(b)))*c) + I*sqrt(pi)*a*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(a
bs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c*(
I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)*a*erf(1/2*I*
sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*
x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(c*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*
sqrt(abs(b)))) + 1/2*sqrt(b*arccos(c*x) + a)*e^(I*arccos(c*x))/c + 1/2*sqrt
(b*arccos(c*x) + a)*e^(-I*arccos(c*x))/c
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b \arccos(cx)}\, dx = \int \sqrt{a + b \operatorname{acos}(c\, x)}\, dx$$

```
[In] int((a + b*acos(c*x))^(1/2),x)
[Out] int((a + b*acos(c*x))^(1/2), x)
```

3.176 $\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx = \operatorname{Int}\left(\frac{\sqrt{a+b\arccos(cx)}}{x},x\right)$$

```
[Out] Unintegrable((a+b*arccos(c*x))^(1/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx = \int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx$$

```
[In] Int[Sqrt[a + b*ArcCos[c*x]]/x,x]
```

```
[Out] Defer[Int][Sqrt[a + b*ArcCos[c*x]]/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx = \int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx$$

```
[In] Integrate[Sqrt[a + b*ArcCos[c*x]]/x,x]
[Out] Integrate[Sqrt[a + b*ArcCos[c*x]]/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\arccos\left(cx\right)}}{x}dx$$

```
[In] int((a+b*arccos(c*x))^(1/2)/x,x)
[Out] int((a+b*arccos(c*x))^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx = \int \frac{\sqrt{a+b\operatorname{acos}\left(cx\right)}}{x}\,dx$$

```
[In] integrate((a+b*acos(c*x))**(1/2)/x,x)
[Out] Integral(sqrt(a + b*acos(c*x))/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx = \int \frac{\sqrt{b\arccos(cx)+a}}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x,x, algorithm="maxima")
[Out] integrate(sqrt(b*arccos(c*x) + a)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.92 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx = \int \frac{\sqrt{b\arccos(cx)+a}}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x,x, algorithm="giac")
[Out] integrate(sqrt(b*arccos(c*x) + a)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x}\,dx = \int \frac{\sqrt{a+b\operatorname{acos}(c\,x)}}{x}\,dx$$

```
[In] int((a + b*acos(c*x))^(1/2)/x,x)
[Out] int((a + b*acos(c*x))^(1/2)/x, x)
```

3.177 $\int \frac{\sqrt{a+b \arccos(cx)}}{x^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\sqrt{a+b \arccos(cx)}}{x^2} dx = \text{Int}\left(\frac{\sqrt{a+b \arccos(cx)}}{x^2}, x\right)$$

```
[Out] Unintegrable((a+b*arccos(c*x))^(1/2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{a+b \arccos(cx)}}{x^2} dx = \int \frac{\sqrt{a+b \arccos(cx)}}{x^2} dx$$

```
[In] Int[Sqrt[a + b*ArcCos[c*x]]/x^2,x]
[Out] Defer[Int][Sqrt[a + b*ArcCos[c*x]]/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{a+b \arccos(cx)}}{x^2} dx$$

Mathematica [N/A]

Not integrable

Time = 7.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x^2}\,dx = \int \frac{\sqrt{a+b\arccos(cx)}}{x^2}\,dx$$

```
[In] Integrate[Sqrt[a + b*ArcCos[c*x]]/x^2,x]
[Out] Integrate[Sqrt[a + b*ArcCos[c*x]]/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{a+b\arccos\left(cx\right)}}{x^2}dx$$

```
[In] int((a+b*arccos(c*x))^(1/2)/x^2,x)
[Out] int((a+b*arccos(c*x))^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{a+b\arccos(cx)}}{x^2}\,dx = \int \frac{\sqrt{a+b\operatorname{acos}\left(cx\right)}}{x^2}\,dx$$

```
[In] integrate((a+b*acos(c*x))**(1/2)/x**2,x)
[Out] Integral(sqrt(a + b*acos(c*x))/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.55 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b \arccos(cx)}}{x^2} \, dx = \int \frac{\sqrt{b \arccos{(cx)} + a}}{x^2} \, dx$$

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(b*arccos(c*x) + a)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b \arccos(cx)}}{x^2} \, dx = \int \frac{\sqrt{b \arccos{(cx)} + a}}{x^2} \, dx$$

```
[In] integrate((a+b*arccos(c*x))^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(b*arccos(c*x) + a)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a + b \arccos(cx)}}{x^2} \, dx = \int \frac{\sqrt{a + b \operatorname{acos}{(c\,x)}}}{x^2} \, dx$$

```
[In] int((a + b*acos(c*x))^(1/2)/x^2,x)
[Out] int((a + b*acos(c*x))^(1/2)/x^2, x)
```

3.178 $\int x^2(a+b\arccos(cx))^{3/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 313

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx = -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c} + \frac{1}{3}x^3(a+b\arccos(cx))^{3/2} + \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8c^3} + \frac{b^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{24c^3} - \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{8c^3} - \frac{b^{3/2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{24c^3}$$

```
[Out] 1/3*x^3*(a+b*arccos(c*x))^(3/2)+1/144*b^(3/2)*cos(3*a/b)*FresnelS(6^(1/2)/P
i^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/c^3-1/144*b^(3/2)
*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*a/b)*6^(1
/2)*Pi^(1/2)/c^3+3/16*b^(3/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arcco
s(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/c^3-3/16*b^(3/2)*FresnelC(2^(1/2)/P
i^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/c^3-1/3*
b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(1/2)/c^3-1/6*b*x^2*(-c^2*x^2+1)^(1/
2)*(a+b*arccos(c*x))^(1/2)/c
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {4726, 4796, 4768, 4720, 3387, 3386, 3432, 3385, 3433, 4732, 4491}

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx = -\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8c^3} - \frac{\sqrt{\frac{\pi}{6}}b^{3/2}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{24c^3} + \frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8c^3} + \frac{\sqrt{\frac{\pi}{6}}b^{3/2}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{24c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c} - \frac{b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^3} + \frac{1}{3}x^3(a+b\arccos(cx))^{3/2}$$

```
[In] Int[x^2*(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] -1/3*(b*Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcCos[c*x]])/c^3 - (b*x^2*Sqrt[1 - c^
2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(6*c) + (x^3*(a + b*ArcCos[c*x])^(3/2))/3 +
 (3*b^(3/2)*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]
])/Sqrt[b]])/(8*c^3) + (b^(3/2)*Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi
]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(24*c^3) - (3*b^(3/2)*Sqrt[Pi/2]*Fresn
elC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(8*c^3) - (b^(3
/2)*Sqrt[Pi/6]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(
3*a)/b])/(24*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
```

```
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3(a+b\arccos(cx))^{3/2}+\frac{1}{2}(bc)\int\frac{x^3\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c}+\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}\\
&\quad -\frac{1}{12}b^2\int\frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx+\frac{b\int\frac{x\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}\,dx}{3c}\\
&= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^3}\\
&\quad -\frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c}+\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}\\
&\quad -\frac{b\text{Subst}\left(\int\frac{\cos^2\left(\frac{a}{b}-\frac{x}{b}\right)\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{12c^3}-\frac{b^2\int\frac{1}{\sqrt{a+b\arccos(cx)}}\,dx}{6c^2}\\
&= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^3}\\
&\quad -\frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c}+\frac{1}{3}x^3(a+b\arccos(cx))^{3/2}\\
&\quad -\frac{b\text{Subst}\left(\int\left(\frac{\sin\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}}+\frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx,x,a+b\arccos(cx)\right)}{12c^3}\\
&\quad -\frac{b\text{Subst}\left(\int\frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{6c^3}
\end{aligned}
$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c} + \frac{1}{3}x^3(a+b\arccos(cx))^{3/2} - \frac{b\text{Subst}\left(\int \frac{\sin\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{48c^3} - \frac{b\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{48c^3} + \frac{\left(b\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{6c^3} - \frac{\left(b\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{6c^3}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c} + \frac{1}{3}x^3(a+b\arccos(cx))^{3/2} + \frac{\left(b\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{48c^3} + \frac{\left(b\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{3c^3} + \frac{\left(b\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{48c^3} - \frac{\left(b\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{48c^3} - \frac{\left(b\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{3c^3} - \frac{\left(b\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{48c^3}$$

$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c}$$
$$+ \frac{1}{3}x^3(a+b\arccos(cx))^{3/2} + \frac{b^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3c^3}$$
$$- \frac{b^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{3c^3}$$
$$+ \frac{\left(b\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{24c^3}$$
$$+ \frac{\left(b\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\sin\left(\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{24c^3}$$
$$- \frac{\left(b\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{24c^3}$$
$$- \frac{\left(b\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\cos\left(\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{24c^3}$$
$$= -\frac{b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{3c^3} - \frac{bx^2\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{6c}$$
$$+ \frac{1}{3}x^3(a+b\arccos(cx))^{3/2} + \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{8c^3}$$
$$+ \frac{b^{3/2}\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{24c^3}$$
$$- \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{8c^3}$$
$$- \frac{b^{3/2}\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{24c^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 7.08 (sec) , antiderivative size = 555, normalized size of antiderivative = 1.77

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx =$$

$$-\frac{iabe^{-\frac{3ia}{b}}\left(-9e^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+9e^{\frac{4ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},\frac{i(a+b\arccos(cx))}{b}\right)+\sqrt{3}\left(\right.\right.}{72c^3\sqrt{a+b\arccos(c}}$$

$$-\frac{\sqrt{b}\left(18\sqrt{b}\sqrt{a+b\arccos(cx)}(3\sqrt{1-c^2x^2}-2cx\arccos(cx))-9\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)(3b\cos\left(\right.\right.}{}$$

```
[In] Integrate[x^2*(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] ((-1/72*I)*a*b*(-9*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma
[3/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 9*E^(((4*I)*a)/b)*Sqrt[(I*(a + b*ArcC
os[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(-(Sqrt[((-I)*
(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*ArcCos[c*x]))/b]) + E^(((
6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((3*I)*(a + b*ArcCos[
c*x]))/b])))/(c^3*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]]) - (Sqrt[b]*(18*S
qrt[b]*Sqrt[a + b*ArcCos[c*x]]*(3*Sqrt[1 - c^2*x^2] - 2*c*x*ArcCos[c*x]) -
9*Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(3*b*Co
s[a/b] + 2*a*Sin[a/b]) - 9*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCo
s[c*x]])/Sqrt[b]]*(2*a*Cos[a/b] - 3*b*Sin[a/b]) - Sqrt[6*Pi]*FresnelS[(Sqrt
[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(b*Cos[(3*a)/b] + 2*a*Sin[(3*a)/b]
) - Sqrt[6*Pi]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(2*a*
Cos[(3*a)/b] - b*Sin[(3*a)/b]) + 6*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(-2*ArcC
os[c*x]*Cos[3*ArcCos[c*x]] + Sin[3*ArcCos[c*x]])))/(144*c^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(241) = 482$.

Time = 2.03 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.75

method	result
default	$\frac{-\sqrt{a+b\arccos(cx)}\sqrt{-\frac{3}{b}}\,\text{FresnelC}\left(\frac{3\sqrt{2}\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{-\frac{3}{b}}\,b}\right)\sin\left(\frac{3a}{b}\right)\sqrt{\pi}\sqrt{2}\,b^2-27\sqrt{a+b\arccos(cx)}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{2}\sqrt{a+b\arccos(cx}}{\sqrt{\pi}\sqrt{-\frac{1}{b}}\,b}\right.}{}$

```
[In] int(x^2*(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/144/c^3/(a+b*arccos(c*x))^(1/2)*(-(a+b*arccos(c*x))^(1/2)*(-3/b)^(1/2)*Fr
esnelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*sin(3*a/b
)*Pi^(1/2)*2^(1/2)*b^2-27*(a+b*arccos(c*x))^(1/2)*cos(a/b)*FresnelS(2^(1/2)
/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-1/b)^(1/2)*Pi^(1/2)*2^(
```

```
1/2)*b^2-27*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/
b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*b^2-(a+b*
arccos(c*x))^(1/2)*(-3/b)^(1/2)*cos(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/
b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*Pi^(1/2)*2^(1/2)*b^2+36*arccos(c*x)^2*c
os(-(a+b*arccos(c*x))/b+a/b)*b^2+12*arccos(c*x)^2*cos(-3*(a+b*arccos(c*x))/
b+3*a/b)*b^2+72*arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*a*b+54*arccos(c*x
)*sin(-(a+b*arccos(c*x))/b+a/b)*b^2+24*arccos(c*x)*cos(-3*(a+b*arccos(c*x))
/b+3*a/b)*a*b+6*arccos(c*x)*sin(-3*(a+b*arccos(c*x))/b+3*a/b)*b^2+36*cos(-(
a+b*arccos(c*x))/b+a/b)*a^2+54*sin(-(a+b*arccos(c*x))/b+a/b)*a*b+12*cos(-3*
(a+b*arccos(c*x))/b+3*a/b)*a^2+6*sin(-3*(a+b*arccos(c*x))/b+3*a/b)*a*b)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx = \int x^2(a+b\operatorname{acos}(cx))^{\frac{3}{2}}\,dx$$

```
[In] integrate(x**2*(a+b*acos(c*x))**(3/2),x)
[Out] Integral(x**2*(a + b*acos(c*x))**(3/2), x)
```

Maxima [F]

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx = \int (b\arccos(cx)+a)^{\frac{3}{2}}x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccos(c*x) + a)^(3/2)*x^2, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.71 (sec) , antiderivative size = 1967, normalized size of antiderivative = 6.28

$$\int x^2(a + b\arccos(cx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*I*sqrt(2)*sqrt(pi)*a^2*b^2*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/
sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b
)/((I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c^3) + 1/8*sqrt(2)*sqrt(pi)*a*b^
3*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqr
t(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^3/sqrt(abs(b)) + b^2*s
qrt(abs(b)))*c^3) + 1/8*I*sqrt(2)*sqrt(pi)*a^2*b^2*erf(1/2*I*sqrt(2)*sqrt(b
*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(a
bs(b))/b)*e^(-I*a/b)/((-I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c^3) + 1/8*s
qrt(2)*sqrt(pi)*a*b^3*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)
) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^3
/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c^3) - 1/4*I*sqrt(pi)*a^2*b^(3/2)*erf(-1/
2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x
) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*b^2 + I*sqrt(6)*b^3/abs(b))*c^
3) + 1/12*sqrt(pi)*a*b^(5/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(
b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sq
rt(6)*b^2 + I*sqrt(6)*b^3/abs(b))*c^3) - 1/8*sqrt(2)*sqrt(pi)*a*b^2*erf(-1/
2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arcco
s(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b))
)*c^3) - 3/32*I*sqrt(2)*sqrt(pi)*b^3*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x)
+ a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(
I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c^3) - 1/8*sqrt(2)*sqrt(pi)*a
*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*s
qrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b
*sqrt(abs(b)))*c^3) + 3/32*I*sqrt(2)*sqrt(pi)*b^3*erf(1/2*I*sqrt(2)*sqrt(b*
arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(ab
s(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c^3) + 1/4*I*sq
rt(pi)*a^2*b^(3/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I
*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*b^2
 - I*sqrt(6)*b^3/abs(b))*c^3) + 1/12*sqrt(pi)*a*b^(5/2)*erf(-1/2*sqrt(6)*sq
rt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(
b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*b^2 - I*sqrt(6)*b^3/abs(b))*c^3) + 1/4*I*
sqrt(pi)*a^2*b*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqr
t(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*b^(3/2)
+ I*sqrt(6)*b^(5/2)/abs(b))*c^3) - 1/12*sqrt(pi)*a*b^2*erf(-1/2*sqrt(6)*sqr
t(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b
```

```
)/abs(b))*e^(3*I*a/b)/((sqrt(6)*b^(3/2) + I*sqrt(6)*b^(5/2)/abs(b))*c^3) +
1/4*I*sqrt(pi)*a^2*b*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)
) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*sqrt(
2)*b^2/sqrt(abs(b)) + sqrt(2)*b*sqrt(abs(b)))*c^3) - 1/4*I*sqrt(pi)*a^2*b*e
rf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*
arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*sqrt(2)*b^2/sqrt(abs(b)) +
 sqrt(2)*b*sqrt(abs(b)))*c^3) - 1/4*I*sqrt(pi)*a^2*b*erf(-1/2*sqrt(6)*sqrt(
b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/
abs(b))*e^(-3*I*a/b)/((sqrt(6)*b^(3/2) - I*sqrt(6)*b^(5/2)/abs(b))*c^3) - 1
/12*sqrt(pi)*a*b^2*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I
*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*b^(
3/2) - I*sqrt(6)*b^(5/2)/abs(b))*c^3) - 1/48*I*sqrt(pi)*b^(5/2)*erf(-1/2*sq
rt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) +
a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*b + I*sqrt(6)*b^2/abs(b))*c^3) + 1
/48*I*sqrt(pi)*b^(5/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1
/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)
*b - I*sqrt(6)*b^2/abs(b))*c^3) + 1/24*sqrt(b*arccos(c*x) + a)*b*arccos(c*x
)*e^(3*I*arccos(c*x))/c^3 + 1/8*sqrt(b*arccos(c*x) + a)*b*arccos(c*x)*e^(I*
arccos(c*x))/c^3 + 1/8*sqrt(b*arccos(c*x) + a)*b*arccos(c*x)*e^(-I*arccos(c
*x))/c^3 + 1/24*sqrt(b*arccos(c*x) + a)*b*arccos(c*x)*e^(-3*I*arccos(c*x))/
c^3 + 1/24*sqrt(b*arccos(c*x) + a)*a*e^(3*I*arccos(c*x))/c^3 + 1/48*I*sqrt(
b*arccos(c*x) + a)*b*e^(3*I*arccos(c*x))/c^3 + 1/8*sqrt(b*arccos(c*x) + a)*
a*e^(I*arccos(c*x))/c^3 + 3/16*I*sqrt(b*arccos(c*x) + a)*b*e^(I*arccos(c*x)
)/c^3 + 1/8*sqrt(b*arccos(c*x) + a)*a*e^(-I*arccos(c*x))/c^3 - 3/16*I*sqrt(
b*arccos(c*x) + a)*b*e^(-I*arccos(c*x))/c^3 + 1/24*sqrt(b*arccos(c*x) + a)*
a*e^(-3*I*arccos(c*x))/c^3 - 1/48*I*sqrt(b*arccos(c*x) + a)*b*e^(-3*I*arcco
s(c*x))/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arccos(cx))^{3/2}\,dx=\int x^2\,\left(a+b\,\mathrm{acos}(c\,x)\right)^{3/2}\,dx$$

```
[In] int(x^2*(a + b*acos(c*x))^(3/2),x)
```

```
[Out] int(x^2*(a + b*acos(c*x))^(3/2), x)
```

3.179 $\int x(a+b\arccos(cx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 172

$$\int x(a+b\arccos(cx))^{3/2}\,dx = -\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{8c}-\frac{(a+b\arccos(cx))^{3/2}}{4c^2}+\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}+\frac{3b^{3/2}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{32c^2}-\frac{3b^{3/2}\sqrt{\pi}\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{32c^2}$$

```
[Out] -1/4*(a+b*arccos(c*x))^(3/2)/c^2+1/2*x^2*(a+b*arccos(c*x))^(3/2)+3/32*b^(3/
2)*cos(2*a/b)*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*Pi^(1/2)
/c^2-3/32*b^(3/2)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(
2*a/b)*Pi^(1/2)/c^2-3/8*b*x*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(1/2)/c
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4726, 4796, 4738, 4732, 4491, 12, 3387, 3386, 3432, 3385, 3433}

$$\int x(a+b\arccos(cx))^{3/2}\,dx = -\frac{3\sqrt{\pi}b^{3/2}\sin\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{32c^2}+\frac{3\sqrt{\pi}b^{3/2}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{32c^2}-\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{8c}-\frac{(a+b\arccos(cx))^{3/2}}{4c^2}+\frac{1}{2}x^2(a+b\arccos(cx))^{3/2}$$

```
[In] Int[x*(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (-3*b*x*Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(8*c) - (a + b*ArcCos[c*
x])^(3/2)/(4*c^2) + (x^2*(a + b*ArcCos[c*x])^(3/2))/2 + (3*b^(3/2)*Sqrt[Pi]
*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/(32
*c^2) - (3*b^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*S
qrt[Pi])]*Sin[(2*a)/b])/(32*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{2}x^2(a+b\arccos(cx))^{3/2} + \frac{1}{4}(3bc)\int \frac{x^2\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}\,dx \\
&= -\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{8c} + \frac{1}{2}x^2(a+b\arccos(cx))^{3/2} \\
&\quad - \frac{1}{16}(3b^2)\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx + \frac{(3b)\int \frac{\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}\,dx}{8c}
\end{aligned}
$$

$$= -\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{8c} - \frac{(a+b\arccos(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{3/2} - \frac{(3b)\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{16c^2}$$

$$= -\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{8c} - \frac{(a+b\arccos(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{3/2} - \frac{(3b)\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{16c^2}$$

$$= -\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{8c} - \frac{(a+b\arccos(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{3/2} - \frac{(3b)\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{32c^2}$$

$$= -\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{8c} - \frac{(a+b\arccos(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{3/2} + \frac{\left(3b\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{32c^2} - \frac{\left(3b\sin\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{32c^2}$$

$$= -\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{8c} - \frac{(a+b\arccos(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{3/2} + \frac{\left(3b\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \sin\left(\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{16c^2} - \frac{\left(3b\sin\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \cos\left(\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{16c^2}$$

$$= -\frac{3bx\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{8c} - \frac{(a+b\arccos(cx))^{3/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{3/2} + \frac{3b^{3/2}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{32c^2} - \frac{3b^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{32c^2}$$

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.84

$$\int x(a+b\arccos(cx))^{3/2}\,dx = \frac{3b^{3/2}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)-3b^{3/2}\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2\ldots}{\ldots}\right.}{\ldots}$$

```
[In] Integrate[x*(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (3*b^(3/2)*Sqrt[Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt
[b]*Sqrt[Pi])] - 3*b^(3/2)*Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(S
qrt[b]*Sqrt[Pi])]*Sin[(2*a)/b] + 2*Sqrt[a + b*ArcCos[c*x]]*(4*a*Cos[2*ArcCo
s[c*x]] + 4*b*ArcCos[c*x]*Cos[2*ArcCos[c*x]] - 3*b*Sin[2*ArcCos[c*x]]))/(32
*c^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. 2(134) = 268.

Time = 1.96 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.63

method	result
default	$\frac{-3\sqrt{a+b\arccos(cx)}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\;\text{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)\sin\left(\frac{2a}{b}\right)b^2-3\sqrt{a+b\arccos(cx)}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\;\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{a+b\ldots}}{\sqrt{\pi}\,\sqrt{\ldots}}\right.}{\ldots}$

```
[In] int(x*(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/32/c^2/(a+b*arccos(c*x))^(1/2)*(-3*(a+b*arccos(c*x))^(1/2)*Pi^(1/2)*(-1/b
)^(1/2)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)
*sin(2*a/b)*b^2-3*(a+b*arccos(c*x))^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*cos(2*a/b)*
FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^2+8*a
rccos(c*x)^2*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*b^2+16*arccos(c*x)*cos(-2*(a
+b*arccos(c*x))/b+2*a/b)*a*b+6*arccos(c*x)*sin(-2*(a+b*arccos(c*x))/b+2*a/b
)*b^2+8*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*a^2+6*sin(-2*(a+b*arccos(c*x))/b+
2*a/b)*a*b)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+b\arccos(cx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x(a+b\arccos(cx))^{3/2}\,dx = \int x(a+b\operatorname{acos}(cx))^{\frac{3}{2}}\,dx$$

```
[In] integrate(x*(a+b*acos(c*x))**(3/2),x)
[Out] Integral(x*(a + b*acos(c*x))**(3/2), x)
```

Maxima [F]

$$\int x(a+b\arccos(cx))^{3/2}\,dx = \int (b\arccos(cx)+a)^{\frac{3}{2}}x\,dx$$

```
[In] integrate(x*(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccos(c*x) + a)^(3/2)*x, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 845, normalized size of antiderivative = 4.91

$$\int x(a+b\arccos(cx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate(x*(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
[Out] -1/4*I*sqrt(pi)*a^2*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b
*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^2 + I*b^3/abs(b))*c^2) +
1/8*sqrt(pi)*a*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arcc
```

```
os(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^2 + I*b^3/abs(b))*c^2) + 1/4*I
*sqrt(pi)*a^2*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arcco
s(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^2 - I*b^3/abs(b))*c^2) + 1/8*s
qrt(pi)*a*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*
x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^2 - I*b^3/abs(b))*c^2) - 1/8*sqrt(
pi)*a*b^2*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos(c*x) + a)*
sqrt(b)/abs(b))*e^(2*I*a/b)/((b^(3/2) + I*b^(5/2)/abs(b))*c^2) - 1/4*I*sqrt
(pi)*a^2*b*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) + a)
*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^(3/2) - I*b^(5/2)/abs(b))*c^2) - 1/8*sqrt
(pi)*a*b^2*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) + a)
*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^(3/2) - I*b^(5/2)/abs(b))*c^2) + 1/4*I*sq
rt(pi)*a^2*sqrt(b)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos(c
*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2) - 3/64*I*sqrt
(pi)*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos(c*x) +
a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2) + 3/64*I*sqrt(pi)*b
^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) + a)*sqr
t(b)/abs(b))*e^(-2*I*a/b)/((b - I*b^2/abs(b))*c^2) + 1/8*sqrt(b*arccos(c*x)
 + a)*b*arccos(c*x)*e^(2*I*arccos(c*x))/c^2 + 1/8*sqrt(b*arccos(c*x) + a)*b
*arccos(c*x)*e^(-2*I*arccos(c*x))/c^2 + 1/8*sqrt(b*arccos(c*x) + a)*a*e^(2*
I*arccos(c*x))/c^2 + 3/32*I*sqrt(b*arccos(c*x) + a)*b*e^(2*I*arccos(c*x))/c
^2 + 1/8*sqrt(b*arccos(c*x) + a)*a*e^(-2*I*arccos(c*x))/c^2 - 3/32*I*sqrt(b
*arccos(c*x) + a)*b*e^(-2*I*arccos(c*x))/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\arccos(cx))^{3/2}\,dx=\int x\,(a+b\,\mathrm{acos}(c\,x))^{3/2}\,dx$$

```
[In] int(x*(a + b*acos(c*x))^(3/2),x)
[Out] int(x*(a + b*acos(c*x))^(3/2), x)
```

3.180 $\int (a+b\arccos(cx))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 159

$$\int (a+b\arccos(cx))^{3/2}\,dx = -\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c} + x(a+b\arccos(cx))^{3/2} + \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c} - \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{2c}$$

```
[Out] x*(a+b*arccos(c*x))^(3/2)+3/4*b^(3/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a
+b*arccos(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/c-3/4*b^(3/2)*FresnelC(2^(1
/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/c-3
/2*b*(-c^2*x^2+1)^(1/2)*(a+b*arccos(c*x))^(1/2)/c
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {4716, 4768, 4720, 3387, 3386, 3432, 3385, 3433}

$$\int (a+b\arccos(cx))^{3/2}\,dx = -\frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c} + \frac{3\sqrt{\frac{\pi}{2}}b^{3/2}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c} - \frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c} + x(a+b\arccos(cx))^{3/2}$$

```
[In] Int[(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (-3*b*Sqrt[1 - c^2*x^2]*Sqrt[a + b*ArcCos[c*x]])/(2*c) + x*(a + b*ArcCos[c*
x])^(3/2) + (3*b^(3/2)*Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*
ArcCos[c*x]])/Sqrt[b]])/(2*c) - (3*b^(3/2)*Sqrt[Pi/2]*FresnelC[(Sqrt[2/Pi]*
Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*c)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rubi steps

$$\text{integral} = x(a+b\arccos(cx))^{3/2} + \frac{1}{2}(3bc)\int \frac{x\sqrt{a+b\arccos(cx)}}{\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c} + x(a+b\arccos(cx))^{3/2} - \frac{1}{4}(3b^2)\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx$$

$$= -\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c} + x(a+b\arccos(cx))^{3/2} - \frac{(3b)\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{4c}$$

$$= -\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c} + x(a+b\arccos(cx))^{3/2} + \frac{\left(3b\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{4c} - \frac{\left(3b\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{4c}$$

$$= -\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c} + x(a+b\arccos(cx))^{3/2} + \frac{\left(3b\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{2c} - \frac{\left(3b\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{2c}$$

$$= -\frac{3b\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}{2c} + x(a+b\arccos(cx))^{3/2} + \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2c} - \frac{3b^{3/2}\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{2c}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.96 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.82

$$\int(a+b\arccos(cx))^{3/2}\,dx = \frac{\sqrt{b}\left(2\sqrt{b}\sqrt{a+b\arccos(cx)}\left(-3\sqrt{1-c^2x^2}+2cx\arccos(cx)\right)+\frac{2ia\sqrt{b}e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\right.}{\cdots}\right.}{\cdots}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (Sqrt[b]*(2*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(-3*Sqrt[1 - c^2*x^2] + 2*c*x*A
rcCos[c*x]) + ((2*I)*a*Sqrt[b]*(Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/
2, ((-I)*(a + b*ArcCos[c*x]))/b] - E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*
x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b]))/(E^((I*a)/b)*Sqrt[a + b*Arc
Cos[c*x]]) + Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[
b]]*(3*b*Cos[a/b] + 2*a*Sin[a/b]) + Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a
+ b*ArcCos[c*x]])/Sqrt[b]]*(2*a*Cos[a/b] - 3*b*Sin[a/b])))/(4*c)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(123) = 246$.

Time = 2.07 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.75

method	result
default	$\frac{-3\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{2}\,b^2-3\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\cdots}{\cdots}$

```
[In] int((a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/c/(a+b*arccos(c*x))^(1/2)*(-3*(a+b*arccos(c*x))^(1/2)*cos(a/b)*FresnelS
(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-1/b)^(1/2)*Pi^(
1/2)*2^(1/2)*b^2-3*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/
2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-1/b)^(1/2)*Pi^(1/2)*2^(1/2)*b^
2+4*arccos(c*x)^2*cos(-(a+b*arccos(c*x))/b+a/b)*b^2+8*arccos(c*x)*cos(-(a+b
*arccos(c*x))/b+a/b)*a*b+6*arccos(c*x)*sin(-(a+b*arccos(c*x))/b+a/b)*b^2+4*
cos(-(a+b*arccos(c*x))/b+a/b)*a^2+6*sin(-(a+b*arccos(c*x))/b+a/b)*a*b)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\arccos(cx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a+b\arccos(cx))^{3/2}\,dx = \int (a+b\operatorname{acos}(cx))^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*acos(c*x))**(3/2),x)
[Out] Integral((a + b*acos(c*x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\arccos(cx))^{3/2}\,dx = \int (b\arccos(cx)+a)^{\frac{3}{2}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccos(c*x) + a)^(3/2), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 993, normalized size of antiderivative = 6.25

$$\int (a+b\arccos(cx))^{3/2}\,dx = \text{Too large to display}$$

```
[In] integrate((a+b*arccos(c*x))^(3/2),x, algorithm="giac")
[Out] -1/2*I*sqrt(2)*sqrt(pi)*a^2*b^2*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/
sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b
)/((I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 1/2*sqrt(2)*sqrt(pi)*a*b^3*
```

```
erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(
b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^3/sqrt(abs(b)) + b^2*sqr
t(abs(b)))*c) + 1/2*I*sqrt(2)*sqrt(pi)*a^2*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arc
cos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b
))/b)*e^(-I*a/b)/((-I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 1/2*sqrt(2)
*sqrt(pi)*a*b^3*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/
2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^3/sqrt(
abs(b)) + b^2*sqrt(abs(b)))*c) - 1/2*sqrt(2)*sqrt(pi)*a*b^2*erf(-1/2*I*sqrt
(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) +
 a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) - 3
/8*I*sqrt(2)*sqrt(pi)*b^3*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(a
bs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*
b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) - 1/2*sqrt(2)*sqrt(pi)*a*b^2*erf(1/2*
I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(
c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b))
)*c) + 3/8*I*sqrt(2)*sqrt(pi)*b^3*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)
/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a
/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) + I*sqrt(pi)*a^2*b*erf(-1/2*
I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(
c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*sqrt(2)*b^2/sqrt(abs(b)) + sqrt(2)*
b*sqrt(abs(b)))*c) - I*sqrt(pi)*a^2*b*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x)
+ a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(
-I*a/b)/((-I*sqrt(2)*b^2/sqrt(abs(b)) + sqrt(2)*b*sqrt(abs(b)))*c) + 1/2*sq
rt(b*arccos(c*x) + a)*b*arccos(c*x)*e^(I*arccos(c*x))/c + 1/2*sqrt(b*arccos
(c*x) + a)*b*arccos(c*x)*e^(-I*arccos(c*x))/c + 1/2*sqrt(b*arccos(c*x) + a)
*a*e^(I*arccos(c*x))/c + 3/4*I*sqrt(b*arccos(c*x) + a)*b*e^(I*arccos(c*x))/
c + 1/2*sqrt(b*arccos(c*x) + a)*a*e^(-I*arccos(c*x))/c - 3/4*I*sqrt(b*arcco
s(c*x) + a)*b*e^(-I*arccos(c*x))/c
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\arccos(cx))^{3/2}\,dx=\int (a+b\,\mathrm{acos}(c\,x))^{3/2}\,dx$$

```
[In] int((a + b*acos(c*x))^(3/2),x)
[Out] int((a + b*acos(c*x))^(3/2), x)
```

3.181 $\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx=\text{Int}\left(\frac{(a+b\arccos(cx))^{3/2}}{x},x\right)$$

```
[Out] Unintegrable((a+b*arccos(c*x))^(3/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx=\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx$$

```
[In] Int[(a + b*ArcCos[c*x])^(3/2)/x,x]
[Out] Defer[Int][(a + b*ArcCos[c*x])^(3/2)/x, x]
```

Rubi steps

$$\text{integral}=\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx=\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx$$

```
[In] Integrate[(a + b*ArcCos[c*x])^(3/2)/x,x]
[Out] Integrate[(a + b*ArcCos[c*x])^(3/2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos{(cx)})^{\frac{3}{2}}}{x}dx$$

```
[In] int((a+b*arccos(c*x))^(3/2)/x,x)
[Out] int((a+b*arccos(c*x))^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 15.35 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx=\int \frac{(a+b\operatorname{acos}{(cx)})^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((a+b*acos(c*x))**(3/2)/x,x)
[Out] Integral((a + b*acos(c*x))**(3/2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x,x, algorithm="maxima")
[Out] integrate((b*arccos(c*x) + a)^(3/2)/x, x)
```

Giac [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{3}{2}}}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x,x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^(3/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^{3/2}}{x}\,dx$$

```
[In] int((a + b*acos(c*x))^(3/2)/x,x)
[Out] int((a + b*acos(c*x))^(3/2)/x, x)
```

3.182 $\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx = \operatorname{Int}\left(\frac{(a+b\arccos(cx))^{3/2}}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccos(c*x))^(3/2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx = \int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx$$

```
[In] Int[(a + b*ArcCos[c*x])^(3/2)/x^2,x]
```

```
[Out] Defer[Int][(a + b*ArcCos[c*x])^(3/2)/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx=\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx$$

```
[In] Integrate[(a + b*ArcCos[c*x])^(3/2)/x^2,x]
[Out] Integrate[(a + b*ArcCos[c*x])^(3/2)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos(cx))^{\frac{3}{2}}}{x^2}dx$$

```
[In] int((a+b*arccos(c*x))^(3/2)/x^2,x)
[Out] int((a+b*arccos(c*x))^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 3.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx=\int \frac{(a+b\operatorname{acos}(cx))^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b*acos(c*x))**(3/2)/x**2,x)
[Out] Integral((a + b*acos(c*x))**(3/2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x^2,x, algorithm="maxima")
[Out] integrate((b*arccos(c*x) + a)^(3/2)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{3}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^(3/2)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{3/2}}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^{3/2}}{x^2}\,dx$$

```
[In] int((a + b*acos(c*x))^(3/2)/x^2,x)
[Out] int((a + b*acos(c*x))^(3/2)/x^2, x)
```

3.183 $\int x^2(a+b\arccos(cx))^{5/2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 358

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx = -\frac{5b^2x\sqrt{a+b\arccos(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a+b\arccos(cx)} - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{18c} + \frac{1}{3}x^3(a+b\arccos(cx))^{5/2} + \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16c^3} + \frac{5b^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\sqrt{\ldots}\right.}{144c^3}$$

```
[Out] 1/3*x^3*(a+b*arccos(c*x))^(5/2)+5/864*b^(5/2)*cos(3*a/b)*FresnelC(6^(1/2)/P
i^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/c^3+5/864*b^(5/2)
*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*a/b)*6^(1
/2)*Pi^(1/2)/c^3+15/32*b^(5/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arcc
os(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/c^3+15/32*b^(5/2)*FresnelS(2^(1/2)
/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/c^3-5/
9*b*(a+b*arccos(c*x))^(3/2)*(-c^2*x^2+1)^(1/2)/c^3-5/18*b*x^2*(a+b*arccos(c
*x))^(3/2)*(-c^2*x^2+1)^(1/2)/c-5/6*b^2*x*(a+b*arccos(c*x))^(1/2)/c^2-5/36*
b^2*x^3*(a+b*arccos(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.688, Rules used = {4726, 4796, 4768, 4716, 4810, 3387, 3386, 3432, 3385, 3433, 3393}

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx=\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{5\sqrt{\frac{\pi}{6}}b^{5/2}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{144c^3}+\frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16c^3}+\frac{5\sqrt{\frac{\pi}{6}}b^{5/2}\sin\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{144c^3}-\frac{5b^2x\sqrt{a+b\arccos(cx)}}{6c^2}-\frac{5}{36}b^2x^3\sqrt{a+b\arccos(cx)}-\frac{5bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{18c}-\frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{9c^3}+\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}$$

```
[In] Int[x^2*(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (-5*b^2*x*Sqrt[a + b*ArcCos[c*x]])/(6*c^2) - (5*b^2*x^3*Sqrt[a + b*ArcCos[c
*x]])/36 - (5*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(3/2))/(9*c^3) - (5*b
*x^2*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(3/2))/(18*c) + (x^3*(a + b*ArcC
os[c*x])^(5/2))/3 + (15*b^(5/2)*Sqrt[Pi/2]*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sq
rt[a + b*ArcCos[c*x]])/Sqrt[b]])/(16*c^3) + (5*b^(5/2)*Sqrt[Pi/6]*Cos[(3*a)
/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(144*c^3) + (15
*b^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*
Sin[a/b])/(16*c^3) + (5*b^(5/2)*Sqrt[Pi/6]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*
ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(144*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
```

```
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{1}{3}x^3(a+b\arccos(cx))^{5/2}+\frac{1}{6}(5bc)\int\frac{x^3(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{5bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{18c}+\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}\\
&\quad-\frac{1}{12}\left(5b^2\right)\int x^2\sqrt{a+b\arccos(cx)}\,dx+\frac{(5b)\int\frac{x(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}\,dx}{9c}\\
&= -\frac{5}{36}b^2x^3\sqrt{a+b\arccos(cx)}-\frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{9c^3}\\
&\quad-\frac{5bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{18c}+\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}\\
&\quad-\frac{\left(5b^2\right)\int\sqrt{a+b\arccos(cx)}\,dx}{6c^2}-\frac{1}{72}\left(5b^3c\right)\int\frac{x^3}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}\,dx\\
&= -\frac{5b^2x\sqrt{a+b\arccos(cx)}}{6c^2}-\frac{5}{36}b^2x^3\sqrt{a+b\arccos(cx)}\\
&\quad-\frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{9c^3}-\frac{5bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{18c}\\
&\quad+\frac{1}{3}x^3(a+b\arccos(cx))^{5/2}+\frac{\left(5b^2\right)\operatorname{Subst}\left(\int\frac{\cos^3\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{72c^3}\\
&\quad-\frac{\left(5b^3\right)\int\frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}\,dx}{12c}
\end{aligned}
$$

$$= -\frac{5b^2x\sqrt{a+b\arccos(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a+b\arccos(cx)} - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{18c} + \frac{1}{3}x^3(a+b\arccos(cx))^{5/2} + \frac{(5b^2)\,\text{Subst}\left(\int\left(\frac{\cos\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{3\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx, x, a+b\arccos(cx)\right)}{72c^3} + \frac{(5b^2)\,\text{Subst}\left(\int\frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}dx, x, a+b\arccos(cx)\right)}{12c^3}$$

$$= -\frac{5b^2x\sqrt{a+b\arccos(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a+b\arccos(cx)} - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{18c} + \frac{1}{3}x^3(a+b\arccos(cx))^{5/2} + \frac{(5b^2)\,\text{Subst}\left(\int\frac{\cos\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}dx, x, a+b\arccos(cx)\right)}{288c^3} + \frac{(5b^2)\,\text{Subst}\left(\int\frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}dx, x, a+b\arccos(cx)\right)}{96c^3} + \frac{\left(5b^2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}dx, x, a+b\arccos(cx)\right)}{12c^3} + \frac{\left(5b^2\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}dx, x, a+b\arccos(cx)\right)}{12c^3}$$

$$
\begin{aligned}
&= -\frac{5b^2x\sqrt{a+b\arccos(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a+b\arccos(cx)} \\
&\quad - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{18c} \\
&\quad + \frac{1}{3}x^3(a+b\arccos(cx))^{5/2} + \frac{\left(5b^2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{96c^3} \\
&\quad + \frac{\left(5b^2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\cos\left(\frac{x^2}{b}\right)\,dx,x,\sqrt{a+b\arccos(cx)}\right)}{6c^3} \\
&\quad + \frac{\left(5b^2\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{3x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{288c^3} \\
&\quad + \frac{\left(5b^2\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{96c^3} \\
&\quad + \frac{\left(5b^2\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\sin\left(\frac{x^2}{b}\right)\,dx,x,\sqrt{a+b\arccos(cx)}\right)}{6c^3} \\
&\quad + \frac{\left(5b^2\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{3x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{288c^3}
\end{aligned}
$$

$$
\begin{aligned}
&= -\frac{5b^2x\sqrt{a+b\arccos(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a+b\arccos(cx)} \\
&\quad - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{18c} \\
&\quad + \frac{1}{3}x^3(a+b\arccos(cx))^{5/2} + \frac{5b^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{6c^3} \\
&\quad + \frac{5b^{5/2}\sqrt{\frac{\pi}{2}}\,\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{6c^3} \\
&\quad + \frac{\left(5b^2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\cos\left(\frac{x^2}{b}\right)\,dx,x,\sqrt{a+b\arccos(cx)}\right)}{48c^3} \\
&\quad + \frac{\left(5b^2\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\cos\left(\frac{3x^2}{b}\right)\,dx,x,\sqrt{a+b\arccos(cx)}\right)}{144c^3} \\
&\quad + \frac{\left(5b^2\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\sin\left(\frac{x^2}{b}\right)\,dx,x,\sqrt{a+b\arccos(cx)}\right)}{48c^3} \\
&\quad + \frac{\left(5b^2\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\sin\left(\frac{3x^2}{b}\right)\,dx,x,\sqrt{a+b\arccos(cx)}\right)}{144c^3}
\end{aligned}
$$

$$= -\frac{5b^2x\sqrt{a+b\arccos(cx)}}{6c^2} - \frac{5}{36}b^2x^3\sqrt{a+b\arccos(cx)} - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{9c^3} - \frac{5bx^2\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{18c} + \frac{1}{3}x^3(a+b\arccos(cx))^{5/2} + \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{16c^3} + \frac{5b^{5/2}\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{144c^3} + \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{16c^3} + \frac{5b^{5/2}\sqrt{\frac{\pi}{6}}\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{144c^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.53 (sec) , antiderivative size = 956, normalized size of antiderivative = 2.67

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx =$$

$$-\frac{ia^2be^{-\frac{3ia}{b}}\left(-9e^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+9e^{\frac{4ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{3}{2},\frac{i(a+b\arccos(cx))}{b}\right)+\sqrt{\cdots}\right.}{72c^3\sqrt{a+b\arccos\cdots}}$$

$$-\frac{a\sqrt{b}\left(18\sqrt{b}\sqrt{a+b\arccos(cx)}\left(3\sqrt{1-c^2x^2}-2cx\arccos(cx)\right)-9\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)(3b\,\text{co}\cdots\right.}{\cdots}$$

$$-\frac{\sqrt{b}\left(27\left(2\sqrt{b}\sqrt{a+b\arccos(cx)}\left(-2\sqrt{1-c^2x^2}(a-5b\arccos(cx))-bcx(-15+4\arccos(cx)^2)\right)+\sqrt{2\pi}\,\text{Fr}\cdots\right.\right.}{\cdots}$$

```
[In] Integrate[x^2*(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] ((-1/72*I)*a^2*b*(-9*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gam
ma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b] + 9*E^(((4*I)*a)/b)*Sqrt[(I*(a + b*Ar
cCos[c*x]))/b]*Gamma[3/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(-(Sqrt[((-I
)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-3*I)*(a + b*ArcCos[c*x]))/b]) + E^(
((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((3*I)*(a + b*ArcCo
s[c*x]))/b])))/(c^3*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]]) - (a*Sqrt[b]*(
18*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(3*Sqrt[1 - c^2*x^2] - 2*c*x*ArcCos[c*x]
```

```
) - 9*Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(3*
b*Cos[a/b] + 2*a*Sin[a/b]) - 9*Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*A
rcCos[c*x]])/Sqrt[b]]*(2*a*Cos[a/b] - 3*b*Sin[a/b]) - Sqrt[6*Pi]*FresnelS[(
Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(b*Cos[(3*a)/b] + 2*a*Sin[(3*a
)/b]) - Sqrt[6*Pi]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(
2*a*Cos[(3*a)/b] - b*Sin[(3*a)/b]) + 6*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(-2*
ArcCos[c*x]*Cos[3*ArcCos[c*x]] + Sin[3*ArcCos[c*x]])))/(72*c^3) - (Sqrt[b]*
(27*(2*Sqrt[b]*Sqrt[a + b*ArcCos[c*x]]*(-2*Sqrt[1 - c^2*x^2]*(a - 5*b*ArcCo
s[c*x]) - b*c*x*(-15 + 4*ArcCos[c*x]^2)) + Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*
Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*((4*a^2 - 15*b^2)*Cos[a/b] - 12*a*b*Sin[a
/b]) + Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*(1
2*a*b*Cos[a/b] + (4*a^2 - 15*b^2)*Sin[a/b])) + Sqrt[6*Pi]*FresnelC[(Sqrt[6/
Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*((12*a^2 - 5*b^2)*Cos[(3*a)/b] - 12*a
*b*Sin[(3*a)/b]) + Sqrt[6*Pi]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])
/Sqrt[b]]*(12*a*b*Cos[(3*a)/b] + (12*a^2 - 5*b^2)*Sin[(3*a)/b]) + 6*Sqrt[b]
*Sqrt[a + b*ArcCos[c*x]]*(-(b*(-5 + 12*ArcCos[c*x]^2)*Cos[3*ArcCos[c*x]]) -
 2*(a - 5*b*ArcCos[c*x])*Sin[3*ArcCos[c*x]])))/(864*c^3)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. 2(278) = 556.

Time = 2.32 (sec) , antiderivative size = 798, normalized size of antiderivative = 2.23

method	result	size
default	Expression too large to display	798

```
[In] int(x^2*(a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/864/c^3/(a+b*arccos(c*x))^(1/2)*(405*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*
x))^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))
^(1/2)/b)*2^(1/2)*b^3-405*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin
(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*2^(
1/2)*b^3+5*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(3*a/b)*FresnelC(3*2^(1/2)/P
i^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-3/b)^(1/2)*2^(1/2)*b^3-5*
Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3
/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-3/b)^(1/2)*2^(1/2)*b^3+216*arccos(c*
x)^3*cos(-(a+b*arccos(c*x))/b+a/b)*b^3+72*arccos(c*x)^3*cos(-3*(a+b*arccos(
c*x))/b+3*a/b)*b^3+648*arccos(c*x)^2*cos(-(a+b*arccos(c*x))/b+a/b)*a*b^2+54
0*arccos(c*x)^2*sin(-(a+b*arccos(c*x))/b+a/b)*b^3+216*arccos(c*x)^2*cos(-3*
(a+b*arccos(c*x))/b+3*a/b)*a*b^2+60*arccos(c*x)^2*sin(-3*(a+b*arccos(c*x))/
b+3*a/b)*b^3+648*arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*a^2*b-810*arccos
(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*b^3+1080*arccos(c*x)*sin(-(a+b*arccos(c
*x))/b+a/b)*a*b^2+216*arccos(c*x)*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*a^2*b-3
0*arccos(c*x)*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*b^3+120*arccos(c*x)*sin(-3*
(a+b*arccos(c*x))/b+3*a/b)*a*b^2+216*cos(-(a+b*arccos(c*x))/b+a/b)*a^3-810*
```

```
cos(-(a+b*arccos(c*x))/b+a/b)*a*b^2+540*sin(-(a+b*arccos(c*x))/b+a/b)*a^2*b
+72*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*a^3-30*cos(-3*(a+b*arccos(c*x))/b+3*a
/b)*a*b^2+60*sin(-3*(a+b*arccos(c*x))/b+3*a/b)*a^2*b)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx = \int x^2(a+b\,\mathrm{acos}\,(cx))^{\frac{5}{2}}\,dx$$

```
[In] integrate(x**2*(a+b*acos(c*x))**(5/2),x)
[Out] Integral(x**2*(a + b*acos(c*x))**(5/2), x)
```

Maxima [F]

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx = \int (b\arccos{(cx)}+a)^{\frac{5}{2}}x^2\,dx$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arccos(c*x) + a)^(5/2)*x^2, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.38 (sec) , antiderivative size = 2778, normalized size of antiderivative = 7.76

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx = \text{Too large to display}$$

```
[In] integrate(x^2*(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] -1/576*(72*I*sqrt(2)*sqrt(pi)*a^3*b^2*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x)
 + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^
(I*a/b)/(I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b))) - 72*I*sqrt(2)*sqrt(pi)*a^3
*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*s
qrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(-I*b^3/sqrt(abs(b)) + b^
2*sqrt(abs(b))) - 216*sqrt(2)*sqrt(pi)*a^2*b^2*erf(-1/2*I*sqrt(2)*sqrt(b*ar
ccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(
b))/b)*e^(I*a/b)/(I*b^2/sqrt(abs(b)) + b*sqrt(abs(b))) - 216*sqrt(2)*sqrt(p
i)*a^2*b^2*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqr
t(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(-I*b^2/sqrt(abs(b)
) + b*sqrt(abs(b))) - 24*sqrt(b*arccos(c*x) + a)*b^2*arccos(c*x)^2*e^(3*I*a
rccos(c*x)) - 72*sqrt(b*arccos(c*x) + a)*b^2*arccos(c*x)^2*e^(I*arccos(c*x)
) - 72*sqrt(b*arccos(c*x) + a)*b^2*arccos(c*x)^2*e^(-I*arccos(c*x)) - 24*sq
rt(b*arccos(c*x) + a)*b^2*arccos(c*x)^2*e^(-3*I*arccos(c*x)) - 144*I*sqrt(p
i)*a^3*b*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*s
qrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/(sqrt(6)*b^(3/2) + I*sqr
t(6)*b^(5/2)/abs(b)) + 144*I*sqrt(pi)*a^3*b*erf(-1/2*sqrt(6)*sqrt(b*arccos(
c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e
^(-3*I*a/b)/(sqrt(6)*b^(3/2) - I*sqrt(6)*b^(5/2)/abs(b)) + 144*I*sqrt(pi)*a
^3*sqrt(b)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)
*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/(sqrt(6)*b + I*sqrt(6)
*b^2/abs(b)) - 144*sqrt(pi)*a^2*b^(3/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x)
 + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*
I*a/b)/(sqrt(6)*b + I*sqrt(6)*b^2/abs(b)) + 216*sqrt(2)*sqrt(pi)*a^2*b*erf(
-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*ar
ccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(I*b/sqrt(abs(b)) + sqrt(abs(b)))
 + 135*sqrt(2)*sqrt(pi)*b^3*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(
abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(I*
b/sqrt(abs(b)) + sqrt(abs(b))) + 216*sqrt(2)*sqrt(pi)*a^2*b*erf(1/2*I*sqrt(
2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) +
a)*sqrt(abs(b))/b)*e^(-I*a/b)/(-I*b/sqrt(abs(b)) + sqrt(abs(b))) + 135*sqrt
(2)*sqrt(pi)*b^3*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1
/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(-I*b/sqrt(ab
s(b)) + sqrt(abs(b))) - 144*I*sqrt(pi)*a^3*sqrt(b)*erf(-1/2*sqrt(6)*sqrt(b*
arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/ab
s(b))*e^(-3*I*a/b)/(sqrt(6)*b - I*sqrt(6)*b^2/abs(b)) - 144*sqrt(pi)*a^2*b^
(3/2)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt
(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/(sqrt(6)*b - I*sqrt(6)*b^2
/abs(b)) - 48*sqrt(b*arccos(c*x) + a)*a*b*arccos(c*x)*e^(3*I*arccos(c*x)) -
 20*I*sqrt(b*arccos(c*x) + a)*b^2*arccos(c*x)*e^(3*I*arccos(c*x)) - 144*sqr
t(b*arccos(c*x) + a)*a*b*arccos(c*x)*e^(I*arccos(c*x)) - 180*I*sqrt(b*arcco
s(c*x) + a)*b^2*arccos(c*x)*e^(I*arccos(c*x)) - 144*sqrt(b*arccos(c*x) + a)
*a*b*arccos(c*x)*e^(-I*arccos(c*x)) + 180*I*sqrt(b*arccos(c*x) + a)*b^2*arc
cos(c*x)*e^(-I*arccos(c*x)) - 48*sqrt(b*arccos(c*x) + a)*a*b*arccos(c*x)*e^
(-3*I*arccos(c*x)) + 20*I*sqrt(b*arccos(c*x) + a)*b^2*arccos(c*x)*e^(-3*I*a
```

```
rccos(c*x)) - 144*I*sqrt(pi)*a^3*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/s
qrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/
(sqrt(6)*sqrt(b) + I*sqrt(6)*b^(3/2)/abs(b)) + 144*sqrt(pi)*a^2*b*erf(-1/2*
sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x)
+ a)*sqrt(b)/abs(b))*e^(3*I*a/b)/(sqrt(6)*sqrt(b) + I*sqrt(6)*b^(3/2)/abs(b
)) - 36*I*sqrt(pi)*a*b^2*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) -
 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/(sqrt(6)
*sqrt(b) + I*sqrt(6)*b^(3/2)/abs(b)) - 144*I*sqrt(pi)*a^3*erf(-1/2*I*sqrt(2
)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a
)*sqrt(abs(b))/b)*e^(I*a/b)/(I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)
)) + 144*I*sqrt(pi)*a^3*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(
b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(-I*sq
rt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b))) + 144*I*sqrt(pi)*a^3*erf(-1/2*
sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x)
+ a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/(sqrt(6)*sqrt(b) - I*sqrt(6)*b^(3/2)/abs(
b)) + 144*sqrt(pi)*a^2*b*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) +
 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/(sqrt(6
)*sqrt(b) - I*sqrt(6)*b^(3/2)/abs(b)) + 36*I*sqrt(pi)*a*b^2*erf(-1/2*sqrt(6
)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*s
qrt(b)/abs(b))*e^(-3*I*a/b)/(sqrt(6)*sqrt(b) - I*sqrt(6)*b^(3/2)/abs(b)) +
144*I*sqrt(pi)*a^3*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I
*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/(sqrt(b)*(sqrt
(6) + I*sqrt(6)*b/abs(b))) + 36*I*sqrt(pi)*a*b^(3/2)*erf(-1/2*sqrt(6)*sqrt(
b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/
abs(b))*e^(3*I*a/b)/(sqrt(6) + I*sqrt(6)*b/abs(b)) + 10*sqrt(pi)*b^(5/2)*er
f(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arcco
s(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/(sqrt(6) + I*sqrt(6)*b/abs(b)) - 14
4*I*sqrt(pi)*a^3*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*s
qrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/(sqrt(b)*(sqrt(
6) - I*sqrt(6)*b/abs(b))) - 36*I*sqrt(pi)*a*b^(3/2)*erf(-1/2*sqrt(6)*sqrt(b
*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/a
bs(b))*e^(-3*I*a/b)/(sqrt(6) - I*sqrt(6)*b/abs(b)) + 10*sqrt(pi)*b^(5/2)*er
f(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arcco
s(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/(sqrt(6) - I*sqrt(6)*b/abs(b)) - 2
4*sqrt(b*arccos(c*x) + a)*a^2*e^(3*I*arccos(c*x)) - 20*I*sqrt(b*arccos(c*x)
 + a)*a*b*e^(3*I*arccos(c*x)) + 10*sqrt(b*arccos(c*x) + a)*b^2*e^(3*I*arcco
s(c*x)) - 72*sqrt(b*arccos(c*x) + a)*a^2*e^(I*arccos(c*x)) - 180*I*sqrt(b*a
rccos(c*x) + a)*a*b*e^(I*arccos(c*x)) + 270*sqrt(b*arccos(c*x) + a)*b^2*e^(
I*arccos(c*x)) - 72*sqrt(b*arccos(c*x) + a)*a^2*e^(-I*arccos(c*x)) + 180*I*
sqrt(b*arccos(c*x) + a)*a*b*e^(-I*arccos(c*x)) + 270*sqrt(b*arccos(c*x) + a
)*b^2*e^(-I*arccos(c*x)) - 24*sqrt(b*arccos(c*x) + a)*a^2*e^(-3*I*arccos(c*
x)) + 20*I*sqrt(b*arccos(c*x) + a)*a*b*e^(-3*I*arccos(c*x)) + 10*sqrt(b*arc
cos(c*x) + a)*b^2*e^(-3*I*arccos(c*x)))/c^3
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+b\arccos(cx))^{5/2}\,dx=\int x^2\,(a+b\,\mathrm{acos}(c\,x))^{5/2}\,dx$$

```
[In] int(x^2*(a + b*acos(c*x))^(5/2),x)
[Out] int(x^2*(a + b*acos(c*x))^(5/2), x)
```

3.184 $\int x(a+b\arccos(cx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 216

$$\int x(a+b\arccos(cx))^{5/2}\,dx=\frac{15b^2\sqrt{a+b\arccos(cx)}}{64c^2}-\frac{15}{32}b^2x^2\sqrt{a+b\arccos(cx)}-\frac{5bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{8c}-\frac{(a+b\arccos(cx))^{5/2}}{4c^2}+\frac{1}{2}x^2(a+b\arccos(cx))^{5/2}+\frac{15b^{5/2}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{128c^2}+\frac{15b^{5/2}\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\ldots}{128c^2}$$

```
[Out] -1/4*(a+b*arccos(c*x))^(5/2)/c^2+1/2*x^2*(a+b*arccos(c*x))^(5/2)+15/128*b^(
5/2)*cos(2*a/b)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*Pi^(1/
2)/c^2+15/128*b^(5/2)*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*
sin(2*a/b)*Pi^(1/2)/c^2-5/8*b*x*(a+b*arccos(c*x))^(3/2)*(-c^2*x^2+1)^(1/2)/
c+15/64*b^2*(a+b*arccos(c*x))^(1/2)/c^2-15/32*b^2*x^2*(a+b*arccos(c*x))^(1/
2)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {4726, 4796, 4738, 4810, 3393, 3387, 3386, 3432, 3385, 3433}

$$\int x(a+b\arccos(cx))^{5/2}\,dx=\frac{15\sqrt{\pi}b^{5/2}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{128c^2}+\frac{15\sqrt{\pi}b^{5/2}\sin\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{128c^2}+\frac{15b^2\sqrt{a+b\arccos(cx)}}{64c^2}-\frac{15}{32}b^2x^2\sqrt{a+b\arccos(cx)}-\frac{5bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{8c}-\frac{(a+b\arccos(cx))^{5/2}}{4c^2}+\frac{1}{2}x^2(a+b\arccos(cx))^{5/2}$$

```
[In] Int[x*(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (15*b^2*Sqrt[a + b*ArcCos[c*x]])/(64*c^2) - (15*b^2*x^2*Sqrt[a + b*ArcCos[c
*x]])/32 - (5*b*x*Sqrt[1 - c^2*x^2]*(a + b*ArcCos[c*x])^(3/2))/(8*c) - (a +
 b*ArcCos[c*x])^(5/2)/(4*c^2) + (x^2*(a + b*ArcCos[c*x])^(5/2))/2 + (15*b^(
5/2)*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sq
rt[Pi])])/(128*c^2) + (15*b^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*
x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/(128*c^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4726

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[x
^(m + 1)*((a + b*ArcCos[c*x])^n/(m + 1)), x] + Dist[b*c*(n/(m + 1)), Int[x^
(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[{a
, b, c}, x] && IGtQ[m, 0] && GtQ[n, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4796

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((f_.)*(x_))^(m_)*((d_) + (e_.
)*(x_)^2)^(p_), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(d + e*x^2)^(p + 1)*((a +
 b*ArcCos[c*x])^n/(e*(m + 2*p + 1))), x] + (Dist[f^2*((m - 1)/(c^2*(m + 2*p
 + 1))), Int[(f*x)^(m - 2)*(d + e*x^2)^p*(a + b*ArcCos[c*x])^n, x], x] - Di
st[b*f*(n/(c*(m + 2*p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], Int[(f*x)
^(m - 1)*(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x]) /; Fr
eeQ[{a, b, c, d, e, f, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && IGtQ[m,
1] && NeQ[m + 2*p + 1, 0]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{1}{2}x^2(a+b\arccos(cx))^{5/2} + \frac{1}{4}(5bc)\int \frac{x^2(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}\,dx$$

$$= -\frac{5bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{8c} + \frac{1}{2}x^2(a+b\arccos(cx))^{5/2} - \frac{1}{16}(15b^2)\int x\sqrt{a+b\arccos(cx)}\,dx + \frac{(5b)\int \frac{(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}\,dx}{8c}$$

$$= -\frac{15}{32}b^2x^2\sqrt{a+b\arccos(cx)} - \frac{5bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{8c} - \frac{(a+b\arccos(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{5/2} - \frac{1}{64}(15b^3c)\int \frac{x^2}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}\,dx$$

$$= -\frac{15}{32}b^2x^2\sqrt{a+b\arccos(cx)} - \frac{5bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{8c} - \frac{(a+b\arccos(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{5/2} + \frac{(15b^2)\,\text{Subst}\left(\int \frac{\cos^2\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{64c^2}$$

$$= -\frac{15}{32}b^2x^2\sqrt{a+b\arccos(cx)} - \frac{5bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{8c} - \frac{(a+b\arccos(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{5/2} + \frac{(15b^2)\,\text{Subst}\left(\int \left(\frac{1}{2\sqrt{x}} + \frac{\cos\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\right)dx, x, a+b\arccos(cx)\right)}{64c^2}$$

$$= \frac{15b^2\sqrt{a+b\arccos(cx)}}{64c^2} - \frac{15}{32}b^2x^2\sqrt{a+b\arccos(cx)} - \frac{5bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{8c} - \frac{(a+b\arccos(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{5/2} + \frac{(15b^2)\,\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{128c^2}$$

$$= \frac{15b^2\sqrt{a+b\arccos(cx)}}{64c^2} - \frac{15}{32}b^2x^2\sqrt{a+b\arccos(cx)} - \frac{5bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{8c} - \frac{(a+b\arccos(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{5/2} + \frac{\left(15b^2\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{128c^2} + \frac{\left(15b^2\sin\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{128c^2}$$

$$= \frac{15b^2\sqrt{a+b\arccos(cx)}}{64c^2} - \frac{15}{32}b^2x^2\sqrt{a+b\arccos(cx)} - \frac{5bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{8c} - \frac{(a+b\arccos(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{5/2} + \frac{\left(15b^2\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int\cos\left(\frac{2x^2}{b}\right)dx, x, \sqrt{a+b\arccos(cx)}\right)}{64c^2} + \frac{\left(15b^2\sin\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int\sin\left(\frac{2x^2}{b}\right)dx, x, \sqrt{a+b\arccos(cx)}\right)}{64c^2}$$

$$= \frac{15b^2\sqrt{a+b\arccos(cx)}}{64c^2} - \frac{15}{32}b^2x^2\sqrt{a+b\arccos(cx)} - \frac{5bx\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{8c} - \frac{(a+b\arccos(cx))^{5/2}}{4c^2} + \frac{1}{2}x^2(a+b\arccos(cx))^{5/2} + \frac{15b^{5/2}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{128c^2} + \frac{15b^{5/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{128c^2}$$

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int x(a+b\arccos(cx))^{5/2}\,dx = \frac{15b^{5/2}\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right) + 15b^{5/2}\sqrt{\pi}\,\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\text{si}$$

```
[In] Integrate[x*(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (15*b^(5/2)*Sqrt[Pi]*Cos[(2*a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqr
t[b]*Sqrt[Pi])] + 15*b^(5/2)*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/
(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b] + 2*Sqrt[a + b*ArcCos[c*x]]*((16*a^2 - 15*
b^2)*Cos[2*ArcCos[c*x]] + 16*b^2*ArcCos[c*x]^2*Cos[2*ArcCos[c*x]] - 20*a*b*
Sin[2*ArcCos[c*x]] + 4*b*ArcCos[c*x]*(8*a*Cos[2*ArcCos[c*x]] - 5*b*Sin[2*Ar
cCos[c*x]])))/(128*c^2)
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(170) = 340$.

Time = 2.01 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.89

method	result
default	$\frac{15\sqrt{a+b\arccos(cx)}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)b^3-15\sqrt{a+b\arccos(cx)}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\sin\left(\frac{2a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{a+b}}{\sqrt{\pi}\,\sqrt{}}\right.}{}$

```
[In] int(x*(a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/128/c^2/(a+b*arccos(c*x))^(1/2)*(15*(a+b*arccos(c*x))^(1/2)*Pi^(1/2)*(-1/
b)^(1/2)*cos(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*
x))^(1/2)/b)*b^3-15*(a+b*arccos(c*x))^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*sin(2*a/b
)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b^3+3
2*arccos(c*x)^3*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*b^3+96*arccos(c*x)^2*cos(
-2*(a+b*arccos(c*x))/b+2*a/b)*a*b^2+40*arccos(c*x)^2*sin(-2*(a+b*arccos(c*x
))/b+2*a/b)*b^3+96*arccos(c*x)*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*a^2*b-30*a
rccos(c*x)*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*b^3+80*arccos(c*x)*sin(-2*(a+b
*arccos(c*x))/b+2*a/b)*a*b^2+32*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*a^3-30*co
s(-2*(a+b*arccos(c*x))/b+2*a/b)*a*b^2+40*sin(-2*(a+b*arccos(c*x))/b+2*a/b)*
a^2*b)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+b\arccos(cx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x*(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int x(a+b\arccos(cx))^{5/2}\,dx = \int x(a+b\operatorname{acos}(cx))^{\frac{5}{2}}\,dx$$

```
[In] integrate(x*(a+b*acos(c*x))**(5/2),x)
```

```
[Out] Integral(x*(a + b*acos(c*x))**(5/2), x)
```

Maxima [F]

$$\int x(a+b\arccos(cx))^{5/2}\,dx=\int (b\arccos{(cx)}+a)^{\frac{5}{2}}x\,dx$$

```
[In] integrate(x*(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(5/2)*x, x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.29 (sec) , antiderivative size = 1307, normalized size of antiderivative = 6.05

$$\int x(a+b\arccos(cx))^{5/2}\,dx=\text{Too large to display}$$

```
[In] integrate(x*(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
```

```
[Out] -1/4*I*sqrt(pi)*a^3*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b
*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^2 + I*b^3/abs(b))*c^2) +
3/8*sqrt(pi)*a^2*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*ar
ccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^2 + I*b^3/abs(b))*c^2) + 1/4
*I*sqrt(pi)*a^3*b^(3/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arc
cos(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^2 - I*b^3/abs(b))*c^2) + 3/8
*sqrt(pi)*a^2*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arcco
s(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^2 - I*b^3/abs(b))*c^2) + 1/8*s
qrt(b*arccos(c*x) + a)*b^2*arccos(c*x)^2*e^(2*I*arccos(c*x))/c^2 + 1/8*sqrt
(b*arccos(c*x) + a)*b^2*arccos(c*x)^2*e^(-2*I*arccos(c*x))/c^2 - 3/8*sqrt(p
i)*a^2*b^2*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos(c*x) + a)
*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^(3/2) + I*b^(5/2)/abs(b))*c^2) + 9/64*I*sq
rt(pi)*a*b^3*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos(c*x) +
a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b^(3/2) + I*b^(5/2)/abs(b))*c^2) - 1/4*I*s
qrt(pi)*a^3*b*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) +
 a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^(3/2) - I*b^(5/2)/abs(b))*c^2) - 3/8*s
qrt(pi)*a^2*b^2*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x)
 + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^(3/2) - I*b^(5/2)/abs(b))*c^2) - 9/6
4*I*sqrt(pi)*a*b^3*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c
*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b^(3/2) - I*b^(5/2)/abs(b))*c^2) +
1/4*I*sqrt(pi)*a^3*sqrt(b)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*
arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2) - 9/6
4*I*sqrt(pi)*a*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arcc
os(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2) - 15/256*
sqrt(pi)*b^(7/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos(c*x
```

```
) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/((b + I*b^2/abs(b))*c^2) + 9/64*I*sqrt(p
i)*a*b^(5/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) +
a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/((b - I*b^2/abs(b))*c^2) - 15/256*sqrt(pi)*
b^(7/2)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) + a)*sq
rt(b)/abs(b))*e^(-2*I*a/b)/((b - I*b^2/abs(b))*c^2) + 1/4*sqrt(b*arccos(c*x
) + a)*a*b*arccos(c*x)*e^(2*I*arccos(c*x))/c^2 + 5/32*I*sqrt(b*arccos(c*x)
+ a)*b^2*arccos(c*x)*e^(2*I*arccos(c*x))/c^2 + 1/4*sqrt(b*arccos(c*x) + a)*
a*b*arccos(c*x)*e^(-2*I*arccos(c*x))/c^2 - 5/32*I*sqrt(b*arccos(c*x) + a)*b
^2*arccos(c*x)*e^(-2*I*arccos(c*x))/c^2 + 1/8*sqrt(b*arccos(c*x) + a)*a^2*e
^(2*I*arccos(c*x))/c^2 + 5/32*I*sqrt(b*arccos(c*x) + a)*a*b*e^(2*I*arccos(c
*x))/c^2 - 15/128*sqrt(b*arccos(c*x) + a)*b^2*e^(2*I*arccos(c*x))/c^2 + 1/8
*sqrt(b*arccos(c*x) + a)*a^2*e^(-2*I*arccos(c*x))/c^2 - 5/32*I*sqrt(b*arcco
s(c*x) + a)*a*b*e^(-2*I*arccos(c*x))/c^2 - 15/128*sqrt(b*arccos(c*x) + a)*b
^2*e^(-2*I*arccos(c*x))/c^2
```

Mupad [F(-1)]

Timed out.

$$\int x(a+b\arccos(cx))^{5/2}\,dx=\int x\,\left(a+b\,\mathrm{acos}(c\,x)\right)^{5/2}\,dx$$

```
[In] int(x*(a + b*acos(c*x))^(5/2),x)
[Out] int(x*(a + b*acos(c*x))^(5/2), x)
```

3.185 $\int (a+b \arccos(cx))^{5/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 179

$$\int (a+b\arccos(cx))^{5/2}\,dx = -\frac{15}{4}b^2x\sqrt{a+b\arccos(cx)} - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{2c} + x(a+b\arccos(cx))^{5/2} + \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c} + \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{4c}$$

```
[Out] x*(a+b*arccos(c*x))^(5/2)+15/8*b^(5/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(
a+b*arccos(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/c+15/8*b^(5/2)*FresnelS(2^
(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/c
-5/2*b*(a+b*arccos(c*x))^(3/2)*(-c^2*x^2+1)^(1/2)/c-15/4*b^2*x*(a+b*arccos(
c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used

$= \{4716, 4768, 4810, 3387, 3386, 3432, 3385, 3433\}$

$$\int (a+b\arccos(cx))^{5/2}\,dx = \frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c} + \frac{15\sqrt{\frac{\pi}{2}}b^{5/2}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c} - \frac{15}{4}b^2x\sqrt{a+b\arccos(cx)} - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{2c} + x(a+b\arccos(cx))^{5/2}$$

```
[In] Int[(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (-15*b^2*x*Sqrt[a + b*ArcCos[c*x]])/4 - (5*b*Sqrt[1 - c^2*x^2]*(a + b*ArcCo
s[c*x])^(3/2))/(2*c) + x*(a + b*ArcCos[c*x])^(5/2) + (15*b^(5/2)*Sqrt[Pi/2]
*Cos[a/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(4*c) + (
15*b^(5/2)*Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]
]*Sin[a/b])/(4*c)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4716

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> Simp[x*(a + b*Ar
cCos[c*x])^n, x] + Dist[b*c*n, Int[x*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 -
c^2*x^2]), x], x] /; FreeQ[{a, b, c}, x] && GtQ[n, 0]
```

Rule 4768

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)*((d_) + (e_.)*(x_)^2)^(p_
.), x_Symbol] :> Simp[(d + e*x^2)^(p + 1)*((a + b*ArcCos[c*x])^n/(2*e*(p +
1))), x] - Dist[b*(n/(2*c*(p + 1)))*Simp[(d + e*x^2)^p/(1 - c^2*x^2)^p], In
t[(1 - c^2*x^2)^(p + 1/2)*(a + b*ArcCos[c*x])^(n - 1), x], x] /; FreeQ[{a,
b, c, d, e, p}, x] && EqQ[c^2*d + e, 0] && GtQ[n, 0] && NeQ[p, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= x(a+b\arccos(cx))^{5/2}+\frac{1}{2}(5bc)\int\frac{x(a+b\arccos(cx))^{3/2}}{\sqrt{1-c^2x^2}}\,dx\\
&= -\frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{2c}\\
&\quad +x(a+b\arccos(cx))^{5/2}-\frac{1}{4}\left(15b^2\right)\int\sqrt{a+b\arccos(cx)}\,dx\\
&= -\frac{15}{4}b^2x\sqrt{a+b\arccos(cx)}-\frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{2c}\\
&\quad +x(a+b\arccos(cx))^{5/2}-\frac{1}{8}\left(15b^3c\right)\int\frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}\,dx\\
&= -\frac{15}{4}b^2x\sqrt{a+b\arccos(cx)}-\frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{2c}\\
&\quad +x(a+b\arccos(cx))^{5/2}+\frac{\left(15b^2\right)\text{Subst}\left(\int\frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{8c}
\end{aligned}
$$

$$= -\frac{15}{4}b^2x\sqrt{a+b\arccos(cx)} - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{2c} + x(a+b\arccos(cx))^{5/2} + \frac{\left(15b^2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{8c} + \frac{\left(15b^2\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{8c}$$

$$= -\frac{15}{4}b^2x\sqrt{a+b\arccos(cx)} - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{2c} + x(a+b\arccos(cx))^{5/2} + \frac{\left(15b^2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\cos\left(\frac{x^2}{b}\right)\,dx,x,\sqrt{a+b\arccos(cx)}\right)}{4c} + \frac{\left(15b^2\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\sin\left(\frac{x^2}{b}\right)\,dx,x,\sqrt{a+b\arccos(cx)}\right)}{4c}$$

$$= -\frac{15}{4}b^2x\sqrt{a+b\arccos(cx)} - \frac{5b\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}{2c} + x(a+b\arccos(cx))^{5/2} + \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{4c} + \frac{15b^{5/2}\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{4c}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.08

$$\int(a+b\arccos(cx))^{5/2}\,dx = \frac{\sqrt{b}e^{-\frac{ia}{b}}\left(\left(4a^2+15b^2\right)\left(1+e^{\frac{2ia}{b}}\right)\sqrt{2\pi}\sqrt{a+b\arccos(cx)}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\right.}{}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (Sqrt[b]*((4*a^2 + 15*b^2)*(1 + E^(((2*I)*a)/b))*Sqrt[2*Pi]*Sqrt[a + b*ArcC
os[c*x]]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] - I*(4*a^2
+ 15*b^2)*(-1 + E^(((2*I)*a)/b))*Sqrt[2*Pi]*Sqrt[a + b*ArcCos[c*x]]*Fresnel
S[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]] - 4*Sqrt[b]*(E^((I*a)/b)*(a
 + b*ArcCos[c*x])*(5*(3*b*c*x + 2*a*Sqrt[1 - c^2*x^2]) + (-8*a*c*x + 10*b*S
```

```
qrt[1 - c^2*x^2])*ArcCos[c*x] - 4*b*c*x*ArcCos[c*x]^2) - (2*I)*a^2*Sqrt[((-
I)*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, ((-I)*(a + b*ArcCos[c*x]))/b] + (2*I)
*a^2*E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[3/2, (I*(a + b*A
rcCos[c*x]))/b])))/(16*c*E^((I*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(139) = 278$.

Time = 2.08 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.24

method	result
default	$15\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\sqrt{2}\,b^3-15\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a\cdots}}{\sqrt{\pi}\cdots}\right.$

```
[In] int((a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/c/(a+b*arccos(c*x))^(1/2)*(15*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(
1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2
)/b)*2^(1/2)*b^3-15*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*
FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*2^(1/2)*b
^3+8*arccos(c*x)^3*cos(-(a+b*arccos(c*x))/b+a/b)*b^3+24*arccos(c*x)^2*cos(-
(a+b*arccos(c*x))/b+a/b)*a*b^2+20*arccos(c*x)^2*sin(-(a+b*arccos(c*x))/b+a/
b)*b^3+24*arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*a^2*b-30*arccos(c*x)*co
s(-(a+b*arccos(c*x))/b+a/b)*b^3+40*arccos(c*x)*sin(-(a+b*arccos(c*x))/b+a/b
)*a*b^2+8*cos(-(a+b*arccos(c*x))/b+a/b)*a^3-30*cos(-(a+b*arccos(c*x))/b+a/b
)*a*b^2+20*sin(-(a+b*arccos(c*x))/b+a/b)*a^2*b)
```

Fricas [F(-2)]

Exception generated.

$$\int (a+b\arccos(cx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (a+b\arccos(cx))^{5/2}\,dx=\int (a+b\operatorname{acos}(cx))^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*acos(c*x))**(5/2),x)
[Out] Integral((a + b*acos(c*x))**(5/2), x)
```

Maxima [F]

$$\int (a+b\arccos(cx))^{5/2}\,dx=\int (b\arccos(cx)+a)^{\frac{5}{2}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
[Out] integrate((b*arccos(c*x) + a)^(5/2), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.58 (sec) , antiderivative size = 1177, normalized size of antiderivative = 6.58

$$\int (a+b\arccos(cx))^{5/2}\,dx=\text{Too large to display}$$

```
[In] integrate((a+b*arccos(c*x))^(5/2),x, algorithm="giac")
[Out] -1/2*I*sqrt(2)*sqrt(pi)*a^3*b^3*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/
sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b
)/((I*b^4/sqrt(abs(b)) + b^3*sqrt(abs(b)))*c) + 1/2*I*sqrt(2)*sqrt(pi)*a^3*
b^3*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sq
rt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^4/sqrt(abs(b)) + b^
3*sqrt(abs(b)))*c) + 3/2*sqrt(2)*sqrt(pi)*a^2*b^3*erf(-1/2*I*sqrt(2)*sqrt(b
*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(a
bs(b))/b)*e^(I*a/b)/((I*b^3/sqrt(abs(b)) + b^2*sqrt(abs(b)))*c) + 3/2*sqrt(
2)*sqrt(pi)*a^2*b^3*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b))
- 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^3/s
qrt(abs(b)) + b^2*sqrt(abs(b)))*c) - 3/2*sqrt(2)*sqrt(pi)*a^2*b^2*erf(-1/2*
I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(
c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*
c) - 15/16*sqrt(2)*sqrt(pi)*b^4*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/
sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b
```

```
)/((I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) - 3/2*sqrt(2)*sqrt(pi)*a^2*b^2*
erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b
*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt
(abs(b)))*c) - 15/16*sqrt(2)*sqrt(pi)*b^4*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c
*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)
*e^(-I*a/b)/((-I*b^2/sqrt(abs(b)) + b*sqrt(abs(b)))*c) + 1/2*sqrt(b*arccos(
c*x) + a)*b^2*arccos(c*x)^2*e^(I*arccos(c*x))/c + 1/2*sqrt(b*arccos(c*x) +
a)*b^2*arccos(c*x)^2*e^(-I*arccos(c*x))/c + I*sqrt(pi)*a^3*b*erf(-1/2*I*sqr
t(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x)
+ a)*sqrt(abs(b))/b)*e^(I*a/b)/((I*sqrt(2)*b^2/sqrt(abs(b)) + sqrt(2)*b*sqr
t(abs(b)))*c) - I*sqrt(pi)*a^3*b*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/
sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/
b)/((-I*sqrt(2)*b^2/sqrt(abs(b)) + sqrt(2)*b*sqrt(abs(b)))*c) + sqrt(b*arcc
os(c*x) + a)*a*b*arccos(c*x)*e^(I*arccos(c*x))/c + 5/4*I*sqrt(b*arccos(c*x)
 + a)*b^2*arccos(c*x)*e^(I*arccos(c*x))/c + sqrt(b*arccos(c*x) + a)*a*b*arc
cos(c*x)*e^(-I*arccos(c*x))/c - 5/4*I*sqrt(b*arccos(c*x) + a)*b^2*arccos(c*
x)*e^(-I*arccos(c*x))/c + 1/2*sqrt(b*arccos(c*x) + a)*a^2*e^(I*arccos(c*x))
/c + 5/4*I*sqrt(b*arccos(c*x) + a)*a*b*e^(I*arccos(c*x))/c - 15/8*sqrt(b*ar
ccos(c*x) + a)*b^2*e^(I*arccos(c*x))/c + 1/2*sqrt(b*arccos(c*x) + a)*a^2*e^
(-I*arccos(c*x))/c - 5/4*I*sqrt(b*arccos(c*x) + a)*a*b*e^(-I*arccos(c*x))/c
 - 15/8*sqrt(b*arccos(c*x) + a)*b^2*e^(-I*arccos(c*x))/c
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\arccos(cx))^{5/2}\,dx=\int (a+b\,\mathrm{acos}(c\,x))^{5/2}\,dx$$

```
[In] int((a + b*acos(c*x))^(5/2),x)
[Out] int((a + b*acos(c*x))^(5/2), x)
```

3.186 $\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx = \operatorname{Int}\left(\frac{(a+b\arccos(cx))^{5/2}}{x},x\right)$$

```
[Out] Unintegrable((a+b*arccos(c*x))^(5/2)/x,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx = \int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx$$

```
[In] Int[(a + b*ArcCos[c*x])^(5/2)/x,x]
```

```
[Out] Defer[Int][(a + b*ArcCos[c*x])^(5/2)/x, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx$$

Mathematica [N/A]

Not integrable

Time = 0.79 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx = \int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx$$

```
[In] Integrate[(a + b*ArcCos[c*x])^(5/2)/x,x]
[Out] Integrate[(a + b*ArcCos[c*x])^(5/2)/x, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos(cx))^{\frac{5}{2}}}{x}dx$$

```
[In] int((a+b*arccos(c*x))^(5/2)/x,x)
[Out] int((a+b*arccos(c*x))^(5/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 38.82 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate((a+b*acos(c*x))**(5/2)/x,x)
[Out] Integral((a + b*acos(c*x))**(5/2)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x,x, algorithm="maxima")
[Out] integrate((b*arccos(c*x) + a)^(5/2)/x, x)
```

Giac [N/A]

Not integrable

Time = 1.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{5}{2}}}{x}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x,x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^(5/2)/x, x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^{5/2}}{x}\,dx$$

```
[In] int((a + b*acos(c*x))^(5/2)/x,x)
[Out] int((a + b*acos(c*x))^(5/2)/x, x)
```

3.187 $\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx = \operatorname{Int}\left(\frac{(a+b\arccos(cx))^{5/2}}{x^2},x\right)$$

```
[Out] Unintegrable((a+b*arccos(c*x))^(5/2)/x^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx = \int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx$$

```
[In] Int[(a + b*ArcCos[c*x])^(5/2)/x^2,x]
```

```
[Out] Defer[Int][(a + b*ArcCos[c*x])^(5/2)/x^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 6.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx = \int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx$$

```
[In] Integrate[(a + b*ArcCos[c*x])^(5/2)/x^2,x]
[Out] Integrate[(a + b*ArcCos[c*x])^(5/2)/x^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.16 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{(a+b\arccos(cx))^{\frac{5}{2}}}{x^2}dx$$

```
[In] int((a+b*arccos(c*x))^(5/2)/x^2,x)
[Out] int((a+b*arccos(c*x))^(5/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x^2,x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 24.82 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx = \int \frac{(a+b\operatorname{acos}(cx))^{\frac{5}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b*acos(c*x))**(5/2)/x**2,x)
[Out] Integral((a + b*acos(c*x))**(5/2)/x**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{5}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x^2,x, algorithm="maxima")
[Out] integrate((b*arccos(c*x) + a)^(5/2)/x^2, x)
```

Giac [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx = \int \frac{(b\arccos(cx)+a)^{\frac{5}{2}}}{x^2}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^(5/2)/x^2,x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^(5/2)/x^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^{5/2}}{x^2}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^{5/2}}{x^2}\,dx$$

```
[In] int((a + b*acos(c*x))^(5/2)/x^2,x)
[Out] int((a + b*acos(c*x))^(5/2)/x^2, x)
```

3.188 $\int \frac{x^2}{\sqrt{a+b \arccos(cx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 223

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} - \frac{\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{2}}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{2\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{6}}\,\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{2\sqrt{b}c^3}$$

```
[Out] -1/12*cos(3*a/b)*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))
*6^(1/2)*Pi^(1/2)/c^3/b^(1/2)+1/12*FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*
x))^(1/2)/b^(1/2))*sin(3*a/b)*6^(1/2)*Pi^(1/2)/c^3/b^(1/2)-1/4*cos(a/b)*Fre
snelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*2^(1/2)*Pi^(1/2)/c^
3/b^(1/2)+1/4*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*si
n(a/b)*2^(1/2)*Pi^(1/2)/c^3/b^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {4732, 4491, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{\sqrt{\frac{\pi}{2}}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{6}}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} - \frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} - \frac{\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3}$$

```
[In] Int[x^2/Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] -1/2*(Sqrt[Pi/2]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqr
t[b]])/(Sqrt[b]*c^3) - (Sqrt[Pi/6]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a
 + b*ArcCos[c*x]])/Sqrt[b]])/(2*Sqrt[b]*c^3) + (Sqrt[Pi/2]*FresnelC[(Sqrt[2
/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(2*Sqrt[b]*c^3) + (Sqrt[Pi
/6]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(2
*Sqrt[b]*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \frac{\cos^2\left(\frac{a}{b}-\frac{x}{b}\right)\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{bc^3}\\
&= \frac{\text{Subst}\left(\int \left(\frac{\sin\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}}+\frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)dx, x, a+b\arccos(cx)\right)}{bc^3}\\
&= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{4bc^3}+\frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{4bc^3}\\
&= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{4bc^3}\\
&\quad -\frac{\cos\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{4bc^3}\\
&\quad +\frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{4bc^3}\\
&\quad +\frac{\sin\left(\frac{3a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{4bc^3}
\end{aligned}
$$

$$= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{2bc^3} - \frac{\cos\left(\frac{3a}{b}\right)\text{Subst}\left(\int \sin\left(\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{2bc^3} + \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{2bc^3} + \frac{\sin\left(\frac{3a}{b}\right)\text{Subst}\left(\int \cos\left(\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{2bc^3}$$

$$= -\frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} - \frac{\sqrt{\frac{\pi}{6}}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{2\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{2}}\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{2\sqrt{b}c^3} + \frac{\sqrt{\frac{\pi}{6}}\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{2\sqrt{b}c^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.32 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.01

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{e^{-\frac{3ia}{b}}\left(3e^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+3e^{\frac{4ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2},\frac{i(a+b\arccos(cx))}{b}\right)+\sqrt{3}\left(\sqrt{\right.\right.}{24c^3\sqrt{a+b\arccos(cx}}$$

```
[In] Integrate[x^2/Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] (3*E^(((2*I)*a)/b)*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a +
 b*ArcCos[c*x]))/b] + 3*E^(((4*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gam
ma[1/2, (I*(a + b*ArcCos[c*x]))/b] + Sqrt[3]*(Sqrt[((-I)*(a + b*ArcCos[c*x]
))/b]*Gamma[1/2, ((-3*I)*(a + b*ArcCos[c*x]))/b] + E^(((6*I)*a)/b)*Sqrt[(I*
(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos[c*x]))/b]))/(24*c^3
*E^(((3*I)*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.89

method	result
default	$\frac{\sqrt{\pi}\,\sqrt{2}\,\sqrt{-\frac{1}{b}}\left(3\sin\left(\frac{a}{b}\right)\mathrm{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)+3\cos\left(\frac{a}{b}\right)\mathrm{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)-\sqrt{-\frac{1}{b}}\,\sqrt{-\frac{3}{b}}\,\cos\left(\frac{3a}{b}\right)\mathrm{FresnelS}\left(3\sqrt{}\right.\right.}{12c^{3}}$

[In] int(x^2/(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/12/c^3*Pi^(1/2)*2^(1/2)*(-1/b)^(1/2)*(3*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2
)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+3*cos(a/b)*FresnelS(2^(1/2)/Pi^(1
/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-(-1/b)^(1/2)*(-3/b)^(1/2)*cos(3
*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b
-(-1/b)^(1/2)*(-3/b)^(1/2)*sin(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1
/2)*(a+b*arccos(c*x))^(1/2)/b)*b)

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x^2}{\sqrt{a+b\operatorname{acos}(cx)}}\,dx$$

[In] integrate(x**2/(a+b*acos(c*x))**(1/2),x)

[Out] Integral(x**2/sqrt(a + b*acos(c*x)), x)

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x^2}{\sqrt{b\arccos(cx)+a}}\,dx$$

[In] integrate(x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")

[Out] integrate(x^2/sqrt(b*arccos(c*x) + a), x)

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.42

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{b\arccos(cx)+a}}{2\sqrt{b}} - \frac{i\sqrt{6}\sqrt{b\arccos(cx)+a}\sqrt{b}}{2|b|}\right)e^{\left(\frac{3ia}{b}\right)}}{4\left(\sqrt{6}\sqrt{b}+\frac{i\sqrt{6}b^{\frac{3}{2}}}{|b|}\right)c^3} + \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2b}\right)e^{\left(\frac{ia}{b}\right)}}{4c^3\left(\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)} - \frac{i\sqrt{\pi}\,\mathrm{erf}\left(\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2b}\right)e^{\left(-\frac{ia}{b}\right)}}{4c^3\left(-\frac{i\sqrt{2}b}{\sqrt{|b|}}+\sqrt{2}\sqrt{|b|}\right)} - \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{6}\sqrt{b\arccos(cx)+a}}{2\sqrt{b}} + \frac{i\sqrt{6}\sqrt{b\arccos(cx)+a}\sqrt{b}}{2|b|}\right)e^{\left(-\frac{3ia}{b}\right)}}{4\left(\sqrt{6}\sqrt{b}-\frac{i\sqrt{6}b^{\frac{3}{2}}}{|b|}\right)c^3}$$

[In] integrate(x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")

[Out] 1/4*I*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) - 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(3*I*a/b)/((sqrt(6)*sqrt(b) + I*sqrt(6)*b^(3/2)/abs(b))*c^3) + 1/4*I*sqrt(pi)*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c^3*(I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - 1/4*I*sqrt(pi)*erf(1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(-I*a/b)/(c^3*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b)))) - 1/4*I*sqrt(pi)*erf(-1/2*sqrt(6)*sqrt(b*arccos(c*x) + a)/sqrt(b) + 1/2*I*sqrt(6)*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-3*I*a/b)/((sqrt(6)*sqrt(b) - I*sqrt(6)*b^(3/2)/abs(b))*c^3)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x^2}{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}\,dx$$

```
[In] int(x^2/(a + b*acos(c*x))^(1/2),x)
[Out] int(x^2/(a + b*acos(c*x))^(1/2), x)
```

3.189 $\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = -\frac{\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}c^2} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{2\sqrt{b}c^2}$$

```
[Out] -1/2*cos(2*a/b)*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*Pi^(1/2)/c^2/b^(1/2)+1/2*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(2*a/b)*Pi^(1/2)/c^2/b^(1/2)
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {4732, 4491, 12, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{\sqrt{\pi}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}c^2} - \frac{\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}c^2}$$

```
[In] Int[x/Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] -1/2*(Sqrt[Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/(Sqrt[b]*c^2) + (Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*a)/b])/(2*Sqrt[b]*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{bc^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{bc^2}$$
$$= \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{2bc^2}$$
$$= -\frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{2bc^2} + \frac{\sin\left(\frac{2a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{2bc^2}$$
$$= -\frac{\cos\left(\frac{2a}{b}\right)\text{Subst}\left(\int \sin\left(\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{bc^2} + \frac{\sin\left(\frac{2a}{b}\right)\text{Subst}\left(\int \cos\left(\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{bc^2}$$
$$= -\frac{\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{2\sqrt{b}c^2} + \frac{\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{2\sqrt{b}c^2}$$

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.86

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{\sqrt{\pi}\left(-\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)+\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)\right)}{2\sqrt{b}c^2}$$

```
[In] Integrate[x/Sqrt[a + b*ArcCos[c*x]],x]
[Out] (Sqrt[Pi]*(-(Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqr
t[Pi])]) + FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*Sin[(2*
a)/b]))/(2*Sqrt[b]*c^2)
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,\left(\cos\left(\frac{2a}{b}\right)\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)+\sin\left(\frac{2a}{b}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)\right)}{2c^{2}}$	91

```
[In] int(x/(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*Pi^(1/2)*(-1/b)^(1/2)*(cos(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1
/2)*(a+b*arccos(c*x))^(1/2)/b)+sin(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b
)^(1/2)*(a+b*arccos(c*x))^(1/2)/b))/c^2
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x}{\sqrt{a+b\operatorname{acos}(cx)}}\,dx$$

```
[In] integrate(x/(a+b*acos(c*x))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x}{\sqrt{b\arccos(cx)+a}}\,dx$$

[In] integrate(x/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")

[Out] integrate(x/sqrt(b*arccos(c*x) + a), x)

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.33

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = -\frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}+\frac{i\sqrt{b\arccos(cx)+a}\sqrt{b}}{|b|}\right)e^{\left(-\frac{2ia}{b}\right)}}{4c^2\left(\sqrt{b}-\frac{ib^{\frac{3}{2}}}{|b|}\right)} + \frac{i\sqrt{\pi}\,\mathrm{erf}\left(-\frac{\sqrt{b\arccos(cx)+a}}{\sqrt{b}}-\frac{i\sqrt{b\arccos(cx)+a}\sqrt{b}}{|b|}\right)e^{\left(\frac{2ia}{b}\right)}}{4\sqrt{b}c^2\left(\frac{ib}{|b|}+1\right)}$$

[In] integrate(x/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")

[Out] -1/4*I*sqrt(pi)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) + I*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(-2*I*a/b)/(c^2*(sqrt(b) - I*b^(3/2)/abs(b))) + 1/4*I*sqrt(pi)*erf(-sqrt(b*arccos(c*x) + a)/sqrt(b) - I*sqrt(b*arccos(c*x) + a)*sqrt(b)/abs(b))*e^(2*I*a/b)/(sqrt(b)*c^2*(I*b/abs(b) + 1))

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{x}{\sqrt{a+b\,\mathrm{acos}\,(c\,x)}}\,dx$$

[In] int(x/(a + b*acos(c*x))^(1/2),x)

[Out] int(x/(a + b*acos(c*x))^(1/2), x)

3.190 $\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 102

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = -\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c} + \frac{\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{\sqrt{b}c}$$

```
[Out] -cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*2^(1/2
)*Pi^(1/2)/c/b^(1/2)+FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1
/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/c/b^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {4720, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c} - \frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c}$$

```
[In] Int[1/Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] -((Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b
]])/(Sqrt[b]*c)) + (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]]
)/Sqrt[b]]*Sin[a/b])/(Sqrt[b]*c)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rubi steps

$$\text{integral} = \frac{\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{bc}$$

$$= -\frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{bc} + \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{bc}$$

$$= -\frac{(2\cos(\frac{a}{b}))\,\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{bc} + \frac{(2\sin(\frac{a}{b}))\,\text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{bc}$$

$$= -\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{\sqrt{b}c} + \frac{\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{\sqrt{b}c}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{e^{-\frac{ia}{b}}\left(\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\,\Gamma\left(\frac{1}{2},-\frac{i(a+b\arccos(cx))}{b}\right)+e^{\frac{2ia}{b}}\sqrt{\frac{i(a+b\arccos(cx))}{b}}\,\Gamma\left(\frac{1}{2},\frac{i(a+b\arccos(cx))}{b}\right)\right)}{2c\sqrt{a+b\arccos(cx)}}$$

```
[In] Integrate[1/Sqrt[a + b*ArcCos[c*x]],x]
```

```
[Out] (Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b
] + E^(((2*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*Ar
cCos[c*x]))/b])/(2*c*E^((I*a)/b)*Sqrt[a + b*ArcCos[c*x]])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\left(\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)+\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\right)}{c}$	89

```
[In] int(1/(a+b*arccos(c*x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*(cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1
/2)*(a+b*arccos(c*x))^(1/2)/b)+sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1
/2)*(a+b*arccos(c*x))^(1/2)/b))/c
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{a+b\operatorname{acos}(cx)}}\,dx$$

```
[In] integrate(1/(a+b*acos(c*x))**(1/2),x)
[Out] Integral(1/sqrt(a + b*acos(c*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{b\arccos(cx)+a}}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*arccos(c*x) + a), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx = \frac{i\sqrt{\pi}\operatorname{erf}\left(-\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2b}\right)e^{\left(\frac{ia}{b}\right)}}{c\left(\frac{i\sqrt{2}b}{\sqrt{|b|}} + \sqrt{2}\sqrt{|b|}\right)} - \frac{i\sqrt{\pi}\operatorname{erf}\left(\frac{i\sqrt{2}\sqrt{b\arccos(cx)+a}}{2\sqrt{|b|}} - \frac{\sqrt{2}\sqrt{b\arccos(cx)+a}\sqrt{|b|}}{2b}\right)e^{\left(-\frac{ia}{b}\right)}}{c\left(-\frac{i\sqrt{2}b}{\sqrt{|b|}} + \sqrt{2}\sqrt{|b|}\right)}$$

```
[In] integrate(1/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
```

```
[Out] I*sqrt(pi)*erf(-1/2*I*sqrt(2)*sqrt(b*arccos(c*x) + a)/sqrt(abs(b)) - 1/2*sq
rt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(b))/b)*e^(I*a/b)/(c*(I*sqrt(2)*b/sqr
t(abs(b)) + sqrt(2)*sqrt(abs(b)))) - I*sqrt(pi)*erf(1/2*I*sqrt(2)*sqrt(b*ar
ccos(c*x) + a)/sqrt(abs(b)) - 1/2*sqrt(2)*sqrt(b*arccos(c*x) + a)*sqrt(abs(
b))/b)*e^(-I*a/b)/(c*(-I*sqrt(2)*b/sqrt(abs(b)) + sqrt(2)*sqrt(abs(b))))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}\,dx$$

```
[In] int(1/(a + b*acos(c*x))^(1/2),x)
```

```
[Out] int(1/(a + b*acos(c*x))^(1/2), x)
```

3.191 $\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \operatorname{Int}\left(\frac{1}{x\sqrt{a+b\arccos(cx)}},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arccos(c*x))^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx$$

```
[In] Int[1/(x*Sqrt[a + b*ArcCos[c*x]]),x]
[Out] Defer[Int][1/(x*Sqrt[a + b*ArcCos[c*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx$$

```
[In] Integrate[1/(x*Sqrt[a + b*ArcCos[c*x]]),x]
[Out] Integrate[1/(x*Sqrt[a + b*ArcCos[c*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\sqrt{a+b\arccos\left(cx\right)}}dx$$

```
[In] int(1/x/(a+b*arccos(c*x))^(1/2),x)
[Out] int(1/x/(a+b*arccos(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx=\int \frac{1}{x\sqrt{a+b\operatorname{acos}\left(cx\right)}}\,dx$$

```
[In] integrate(1/x/(a+b*acos(c*x))**(1/2),x)
[Out] Integral(1/(x*sqrt(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{b\arccos\left(cx\right)+a}x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*arccos(c*x) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{b\arccos\left(cx\right)+a}x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*arccos(c*x) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{x\,\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}\,dx$$

```
[In] int(1/(x*(a + b*acos(c*x))^(1/2)),x)
[Out] int(1/(x*(a + b*acos(c*x))^(1/2)), x)
```

3.192 $\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \text{Int}\left(\frac{1}{x^2\sqrt{a+b\arccos(cx)}},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*arccos(c*x))^(1/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx$$

```
[In] Int[1/(x^2*Sqrt[a + b*ArcCos[c*x]]),x]
[Out] Defer[Int][1/(x^2*Sqrt[a + b*ArcCos[c*x]]), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.39 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx$$

```
[In] Integrate[1/(x^2*Sqrt[a + b*ArcCos[c*x]]),x]
[Out] Integrate[1/(x^2*Sqrt[a + b*ArcCos[c*x]]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\sqrt{a+b\arccos\left(cx\right)}}dx$$

```
[In] int(1/x^2/(a+b*arccos(c*x))^(1/2),x)
[Out] int(1/x^2/(a+b*arccos(c*x))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{x^2\sqrt{a+b\operatorname{acos}\left(cx\right)}}\,dx$$

```
[In] integrate(1/x**2/(a+b*acos(c*x))**(1/2),x)
[Out] Integral(1/(x**2*sqrt(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.62 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{b\arccos\left(cx\right)+a}x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(b*arccos(c*x) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{\sqrt{b\arccos\left(cx\right)+a}x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(b*arccos(c*x) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+b\arccos(cx)}}\,dx = \int \frac{1}{x^2\,\sqrt{a+b\,\mathrm{acos}\left(c\,x\right)}}\,dx$$

```
[In] int(1/(x^2*(a + b*acos(c*x))^(1/2)),x)
[Out] int(1/(x^2*(a + b*acos(c*x))^(1/2)), x)
```

3.193 $\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 252

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{3\pi}{2}}\operatorname{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{b^{3/2}c^3}$$

```
[Out] -1/2*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*2^
(1/2)*Pi^(1/2)/b^(3/2)/c^3-1/2*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^
(1/2)/b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/b^(3/2)/c^3-1/2*cos(3*a/b)*Fresnel
C(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(3/2
)/c^3-1/2*FresnelS(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*
a/b)*6^(1/2)*Pi^(1/2)/b^(3/2)/c^3+2*x^2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(
c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4728, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = -\frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{\pi}{2}}\sin\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{3\pi}{2}}\sin\left(\frac{3a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3} + \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

```
[In] Int[x^2/(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (2*x^2*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (Sqrt[Pi/2]*Cos[a
/b]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(b^(3/2)*c^3) -
 (Sqrt[(3*Pi)/2]*Cos[(3*a)/b]*FresnelC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])
/Sqrt[b]])/(b^(3/2)*c^3) - (Sqrt[Pi/2]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcC
os[c*x]])/Sqrt[b]]*Sin[a/b])/(b^(3/2)*c^3) - (Sqrt[(3*Pi)/2]*FresnelS[(Sqrt
[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(b^(3/2)*c^3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
```

```
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

integral

$$= \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} + \frac{2\text{Subst}\left(\int \left(-\frac{3\cos\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} - \frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right) dx, x, a+b\arccos(cx)\right)}{b^2c^3}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}} dx, x, a+b\arccos(cx)\right)}{2b^2c^3} - \frac{3\text{Subst}\left(\int \frac{\cos\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}} dx, x, a+b\arccos(cx)\right)}{2b^2c^3}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}} dx, x, a+b\arccos(cx)\right)}{2b^2c^3} - \frac{\left(3\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{3x}{b}\right)}{\sqrt{x}} dx, x, a+b\arccos(cx)\right)}{2b^2c^3} - \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}} dx, x, a+b\arccos(cx)\right)}{2b^2c^3} - \frac{\left(3\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{3x}{b}\right)}{\sqrt{x}} dx, x, a+b\arccos(cx)\right)}{2b^2c^3}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{\cos\left(\frac{a}{b}\right)\text{Subst}\left(\int\cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{b^2c^3}$$
$$- \frac{\left(3\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\cos\left(\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{b^2c^3}$$
$$- \frac{\sin\left(\frac{a}{b}\right)\text{Subst}\left(\int\sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{b^2c^3}$$
$$- \frac{\left(3\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\sin\left(\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{b^2c^3}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{\sqrt{\frac{\pi}{2}}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}$$
$$- \frac{\sqrt{\frac{3\pi}{2}}\cos\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c^3}$$
$$- \frac{\sqrt{\frac{\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c^3} - \frac{\sqrt{\frac{3\pi}{2}}\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{b^{3/2}c^3}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.08

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{e^{-\frac{3ia}{b}}\left(8c^2e^{\frac{3ia}{b}}x^2\sqrt{1-c^2x^2} + ie^{\frac{2ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\Gamma\left(\frac{1}{2}, -\frac{i(a+b\arccos(cx))}{b}\right) - ie^{4}\right.}{}$$

```
[In] Integrate[x^2/(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (8*c^2*E^(((3*I)*a)/b)*x^2*Sqrt[1 - c^2*x^2] + I*E^(((2*I)*a)/b)*Sqrt[((-I)
*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a + b*ArcCos[c*x]))/b] - I*E^(((
4*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, (I*(a + b*ArcCos[c*x]
))/b] + I*Sqrt[3]*Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-3*I)*(a
+ b*ArcCos[c*x]))/b] - I*Sqrt[3]*E^(((6*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]
))/b]*Gamma[1/2, ((3*I)*(a + b*ArcCos[c*x]))/b])/(4*b*c^3*E^(((3*I)*a)/b)*S
qrt[a + b*ArcCos[c*x]])
```

Maple [A] (verified)

Time = 2.07 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.19

method	result
default	$-\frac{\sqrt{-\frac{3}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{3a}{b}\right)\mathrm{FresnelC}\left(\frac{3\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{3}{b}}\,b}\right)-\sqrt{-\frac{3}{b}}\,\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{3a}{b}\right)\mathrm{FresnelS}\left(\frac{3\sqrt{2}}{}\right.}{}$

[In] int(x^2/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2/c^3/b*((-3/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(3*a/b)*FresnelC(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)-(-3/b)^(1/2)*Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(3*a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-1/b)^(1/2)-Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-1/b)^(1/2)+sin(-(a+b*arccos(c*x))/b+a/b)+sin(-3*(a+b*arccos(c*x))/b+3*a/b))/(a+b*arccos(c*x))^(1/2)

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x^2}{(a+b\operatorname{acos}(cx))^{\frac{3}{2}}}\,dx$$

[In] integrate(x**2/(a+b*acos(c*x))**(3/2),x)

[Out] Integral(x**2/(a + b*acos(c*x))**(3/2), x)

Maxima [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x^2/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(x^2/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x^2}{(a+b\,\mathrm{acos}(c\,x))^{3/2}}\,dx$$

```
[In] int(x^2/(a + b*acos(c*x))^(3/2),x)
[Out] int(x^2/(a + b*acos(c*x))^(3/2), x)
```

3.194 $\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 130

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{2\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}c^2} - \frac{2\sqrt{\pi}\operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{b^{3/2}c^2}$$

```
[Out] -2*cos(2*a/b)*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*Pi^(1/2)
/b^(3/2)/c^2-2*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(2*a
/b)*Pi^(1/2)/b^(3/2)/c^2+2*x*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {4728, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = -\frac{2\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\operatorname{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}c^2} - \frac{2\sqrt{\pi}\sin\left(\frac{2a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}c^2} + \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

```
[In] Int[x/(a + b*ArcCos[c*x])^(3/2),x]
```

```
[Out] (2*x*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (2*Sqrt[Pi]*Cos[(2*
a)/b]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/(b^(3/2)*c^
```

```
2) - (2*Sqrt[Pi]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])]*S
in[(2*a)/b])/(b^(3/2)*c^2)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4728

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - D
ist[1/(b^2*c^(m + 1)*(n + 1)), Subst[Int[ExpandTrigReduce[x^(n + 1), Cos[-a
/b + x/b]^(m - 1)*(m - (m + 1)*Cos[-a/b + x/b]^2), x], x], x, a + b*ArcCos[
c*x]], x] /; FreeQ[{a, b, c}, x] && IGtQ[m, 0] && GeQ[n, -2] && LtQ[n, -1]
```

Rubi steps

$$\text{integral} = \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{2\text{Subst}\left(\int \frac{\cos\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{b^2c^2}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{\left(2\cos\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\cos\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{b^2c^2} - \frac{\left(2\sin\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \frac{\sin\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{b^2c^2}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{\left(4\cos\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \cos\left(\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{b^2c^2} - \frac{\left(4\sin\left(\frac{2a}{b}\right)\right)\mathrm{Subst}\left(\int \sin\left(\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{b^2c^2}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{2\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{b^{3/2}c^2} - \frac{2\sqrt{\pi}\,\mathrm{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{b^{3/2}c^2}$$

Mathematica [F]

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx$$

```
[In] Integrate[x/(a + b*ArcCos[c*x])^(3/2),x]
[Out] Integrate[x/(a + b*ArcCos[c*x])^(3/2), x]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.21

method	result
default	$-\frac{2\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{2a}{b}\right)\mathrm{FresnelC}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)-2\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{2a}{b}\right)\mathrm{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{a+b}}{\sqrt{\pi}}\right.}{c^2b\sqrt{a+b\arccos(cx)}}$

```
[In] int(x/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/c^2/b/(a+b*arccos(c*x))^(1/2)*(2*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))
^(1/2)*cos(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x)
)^(1/2)/b)-2*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(2*a/b)*Fresn
elS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)+sin(-2*(a+b*
arccos(c*x))/b+2*a/b))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(x/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x}{(a+b\operatorname{acos}(cx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*acos(c*x))**(3/2),x)
[Out] Integral(x/(a + b*acos(c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(x/(b*arccos(c*x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x}{(b\arccos(cx)+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(x/(b*arccos(c*x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{x}{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(x/(a + b*acos(c*x))^(3/2),x)
[Out] int(x/(a + b*acos(c*x))^(3/2), x)
```

3.195 $\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 137

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}} - \frac{2\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{2\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c}$$

```
[Out] -2*cos(a/b)*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*2^(1
/2)*Pi^(1/2)/b^(3/2)/c-2*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/
b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/b^(3/2)/c+2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*
arccos(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {4718, 4810, 3387, 3386, 3432, 3385, 3433}

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = -\frac{2\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} - \frac{2\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c} + \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}$$

```
[In] Int[(a + b*ArcCos[c*x])^(-3/2),x]
```

```
[Out] (2*Sqrt[1 - c^2*x^2])/(b*c*Sqrt[a + b*ArcCos[c*x]]) - (2*Sqrt[2*Pi]*Cos[a/b
]*FresnelC[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(b^(3/2)*c) - (2*
Sqrt[2*Pi]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])
/(b^(3/2)*c)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4810

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*(x_)^(m_.)*((d_) + (e_.)*(x_)^
2)^(p_.), x_Symbol] :> Dist[(-(b*c^(m + 1))^(-1))*Simp[(d + e*x^2)^p/(1 - c
^2*x^2)^p], Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b]^(2*p + 1), x],
```

```
x, a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^2*d + e,
 0] && IGtQ[2*p + 2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}+\frac{(2c)\int\frac{x}{\sqrt{1-c^2x^2}\sqrt{a+b\arccos(cx)}}\,dx}{b}\\
&= \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\text{Subst}\left(\int\frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{b^2c}\\
&= \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{\left(2\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{b^2c}\\
&\quad-\frac{\left(2\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{b^2c}\\
&= \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{\left(4\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\cos\left(\frac{x^2}{b}\right)\,dx,x,\sqrt{a+b\arccos(cx)}\right)}{b^2c}\\
&\quad-\frac{\left(4\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\sin\left(\frac{x^2}{b}\right)\,dx,x,\sqrt{a+b\arccos(cx)}\right)}{b^2c}\\
&= \frac{2\sqrt{1-c^2x^2}}{bc\sqrt{a+b\arccos(cx)}}-\frac{2\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{3/2}c}\\
&\quad-\frac{2\sqrt{2\pi}\,\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{b^{3/2}c}
\end{aligned}$$

Mathematica [F]

$$\int\frac{1}{(a+b\arccos(cx))^{3/2}}\,dx=\int\frac{1}{(a+b\arccos(cx))^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCos[c*x])^(-3/2),x]
[Out] Integrate[(a + b*ArcCos[c*x])^(-3/2), x]
```

Maple [A] (verified)

Time = 1.96 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15

method	result
default	$-\frac{2\left(\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\sqrt{-\frac{1}{b}}-\sqrt{\pi}\,\sqrt{2}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arc}}{\sqrt{\pi}\,\sqrt{-}}\right.\right.}{cb\sqrt{a+b\arccos(cx)}}$

```
[In] int(1/(a+b*arccos(c*x))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/c/b/(a+b*arccos(c*x))^(1/2)*(Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*co
s(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-
1/b)^(1/2)-Pi^(1/2)*2^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelS(2^(1/
2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-1/b)^(1/2)+sin(-(a+b*
arccos(c*x))/b+a/b))
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(a+b\operatorname{acos}(cx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*acos(c*x))**(3/2),x)
```

```
[Out] Integral((a + b*acos(c*x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{1}{(b\arccos{(cx)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate((b*arccos(c*x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{1}{(b\arccos{(cx)}+a)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(cx))^{3/2}}\,dx=\int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))^{3/2}}\,dx$$

```
[In] int(1/(a + b*acos(c*x))^(3/2),x)
[Out] int(1/(a + b*acos(c*x))^(3/2), x)
```

3.196 $\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \mathrm{Int}\left(\frac{1}{x(a+b\arccos(cx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arccos(c*x))^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx$$

```
[In] Int[1/(x*(a + b*ArcCos[c*x])^(3/2)),x]
[Out] Defer[Int][1/(x*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcCos[c*x])^(3/2)),x]
[Out] Integrate[1/(x*(a + b*ArcCos[c*x])^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.92 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\,(a+b\arccos\,(cx))^{\frac{3}{2}}}dx$$

```
[In] int(1/x/(a+b*arccos(c*x))^(3/2),x)
[Out] int(1/x/(a+b*arccos(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.81 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x\,(a+b\,\mathrm{acos}\,(cx))^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x/(a+b*acos(c*x))**(3/2),x)
[Out] Integral(1/(x*(a + b*acos(c*x))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(b\arccos(cx)+a)^{\frac{3}{2}}x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*arccos(c*x) + a)^(3/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(1/(x*(a + b*acos(c*x))^(3/2)),x)
[Out] int(1/(x*(a + b*acos(c*x))^(3/2)), x)
```

3.197 $\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x^2(a+b\arccos(cx))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*arccos(c*x))^(3/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx$$

```
[In] Int[1/(x^2*(a + b*ArcCos[c*x])^(3/2)),x]
[Out] Defer[Int][1/(x^2*(a + b*ArcCos[c*x])^(3/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 7.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx$$

```
[In] Integrate[1/(x^2*(a + b*ArcCos[c*x])^(3/2)),x]
[Out] Integrate[1/(x^2*(a + b*ArcCos[c*x])^(3/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^2\left(a+b\arccos\left(cx\right)\right)^{\frac{3}{2}}}dx$$

```
[In] int(1/x^2/(a+b*arccos(c*x))^(3/2),x)
[Out] int(1/x^2/(a+b*arccos(c*x))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 2.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x^2\left(a+b\operatorname{acos}\left(cx\right)\right)^{\frac{3}{2}}}\,dx$$

```
[In] integrate(1/x**2/(a+b*acos(c*x))**(3/2),x)
[Out] Integral(1/(x**2*(a + b*acos(c*x))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(b\arccos(cx)+a)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="maxima")
[Out] integrate(1/((b*arccos(c*x) + a)^(3/2)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 1.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{(b\arccos(cx)+a)^{\frac{3}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(3/2),x, algorithm="giac")
[Out] integrate(1/((b*arccos(c*x) + a)^(3/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{3/2}}\,dx = \int \frac{1}{x^2\,\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^{3/2}}\,dx$$

```
[In] int(1/(x^2*(a + b*acos(c*x))^(3/2)),x)
[Out] int(1/(x^2*(a + b*acos(c*x))^(3/2)), x)
```

3.198 $\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 292

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx=\frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}-\frac{8x}{3b^2c^2\sqrt{a+b\arccos(cx)}}+\frac{4x^3}{b^2\sqrt{a+b\arccos(cx)}}+\frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3}+\frac{\sqrt{6\pi}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{5/2}c^3}-\frac{\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{3b^{5/2}c^3}-\frac{\sqrt{6\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{b^{5/2}c^3}$$

```
[Out] 1/3*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(5/2)/c^3-1/3*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(
1/2)/b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/b^(5/2)/c^3+cos(3*a/b)*FresnelS(6^(
1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*6^(1/2)*Pi^(1/2)/b^(5/2)/c^3
-FresnelC(6^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*sin(3*a/b)*6^(1
/2)*Pi^(1/2)/b^(5/2)/c^3+2/3*x^2*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^(
3/2)-8/3*x/b^2/c^2/(a+b*arccos(c*x))^(1/2)+4*x^3/b^2/(a+b*arccos(c*x))^(1/2
)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {4730, 4808, 4732, 4491, 3387, 3386, 3432, 3385, 3433, 4720}

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx = -\frac{\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} - \frac{\sqrt{6\pi}\sin\left(\frac{3a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{5/2}c^3} + \frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} + \frac{\sqrt{6\pi}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{5/2}c^3} - \frac{8x}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{4x^3}{b^2\sqrt{a+b\arccos(cx)}} + \frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

```
[In] Int[x^2/(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (2*x^2*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) - (8*x)/(3*b^2*
c^2*Sqrt[a + b*ArcCos[c*x]]) + (4*x^3)/(b^2*Sqrt[a + b*ArcCos[c*x]]) + (Sqr
t[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]])/(3
*b^(5/2)*c^3) + (Sqrt[6*Pi]*Cos[(3*a)/b]*FresnelS[(Sqrt[6/Pi]*Sqrt[a + b*Ar
cCos[c*x]])/Sqrt[b]])/(b^(5/2)*c^3) - (Sqrt[2*Pi]*FresnelC[(Sqrt[2/Pi]*Sqrt
[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(3*b^(5/2)*c^3) - (Sqrt[6*Pi]*Fresn
elC[(Sqrt[6/Pi]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[(3*a)/b])/(b^(5/2)*c^
3)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
tQ[p, 0]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
```

```
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{4\int \frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}\,dx}{3bc} \\
&\quad + \frac{(2c)\int \frac{x^3}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}\,dx}{b} \\
&= \frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\arccos(cx)}} \\
&\quad + \frac{4x^3}{b^2\sqrt{a+b\arccos(cx)}} - \frac{12\int \frac{x^2}{\sqrt{a+b\arccos(cx)}}\,dx}{b^2} + \frac{8\int \frac{1}{\sqrt{a+b\arccos(cx)}}\,dx}{3b^2c^2} \\
&= \frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\arccos(cx)}} \\
&\quad + \frac{4x^3}{b^2\sqrt{a+b\arccos(cx)}} + \frac{8\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{3b^3c^3} \\
&\quad - \frac{12\text{Subst}\left(\int \frac{\cos^2\left(\frac{a}{b}-\frac{x}{b}\right)\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{b^3c^3} \\
&= \frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{4x^3}{b^2\sqrt{a+b\arccos(cx)}} \\
&\quad - \frac{12\text{Subst}\left(\int \left(\frac{\sin\left(\frac{3a}{b}-\frac{3x}{b}\right)}{4\sqrt{x}} + \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{4\sqrt{x}}\right)\,dx, x, a+b\arccos(cx)\right)}{b^3c^3} \\
&\quad - \frac{\left(8\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{3b^3c^3} \\
&\quad + \frac{\left(8\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{3b^3c^3}
\end{aligned}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{4x^3}{b^2\sqrt{a+b\arccos(cx)}} - \frac{3\text{Subst}\left(\int \frac{\sin\left(\frac{3a}{b}-\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{b^3c^3} - \frac{3\text{Subst}\left(\int \frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{b^3c^3} - \frac{\left(16\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{3b^3c^3} + \frac{\left(16\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{3b^3c^3}$$

$$= \frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{4x^3}{b^2\sqrt{a+b\arccos(cx)}} - \frac{8\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} + \frac{8\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{3b^{5/2}c^3} + \frac{\left(3\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{b^3c^3} + \frac{\left(3\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{b^3c^3} - \frac{\left(3\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{b^3c^3} - \frac{\left(3\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{3x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{b^3c^3}$$

$$
\begin{aligned}
&= \frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\arccos(cx)}} \\
&\quad + \frac{4x^3}{b^2\sqrt{a+b\arccos(cx)}} - \frac{8\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} \\
&\quad + \frac{8\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{3b^{5/2}c^3} \\
&\quad + \frac{\left(6\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{b^3c^3} \\
&\quad + \frac{\left(6\cos\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\sin\left(\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{b^3c^3} \\
&\quad - \frac{\left(6\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{b^3c^3} \\
&\quad - \frac{\left(6\sin\left(\frac{3a}{b}\right)\right)\text{Subst}\left(\int\cos\left(\frac{3x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{b^3c^3} \\
&= \frac{2x^2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{8x}{3b^2c^2\sqrt{a+b\arccos(cx)}} \\
&\quad + \frac{4x^3}{b^2\sqrt{a+b\arccos(cx)}} + \frac{\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c^3} \\
&\quad + \frac{\sqrt{6\pi}\cos\left(\frac{3a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{b^{5/2}c^3} - \frac{\sqrt{2\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{3b^{5/2}c^3} \\
&\quad - \frac{\sqrt{6\pi}\,\text{FresnelC}\left(\frac{\sqrt{\frac{6}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{3a}{b}\right)}{b^{5/2}c^3}
\end{aligned}
$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.88 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.10

$$
\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx =
$$

$$
-\frac{-b\sqrt{1-c^2x^2} - (a+b\arccos(cx))\left(e^{-i\arccos(cx)} + e^{i\arccos(cx)} - e^{-\frac{ia}{b}}\sqrt{-\frac{i(a+b\arccos(cx))}{b}}\,\Gamma\left(\frac{1}{2}, -\frac{i(a+b\arccos(cx))}{b}\right)\right.}{}
$$

```
[In] Integrate[x^2/(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] -1/6*(-(b*Sqrt[1 - c^2*x^2]) - (a + b*ArcCos[c*x])*(E^((-I)*ArcCos[c*x]) +
E^(I*ArcCos[c*x]) - (Sqrt[((-I)*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((-I)*(a
 + b*ArcCos[c*x]))/b])/E^((I*a)/b) - E^((I*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]
))/b]*Gamma[1/2, (I*(a + b*ArcCos[c*x]))/b]) - 3*(a + b*ArcCos[c*x])*(E^((-
3*I)*ArcCos[c*x]) + E^((3*I)*ArcCos[c*x]) - (Sqrt[3]*Sqrt[((-I)*(a + b*ArcC
os[c*x]))/b]*Gamma[1/2, ((-3*I)*(a + b*ArcCos[c*x]))/b])/E^(((3*I)*a)/b) -
Sqrt[3]*E^(((3*I)*a)/b)*Sqrt[(I*(a + b*ArcCos[c*x]))/b]*Gamma[1/2, ((3*I)*(
a + b*ArcCos[c*x]))/b]) - b*Sin[3*ArcCos[c*x]])/(b^2*c^3*(a + b*ArcCos[c*x]
)^(3/2))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(236) = 472$.

Time = 2.20 (sec) , antiderivative size = 673, normalized size of antiderivative = 2.30

method	result
default	$\frac{-2\arccos(cx)\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b}\right)\sqrt{2}\,\sqrt{\pi}\,\sqrt{-\frac{1}{b}}\,b-2\arccos(cx)\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{FresnelC}}{}$

```
[In] int(x^2/(a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/c^3/b^2*(-2*arccos(c*x)*(a+b*arccos(c*x))^(1/2)*cos(a/b)*FresnelS(2^(1/
2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*2^(1/2)*Pi^(1/2)*(-1/b)
^(1/2)*b-2*arccos(c*x)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi
^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2
)*b-6*arccos(c*x)*(a+b*arccos(c*x))^(1/2)*cos(3*a/b)*FresnelS(3*2^(1/2)/Pi^
(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-3/b)^(1/2)*2^(1/2)*Pi^(1/2)
*b-6*arccos(c*x)*(a+b*arccos(c*x))^(1/2)*sin(3*a/b)*FresnelC(3*2^(1/2)/Pi^(
1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-3/b)^(1/2)*2^(1/2)*Pi^(1/2)*
b-2*(a+b*arccos(c*x))^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)
*(a+b*arccos(c*x))^(1/2)/b)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*a-2*(a+b*arccos(c
*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x)
)^(1/2)/b)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*a-6*(a+b*arccos(c*x))^(1/2)*cos(3*
a/b)*FresnelS(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-
3/b)^(1/2)*2^(1/2)*Pi^(1/2)*a-6*(a+b*arccos(c*x))^(1/2)*sin(3*a/b)*FresnelC
(3*2^(1/2)/Pi^(1/2)/(-3/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*(-3/b)^(1/2)*2^
(1/2)*Pi^(1/2)*a+2*arccos(c*x)*cos(-(a+b*arccos(c*x))/b+a/b)*b+6*arccos(c*x
)*cos(-3*(a+b*arccos(c*x))/b+3*a/b)*b-sin(-(a+b*arccos(c*x))/b+a/b)*b+2*cos
(-(a+b*arccos(c*x))/b+a/b)*a-sin(-3*(a+b*arccos(c*x))/b+3*a/b)*b+6*cos(-3*(
a+b*arccos(c*x))/b+3*a/b)*a)/(a+b*arccos(c*x))^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate(x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x^2}{(a+b\operatorname{acos}(cx))^{\frac{5}{2}}}\,dx$$

[In] integrate(x**2/(a+b*acos(c*x))**(5/2),x)

[Out] Integral(x**2/(a + b*acos(c*x))**(5/2), x)

Maxima [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")

[Out] integrate(x^2/(b*arccos(c*x) + a)^(5/2), x)

Giac [F]

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x^2}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

[In] integrate(x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")

[Out] integrate(x^2/(b*arccos(c*x) + a)^(5/2), x)

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+b\arccos(cx))^{5/2}}\,dx=\int \frac{x^2}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^{5/2}} \,d x$$

```
[In] int(x^2/(a + b*acos(c*x))^(5/2),x)
[Out] int(x^2/(a + b*acos(c*x))^(5/2), x)
```

3.199 $\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 180

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx = \frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\arccos(cx)}} + \frac{8\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{3b^{5/2}c^2} - \frac{8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{3b^{5/2}c^2}$$

```
[Out] 8/3*cos(2*a/b)*FresnelS(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*Pi^(1/2
)/b^(5/2)/c^2-8/3*FresnelC(2*(a+b*arccos(c*x))^(1/2)/b^(1/2)/Pi^(1/2))*sin(
2*a/b)*Pi^(1/2)/b^(5/2)/c^2+2/3*x*(-c^2*x^2+1)^(1/2)/b/c/(a+b*arccos(c*x))^
(3/2)-4/3/b^2/c^2/(a+b*arccos(c*x))^(1/2)+8/3*x^2/b^2/(a+b*arccos(c*x))^(1/
2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {4730, 4808, 4732, 4491, 12, 3387, 3386, 3432, 3385, 3433, 4738}

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx = -\frac{8\sqrt{\pi}\sin\left(\frac{2a}{b}\right)\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{3b^{5/2}c^2} + \frac{8\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{3b^{5/2}c^2} - \frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\arccos(cx)}} + \frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

```
[In] Int[x/(a + b*ArcCos[c*x])^(5/2),x]
```

```
[Out] (2*x*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) - 4/(3*b^2*c^2*Sq
rt[a + b*ArcCos[c*x]]) + (8*x^2)/(3*b^2*Sqrt[a + b*ArcCos[c*x]]) + (8*Sqrt[
Pi]*Cos[(2*a)/b]*FresnelS[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]*Sqrt[Pi])])/
(3*b^(5/2)*c^2) - (8*Sqrt[Pi]*FresnelC[(2*Sqrt[a + b*ArcCos[c*x]])/(Sqrt[b]
*Sqrt[Pi])]*Sin[(2*a)/b])/(3*b^(5/2)*c^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4491

```
Int[Cos[(a_.) + (b_.)*(x_)]^(p_.)*((c_.) + (d_.)*(x_))^(m_.)*Sin[(a_.) + (b
_.)*(x_)]^(n_.), x_Symbol] :> Int[ExpandTrigReduce[(c + d*x)^m, Sin[a + b*x
]^n*Cos[a + b*x]^p, x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && IG
```

```
tQ[p, 0]
```

Rule 4730

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Simp[(
-x^m)*Sqrt[1 - c^2*x^2]*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] + (
-Dist[c*((m + 1)/(b*(n + 1))), Int[x^(m + 1)*((a + b*ArcCos[c*x])^(n + 1)/S
qrt[1 - c^2*x^2]), x], x] + Dist[m/(b*c*(n + 1)), Int[x^(m - 1)*((a + b*Arc
Cos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x]) /; FreeQ[{a, b, c}, x] && IGt
Q[m, 0] && LtQ[n, -2]
```

Rule 4732

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*(x_)^(m_.), x_Symbol] :> Dist[-
(b*c^(m + 1))^(-1), Subst[Int[x^n*Cos[-a/b + x/b]^m*Sin[-a/b + x/b], x], x,
 a + b*ArcCos[c*x]], x] /; FreeQ[{a, b, c, n}, x] && IGtQ[m, 0]
```

Rule 4738

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)/Sqrt[(d_) + (e_.)*(x_)^2], x_S
ymbol] :> Simp[(-(b*c*(n + 1))^(-1))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]
]*(a + b*ArcCos[c*x])^(n + 1), x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[c^
2*d + e, 0] && NeQ[n, -1]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{2\int \frac{1}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}\,dx}{3bc} \\
&\quad + \frac{(4c)\int \frac{x^2}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}\,dx}{3b} \\
&= \frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}} \\
&\quad + \frac{8x^2}{3b^2\sqrt{a+b\arccos(cx)}} - \frac{16\int \frac{x}{\sqrt{a+b\arccos(cx)}}\,dx}{3b^2}
\end{aligned}
$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\arccos(cx)}} - \frac{16\text{Subst}\left(\int \frac{\cos\left(\frac{a}{b}-\frac{x}{b}\right)\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{3b^3c^2}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\arccos(cx)}} - \frac{16\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}-\frac{2x}{b}\right)}{2\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{3b^3c^2}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\arccos(cx)}} - \frac{8\text{Subst}\left(\int \frac{\sin\left(\frac{2a}{b}-\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{3b^3c^2}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\arccos(cx)}} + \frac{\left(8\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\sin\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{3b^3c^2} - \frac{\left(8\sin\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \frac{\cos\left(\frac{2x}{b}\right)}{\sqrt{x}}\,dx, x, a+b\arccos(cx)\right)}{3b^3c^2}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\arccos(cx)}} + \frac{\left(16\cos\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \sin\left(\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{3b^3c^2} - \frac{\left(16\sin\left(\frac{2a}{b}\right)\right)\text{Subst}\left(\int \cos\left(\frac{2x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{3b^3c^2}$$

$$= \frac{2x\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} - \frac{4}{3b^2c^2\sqrt{a+b\arccos(cx)}} + \frac{8x^2}{3b^2\sqrt{a+b\arccos(cx)}} + \frac{8\sqrt{\pi}\cos\left(\frac{2a}{b}\right)\text{FresnelS}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)}{3b^{5/2}c^2} - \frac{8\sqrt{\pi}\,\text{FresnelC}\left(\frac{2\sqrt{a+b\arccos(cx)}}{\sqrt{b}\sqrt{\pi}}\right)\sin\left(\frac{2a}{b}\right)}{3b^{5/2}c^2}$$

Mathematica [F]

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx=\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx$$

[In] Integrate[x/(a + b*ArcCos[c*x])^(5/2),x]

[Out] Integrate[x/(a + b*ArcCos[c*x])^(5/2), x]

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(142) = 284$.

Time = 2.08 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.89

method	result
default	$\frac{-8\arccos(cx)\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{2a}{b}\right)\operatorname{FresnelS}\left(\frac{2\sqrt{2}\,\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\,\sqrt{-\frac{2}{b}}\,b}\right)b-8\arccos(cx)\sqrt{-\frac{1}{b}}\,\sqrt{\pi}\,\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{2a}{b}\right)}{}$

[In] int(x/(a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)

[Out] 1/3/c^2/b^2*(-8*arccos(c*x)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b-8*arccos(c*x)*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*b-8*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*cos(2*a/b)*FresnelS(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*a-8*(-1/b)^(1/2)*Pi^(1/2)*(a+b*arccos(c*x))^(1/2)*sin(2*a/b)*FresnelC(2*2^(1/2)/Pi^(1/2)/(-2/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*a+4*arccos(c*x)*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*b-sin(-2*(a+b*arccos(c*x))/b+2*a/b)*b+4*cos(-2*(a+b*arccos(c*x))/b+2*a/b)*a)/(a+b*arccos(c*x))^(3/2)

Fricas [F(-2)]

Exception generated.

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

[In] integrate(x/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Error detected within library code: integrate: implementation incomplete (constant residues)

Sympy [F]

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x}{(a+b\operatorname{acos}(cx))^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(a+b*acos(c*x))**(5/2),x)
[Out] Integral(x/(a + b*acos(c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
[Out] integrate(x/(b*arccos(c*x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(x/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
[Out] integrate(x/(b*arccos(c*x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{x}{(a+b\operatorname{acos}(c\,x))^{5/2}}\,dx$$

```
[In] int(x/(a + b*acos(c*x))^(5/2),x)
[Out] int(x/(a + b*acos(c*x))^(5/2), x)
```

3.200 $\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 163

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = \frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} + \frac{4x}{3b^2\sqrt{a+b\arccos(cx)}} + \frac{4\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} - \frac{4\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{3b^{5/2}c}$$

```
[Out] 4/3*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/2)/b^(1/2))*2^(
1/2)*Pi^(1/2)/b^(5/2)/c-4/3*FresnelC(2^(1/2)/Pi^(1/2)*(a+b*arccos(c*x))^(1/
2)/b^(1/2))*sin(a/b)*2^(1/2)*Pi^(1/2)/b^(5/2)/c+2/3*(-c^2*x^2+1)^(1/2)/b/c/
(a+b*arccos(c*x))^(3/2)+4/3*x/b^2/(a+b*arccos(c*x))^(1/2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used

$= \{4718, 4808, 4720, 3387, 3386, 3432, 3385, 3433\}$

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = -\frac{4\sqrt{2\pi}\sin\left(\frac{a}{b}\right)\text{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} + \frac{4\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\text{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} + \frac{4x}{3b^2\sqrt{a+b\arccos(cx)}} + \frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}$$

```
[In] Int[(a + b*ArcCos[c*x])^(-5/2),x]
```

```
[Out] (2*Sqrt[1 - c^2*x^2])/(3*b*c*(a + b*ArcCos[c*x])^(3/2)) + (4*x)/(3*b^2*Sqrt
[a + b*ArcCos[c*x]]) + (4*Sqrt[2*Pi]*Cos[a/b]*FresnelS[(Sqrt[2/Pi]*Sqrt[a +
 b*ArcCos[c*x]])/Sqrt[b]])/(3*b^(5/2)*c) - (4*Sqrt[2*Pi]*FresnelC[(Sqrt[2/P
i]*Sqrt[a + b*ArcCos[c*x]])/Sqrt[b]]*Sin[a/b])/(3*b^(5/2)*c)
```

Rule 3385

```
Int[sin[Pi/2 + (e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> D
ist[2/d, Subst[Int[Cos[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d
, e, f}, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3386

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d
, Subst[Int[Sin[f*(x^2/d)], x], x, Sqrt[c + d*x]], x] /; FreeQ[{c, d, e, f}
, x] && ComplexFreeQ[f] && EqQ[d*e - c*f, 0]
```

Rule 3387

```
Int[sin[(e_.) + (f_.)*(x_)]/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[Cos
[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] + Dist[Sin[(d
*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/Sqrt[c + d*x], x], x] /; FreeQ[{c, d,
e, f}, x] && ComplexFreeQ[f] && NeQ[d*e - c*f, 0]
```

Rule 3432

```
Int[Sin[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelS[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 3433

```
Int[Cos[(d_.)*((e_.) + (f_.)*(x_))^2], x_Symbol] :> Simp[(Sqrt[Pi/2]/(f*Rt[
d, 2]))*FresnelC[Sqrt[2/Pi]*Rt[d, 2]*(e + f*x)], x] /; FreeQ[{d, e, f}, x]
```

Rule 4718

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-Sqrt[1 - c
^2*x^2])*((a + b*ArcCos[c*x])^(n + 1)/(b*c*(n + 1))), x] - Dist[c/(b*(n + 1
)), Int[x*((a + b*ArcCos[c*x])^(n + 1)/Sqrt[1 - c^2*x^2]), x], x] /; FreeQ[
{a, b, c}, x] && LtQ[n, -1]
```

Rule 4720

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[-(b*c)^(-1),
 Subst[Int[x^n*Sin[-a/b + x/b], x], x, a + b*ArcCos[c*x]], x] /; FreeQ[{a,
b, c, n}, x]
```

Rule 4808

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_)*((f_.)*(x_))^(m_.))/Sqrt[(d_)
+ (e_.)*(x_)^2], x_Symbol] :> Simp[(-(f*x)^m/(b*c*(n + 1)))*Simp[Sqrt[1 - c
^2*x^2]/Sqrt[d + e*x^2]]*(a + b*ArcCos[c*x])^(n + 1), x] + Dist[f*(m/(b*c*(
n + 1)))*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]], Int[(f*x)^(m - 1)*(a + b*
ArcCos[c*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[c^2*
d + e, 0] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}+\frac{(2c)\int\frac{x}{\sqrt{1-c^2x^2}(a+b\arccos(cx))^{3/2}}\,dx}{3b}\\
&= \frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}+\frac{4x}{3b^2\sqrt{a+b\arccos(cx)}}-\frac{4\int\frac{1}{\sqrt{a+b\arccos(cx)}}\,dx}{3b^2}\\
&= \frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}+\frac{4x}{3b^2\sqrt{a+b\arccos(cx)}}-\frac{4\text{Subst}\left(\int\frac{\sin\left(\frac{a}{b}-\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{3b^3c}\\
&= \frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}}+\frac{4x}{3b^2\sqrt{a+b\arccos(cx)}}\\
&\quad+\frac{\left(4\cos\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\sin\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{3b^3c}\\
&\quad-\frac{\left(4\sin\left(\frac{a}{b}\right)\right)\text{Subst}\left(\int\frac{\cos\left(\frac{x}{b}\right)}{\sqrt{x}}\,dx,x,a+b\arccos(cx)\right)}{3b^3c}
\end{aligned}$$

$$
\begin{aligned}
&= \frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} + \frac{4x}{3b^2\sqrt{a+b\arccos(cx)}} \\
&\quad + \frac{\left(8\cos\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int\sin\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{3b^3c} \\
&\quad - \frac{\left(8\sin\left(\frac{a}{b}\right)\right)\operatorname{Subst}\left(\int\cos\left(\frac{x^2}{b}\right)\,dx, x, \sqrt{a+b\arccos(cx)}\right)}{3b^3c} \\
&= \frac{2\sqrt{1-c^2x^2}}{3bc(a+b\arccos(cx))^{3/2}} + \frac{4x}{3b^2\sqrt{a+b\arccos(cx)}} \\
&\quad + \frac{4\sqrt{2\pi}\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)}{3b^{5/2}c} \\
&\quad - \frac{4\sqrt{2\pi}\operatorname{FresnelC}\left(\frac{\sqrt{\frac{2}{\pi}}\sqrt{a+b\arccos(cx)}}{\sqrt{b}}\right)\sin\left(\frac{a}{b}\right)}{3b^{5/2}c}
\end{aligned}
$$

Mathematica [F]

$$
\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx
$$

```
[In] Integrate[(a + b*ArcCos[c*x])^(-5/2),x]
[Out] Integrate[(a + b*ArcCos[c*x])^(-5/2), x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(129) = 258$.

Time = 2.09 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.09

method	result
default	$-\frac{\frac{4\arccos(cx)\sqrt{a+b\arccos(cx)}\,\cos\left(\frac{a}{b}\right)\operatorname{FresnelS}\left(\frac{\sqrt{2}\sqrt{a+b\arccos(cx)}}{\sqrt{\pi}\sqrt{-\frac{1}{b}}\,b}\right)\sqrt{2}\sqrt{\pi}\sqrt{-\frac{1}{b}}\,b}{3} - \frac{4\arccos(cx)\sqrt{a+b\arccos(cx)}\,\sin\left(\frac{a}{b}\right)\operatorname{FresnelC}\left(\frac{\sqrt{2}\sqrt{a+b}}{\sqrt{\pi}\sqrt{}}\right.}{3}}{}$

```
[In] int(1/(a+b*arccos(c*x))^(5/2),x,method=_RETURNVERBOSE)
[Out] 2/3/c/b^2*(-2*arccos(c*x)*(a+b*arccos(c*x))^(1/2)*cos(a/b)*FresnelS(2^(1/2)
/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*2^(1/2)*Pi^(1/2)*(-1/b)^(
1/2)*b-2*arccos(c*x)*(a+b*arccos(c*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(
1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x))^(1/2)/b)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*
b-2*(a+b*arccos(c*x))^(1/2)*cos(a/b)*FresnelS(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)
```

```
*(a+b*arccos(c*x))^(1/2)/b)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*a-2*(a+b*arccos(c
*x))^(1/2)*sin(a/b)*FresnelC(2^(1/2)/Pi^(1/2)/(-1/b)^(1/2)*(a+b*arccos(c*x)
)^(1/2)/b)*2^(1/2)*Pi^(1/2)*(-1/b)^(1/2)*a+2*arccos(c*x)*cos(-(a+b*arccos(c
*x))/b+a/b)*b-sin(-(a+b*arccos(c*x))/b+a/b)*b+2*cos(-(a+b*arccos(c*x))/b+a/
b)*a)/(a+b*arccos(c*x))^(3/2)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(a + b \arccos(cx))^{5/2}} \, dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{1}{(a + b \arccos(cx))^{5/2}} \, dx = \int \frac{1}{(a + b \operatorname{acos}(cx))^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/(a+b*acos(c*x))**(5/2),x)
```

```
[Out] Integral((a + b*acos(c*x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a + b \arccos(cx))^{5/2}} \, dx = \int \frac{1}{(b \arccos(cx) + a)^{\frac{5}{2}}} \, dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*arccos(c*x) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(b\arccos(cx)+a)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}(c\,x))^{5/2}}\,dx$$

```
[In] int(1/(a + b*acos(c*x))^(5/2),x)
[Out] int(1/(a + b*acos(c*x))^(5/2), x)
```

3.201 $\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \text{Int}\left(\frac{1}{x(a+b\arccos(cx))^{5/2}},x\right)$$

```
[Out] Unintegrable(1/x/(a+b*arccos(c*x))^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx$$

```
[In] Int[1/(x*(a + b*ArcCos[c*x])^(5/2)),x]
[Out] Defer[Int][1/(x*(a + b*ArcCos[c*x])^(5/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx=\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx$$

```
[In] Integrate[1/(x*(a + b*ArcCos[c*x])^(5/2)),x]
[Out] Integrate[1/(x*(a + b*ArcCos[c*x])^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.96 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x\left(a+b\arccos\left(cx\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/x/(a+b*arccos(c*x))^(5/2),x)
[Out] int(1/x/(a+b*arccos(c*x))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 8.33 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx=\int \frac{1}{x\left(a+b\operatorname{acos}\left(cx\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x/(a+b*acos(c*x))**(5/2),x)
[Out] Integral(1/(x*(a + b*acos(c*x))**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.64 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(b\arccos(cx)+a)^{\frac{5}{2}}x}\,dx$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*arccos(c*x) + a)^(5/2)*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate(1/x/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{x\,(a+b\,\mathrm{acos}\,(c\,x))^{5/2}}\,dx$$

```
[In] int(1/(x*(a + b*acos(c*x))^(5/2)),x)
[Out] int(1/(x*(a + b*acos(c*x))^(5/2)), x)
```

3.202 $\int \frac{1}{x^2(a+b \arccos(cx))^{5/2}} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{1}{x^2(a+b \arccos(cx))^{5/2}} dx = \text{Int}\left(\frac{1}{x^2(a+b \arccos(cx))^{5/2}}, x\right)$$

```
[Out] Unintegrable(1/x^2/(a+b*arccos(c*x))^(5/2),x)
```

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2(a+b \arccos(cx))^{5/2}} dx = \int \frac{1}{x^2(a+b \arccos(cx))^{5/2}} dx$$

```
[In] Int[1/(x^2*(a + b*ArcCos[c*x])^(5/2)),x]
[Out] Defer[Int][1/(x^2*(a + b*ArcCos[c*x])^(5/2)), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{x^2(a+b \arccos(cx))^{5/2}} dx$$

Mathematica [N/A]

Not integrable

Time = 7.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx$$

```
[In] Integrate[1/(x^2*(a + b*ArcCos[c*x])^(5/2)),x]
[Out] Integrate[1/(x^2*(a + b*ArcCos[c*x])^(5/2)), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{x^{2} \left(a+b\arccos\left(c x\right)\right)^{\frac{5}{2}}}dx$$

```
[In] int(1/x^2/(a+b*arccos(c*x))^(5/2),x)
[Out] int(1/x^2/(a+b*arccos(c*x))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 15.38 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{x^{2} \left(a+b\operatorname{acos}\left(c x\right)\right)^{\frac{5}{2}}}\,dx$$

```
[In] integrate(1/x**2/(a+b*acos(c*x))**(5/2),x)
[Out] Integral(1/(x**2*(a + b*acos(c*x))**(5/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="maxima")
[Out] integrate(1/((b*arccos(c*x) + a)^(5/2)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 1.56 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{(b\arccos{(cx)}+a)^{\frac{5}{2}}x^2}\,dx$$

```
[In] integrate(1/x^2/(a+b*arccos(c*x))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*arccos(c*x) + a)^(5/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+b\arccos(cx))^{5/2}}\,dx = \int \frac{1}{x^2\,{(a+b\,\mathrm{acos}\left(c\,x\right))}^{5/2}}\,dx$$

```
[In] int(1/(x^2*(a + b*acos(c*x))^(5/2)),x)
[Out] int(1/(x^2*(a + b*acos(c*x))^(5/2)), x)
```

3.203 $\int (dx)^{5/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 120

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx = -\frac{20bd^2\sqrt{dx}\sqrt{1-c^2x^2}}{147c^3}-\frac{4b(dx)^{5/2}\sqrt{1-c^2x^2}}{49c}+\frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}+\frac{20bd^{5/2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{147c^{7/2}}$$

```
[Out] 2/7*(d*x)^(7/2)*(a+b*arccos(c*x))/d+20/147*b*d^(5/2)*EllipticF(c^(1/2)*(d*x
)^(1/2)/d^(1/2),I)/c^(7/2)-4/49*b*(d*x)^(5/2)*(-c^2*x^2+1)^(1/2)/c-20/147*b
*d^2*(d*x)^(1/2)*(-c^2*x^2+1)^(1/2)/c^3
```

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4724, 327, 335, 227}

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx = \frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}+\frac{20bd^{5/2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{147c^{7/2}}-\frac{4b\sqrt{1-c^2x^2}(dx)^{5/2}}{49c}-\frac{20bd^2\sqrt{1-c^2x^2}\sqrt{dx}}{147c^3}$$

```
[In] Int[(d*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-20*b*d^2*Sqrt[d*x]*Sqrt[1 - c^2*x^2])/(147*c^3) - (4*b*(d*x)^(5/2)*Sqrt[1
 - c^2*x^2])/(49*c) + (2*(d*x)^(7/2)*(a + b*ArcCos[c*x]))/(7*d) + (20*b*d^(
5/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], -1])/(147*c^(7/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\text{integral} &= \frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}+\frac{(2bc)\int\frac{(dx)^{7/2}}{\sqrt{1-c^2x^2}}\,dx}{7d}\\
&= -\frac{4b(dx)^{5/2}\sqrt{1-c^2x^2}}{49c}+\frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}+\frac{(10bd)\int\frac{(dx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx}{49c}\\
&= -\frac{20bd^2\sqrt{dx}\sqrt{1-c^2x^2}}{147c^3}-\frac{4b(dx)^{5/2}\sqrt{1-c^2x^2}}{49c}\\
&\quad+\frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}+\frac{(10bd^3)\int\frac{1}{\sqrt{dx}\sqrt{1-c^2x^2}}\,dx}{147c^3}\\
&= -\frac{20bd^2\sqrt{dx}\sqrt{1-c^2x^2}}{147c^3}-\frac{4b(dx)^{5/2}\sqrt{1-c^2x^2}}{49c}\\
&\quad+\frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d}+\frac{(20bd^2)\,\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx,x,\sqrt{dx}\right)}{147c^3}
\end{aligned}
$$

$$= -\frac{20bd^2\sqrt{dx}\sqrt{1-c^2x^2}}{147c^3} - \frac{4b(dx)^{5/2}\sqrt{1-c^2x^2}}{49c} + \frac{2(dx)^{7/2}(a+b\arccos(cx))}{7d} + \frac{20bd^{5/2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{147c^{7/2}}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.25 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.32

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx = \frac{2d^2\sqrt{dx}\left(-10b+4bc^2x^2+6bc^4x^4+21ac^3x^3\sqrt{1-c^2x^2}+21bc^3x^3\sqrt{1-c^2x^2}\arccos(cx)+\cdots\right)}{147c^3\sqrt{1-c^2x^2}}$$

```
[In] Integrate[(d*x)^(5/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (2*d^2*Sqrt[d*x]*(-10*b + 4*b*c^2*x^2 + 6*b*c^4*x^4 + 21*a*c^3*x^3*Sqrt[1 -
 c^2*x^2] + 21*b*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + ((10*I)*b*Sqrt[1 -
 1/(c^2*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[-c^(-1)]/Sqrt[x]], -1])/Sqrt
[-c^(-1)]))/(147*c^3*Sqrt[1 - c^2*x^2])
```

Maple [A] (verified)

Time = 2.13 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.20

method	result
derivativedivides	$\frac{\frac{2a(dx)^{\frac{7}{2}}}{7}+2b\left(\frac{(dx)^{\frac{7}{2}}\arccos(cx)}{7}+\frac{2c\left(-\frac{d^2(dx)^{\frac{5}{2}}\sqrt{-c^2x^2+1}}{7c^2}-\frac{5d^4\sqrt{dx}\sqrt{-c^2x^2+1}}{21c^4}+\frac{5d^4\sqrt{-cx+1}\sqrt{cx+1}\operatorname{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)}{21c^4\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right)}{7d}\right)}{d}$
default	$\frac{\frac{2a(dx)^{\frac{7}{2}}}{7}+2b\left(\frac{(dx)^{\frac{7}{2}}\arccos(cx)}{7}+\frac{2c\left(-\frac{d^2(dx)^{\frac{5}{2}}\sqrt{-c^2x^2+1}}{7c^2}-\frac{5d^4\sqrt{dx}\sqrt{-c^2x^2+1}}{21c^4}+\frac{5d^4\sqrt{-cx+1}\sqrt{cx+1}\operatorname{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)}{21c^4\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right)}{7d}\right)}{d}$
parts	$\frac{2a(dx)^{\frac{7}{2}}}{7d}+\frac{2b\left(\frac{(dx)^{\frac{7}{2}}\arccos(cx)}{7}+\frac{2c\left(-\frac{d^2(dx)^{\frac{5}{2}}\sqrt{-c^2x^2+1}}{7c^2}-\frac{5d^4\sqrt{dx}\sqrt{-c^2x^2+1}}{21c^4}+\frac{5d^4\sqrt{-cx+1}\sqrt{cx+1}\operatorname{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)}{21c^4\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right)}{7d}\right)}{d}$

```
[In] int((d*x)^(5/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/7*a*(d*x)^(7/2)+b*(1/7*(d*x)^(7/2)*arccos(c*x)+2/7*c/d*(-1/7/c^2*d^2
*(d*x)^(5/2)*(-c^2*x^2+1)^(1/2)-5/21/c^4*d^4*(d*x)^(1/2)*(-c^2*x^2+1)^(1/2)
+5/21/c^4*d^4/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c*x+1)^(1/2)/(-c^2*x^2+1)^(1/2)*E
llipticF((d*x)^(1/2)*(c/d)^(1/2),I))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx = -\frac{2\left(10\sqrt{-c^2d}bd^2\text{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)-\left(21\,bc^5d^2x^3\arccos\left(cx\right)+21\,ac^5d^2x^3-2\left(3\,bc^4d^2x^2+5\,bc^2\right.\right.\right.}{147\,c^5}$$

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] -2/147*(10*sqrt(-c^2*d)*b*d^2*weierstrassPInverse(4/c^2, 0, x) - (21*b*c^5*
d^2*x^3*arccos(c*x) + 21*a*c^5*d^2*x^3 - 2*(3*b*c^4*d^2*x^2 + 5*b*c^2*d^2)*
sqrt(-c^2*x^2 + 1))*sqrt(d*x))/c^5
```

Sympy [A] (verification not implemented)

Time = 77.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.71

$$\int (dx)^{5/2}(a+b\arccos(cx))\,dx = a\left(\begin{cases}\frac{2(dx)^{\frac{7}{2}}}{7d} & \text{for } d\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + bc\left(\begin{cases}\frac{d^{\frac{5}{2}}x^{\frac{9}{2}}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle| c^2x^2e^{2i\pi}\right)}{7\Gamma\left(\frac{13}{4}\right)} & \text{for } d>-\infty\wedge d<\infty\wedge d\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}\frac{2(dx)^{\frac{7}{2}}}{7d} & \text{for } d\neq 0\\ 0 & \text{otherwise}\end{cases}\right)\text{acos}\,(cx)$$

```
[In] integrate((d*x)**(5/2)*(a+b*acos(c*x)),x)
```

```
[Out] a*Piecewise((2*(d*x)**(7/2)/(7*d), Ne(d, 0)), (0, True)) + b*c*Piecewise((d
**(5/2)*x**(9/2)*gamma(9/4)*hyper((1/2, 9/4), (13/4,), c**2*x**2*exp_polar(
2*I*pi))/(7*gamma(13/4)), (d > -oo) & (d < oo) & Ne(d, 0)), (0, True)) + b*
Piecewise((2*(d*x)**(7/2)/(7*d), Ne(d, 0)), (0, True))*acos(c*x)
```

Maxima [F]

$$\int (dx)^{5/2}(a + b\arccos(cx))\,dx = \int (dx)^{\frac{5}{2}}\left(b\arccos\left(cx\right) + a\right)dx$$

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] 1/147*(42*b*c^4*d^(5/2)*x^(7/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)
- (12*b*c^4*d^2*x^(7/2) + 294*b*c^5*d^2*integrate(1/7*sqrt(c*x + 1)*sqrt(-c
*x + 1)*x^(7/2)/(c^2*x^2 - 1), x) + 28*b*c^2*d^2*x^(3/2) + 21*(2*b*d^2*arct
an(sqrt(c)*sqrt(x)) + b*d^2*log((c*x - 1)/(c*x + 2*sqrt(c)*sqrt(x) + 1)))*s
qrt(c))*sqrt(d))/c^4
```

Giac [F]

$$\int (dx)^{5/2}(a + b\arccos(cx))\,dx = \int (dx)^{\frac{5}{2}}\left(b\arccos\left(cx\right) + a\right)dx$$

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x)),x, algorithm="giac")
```

```
[Out] integrate((d*x)^(5/2)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2}(a + b\arccos(cx))\,dx = \int \left(a + b\,\mathrm{acos}(c\,x)\right)\,{(d\,x)}^{5/2}\,dx$$

```
[In] int((a + b*acos(c*x))*(d*x)^(5/2),x)
```

```
[Out] int((a + b*acos(c*x))*(d*x)^(5/2), x)
```

3.204 $\int (dx)^{3/2}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx = -\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c}+\frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d} + \frac{12bd^{3/2}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{25c^{5/2}} - \frac{12bd^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{25c^{5/2}}$$

```
[Out] 2/5*(d*x)^(5/2)*(a+b*arccos(c*x))/d+12/25*b*d^(3/2)*EllipticE(c^(1/2)*(d*x)
^(1/2)/d^(1/2),I)/c^(5/2)-12/25*b*d^(3/2)*EllipticF(c^(1/2)*(d*x)^(1/2)/d^(
1/2),I)/c^(5/2)-4/25*b*(d*x)^(3/2)*(-c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4724, 327, 335, 313, 227, 1213, 435}

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx = \frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d} - \frac{12bd^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{25c^{5/2}} + \frac{12bd^{3/2}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{25c^{5/2}} - \frac{4b\sqrt{1-c^2x^2}(dx)^{3/2}}{25c}$$

```
[In] Int[(d*x)^(3/2)*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-4*b*(d*x)^(3/2)*Sqrt[1 - c^2*x^2])/(25*c) + (2*(d*x)^(5/2)*(a + b*ArcCos[
c*x]))/(5*d) + (12*b*d^(3/2)*EllipticE[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]],
```

```
 -1])/(25*c^(5/2)) - (12*b*d^(3/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqr
t[d]], -1])/(25*c^(5/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
```

```
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d} + \frac{(2bc)\int \frac{(dx)^{5/2}}{\sqrt{1-c^2x^2}}\,dx}{5d}$$

$$= -\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d} + \frac{(6bd)\int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}}\,dx}{25c}$$

$$= -\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d} + \frac{(12b)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx, x, \sqrt{dx}\right)}{25c}$$

$$= -\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d} - \frac{(12bd)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx, x, \sqrt{dx}\right)}{25c^2} + \frac{(12bd)\text{Subst}\left(\int \frac{1+\frac{cx^2}{d}}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx, x, \sqrt{dx}\right)}{25c^2}$$

$$= -\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d} - \frac{12bd^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right), -1\right)}{25c^{5/2}} + \frac{(12bd)\text{Subst}\left(\int \frac{\sqrt{1+\frac{cx^2}{d}}}{\sqrt{1-\frac{cx^2}{d}}}\,dx, x, \sqrt{dx}\right)}{25c^2}$$

$$= -\frac{4b(dx)^{3/2}\sqrt{1-c^2x^2}}{25c} + \frac{2(dx)^{5/2}(a+b\arccos(cx))}{5d} + \frac{12bd^{3/2}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{25c^{5/2}} - \frac{12bd^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right), -1\right)}{25c^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.53

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx = \frac{2(dx)^{3/2}\left(5acx - 2b\sqrt{1-c^2x^2} + 5bcx\arccos(cx) + 2b\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, c^2x^2\right)\right)}{25c}$$

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x]),x]
[Out] (2*(d*x)^(3/2)*(5*a*c*x - 2*b*Sqrt[1 - c^2*x^2] + 5*b*c*x*ArcCos[c*x] + 2*b
*Hypergeometric2F1[1/2, 3/4, 7/4, c^2*x^2]))/(25*c)
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{\frac{2(dx)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dx)^{\frac{5}{2}}\arccos(cx)}{5}+\frac{2c\left(-\frac{d^2(dx)^{\frac{3}{2}}\sqrt{-c^2x^2+1}}{5c^2}-\frac{3d^3\sqrt{-cx+1}\sqrt{cx+1}\left(\mathrm{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)-\mathrm{EllipticE}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)\right)}{5c^3\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right)}{5d}\right.}{d}$
default	$\frac{\frac{2(dx)^{\frac{5}{2}}a}{5}+2b\left(\frac{(dx)^{\frac{5}{2}}\arccos(cx)}{5}+\frac{2c\left(-\frac{d^2(dx)^{\frac{3}{2}}\sqrt{-c^2x^2+1}}{5c^2}-\frac{3d^3\sqrt{-cx+1}\sqrt{cx+1}\left(\mathrm{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)-\mathrm{EllipticE}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)\right)}{5c^3\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right)}{5d}\right.}{d}$
parts	$\frac{2a(dx)^{\frac{5}{2}}}{5d}+\frac{2b\left(\frac{(dx)^{\frac{5}{2}}\arccos(cx)}{5}+\frac{2c\left(-\frac{d^2(dx)^{\frac{3}{2}}\sqrt{-c^2x^2+1}}{5c^2}-\frac{3d^3\sqrt{-cx+1}\sqrt{cx+1}\left(\mathrm{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)-\mathrm{EllipticE}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)\right.}{5c^3\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right.}{5d}\right.}{d}$

```
[In] int((d*x)^(3/2)*(a+b*arccos(c*x)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/5*(d*x)^(5/2)*a+b*(1/5*(d*x)^(5/2)*arccos(c*x)+2/5*c/d*(-1/5/c^2*d^2
*(d*x)^(3/2)*(-c^2*x^2+1)^(1/2)-3/5/c^3*d^3/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c*x
+1)^(1/2)/(-c^2*x^2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)-Elliptic
E((d*x)^(1/2)*(c/d)^(1/2),I)))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx = \frac{2\left(6\sqrt{-c^2d}bd\mathrm{weierstrassZeta}\left(\frac{4}{c^2},0,\mathrm{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)\right)+\left(5\,bc^3dx^2\arccos\left(cx\right)\right.\right.}{25\,c^3}$$

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x)),x, algorithm="fricas")
```

```
[Out] 2/25*(6*sqrt(-c^2*d)*b*d*weierstrassZeta(4/c^2, 0, weierstrassPInverse(4/c^
2, 0, x)) + (5*b*c^3*d*x^2*arccos(c*x) + 5*a*c^3*d*x^2 - 2*sqrt(-c^2*x^2 +
1)*b*c^2*d*x)*sqrt(d*x))/c^3
```

Sympy [A] (verification not implemented)

Time = 12.55 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx = a\left(\begin{cases}\frac{2(dx)^{\frac{5}{2}}}{5d} & \text{for } d\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + bc\left(\begin{cases}\frac{d^{\frac{3}{2}}x^{\frac{7}{2}}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle| c^2x^2e^{2i\pi}\right)}{5\Gamma\left(\frac{11}{4}\right)} & \text{for } d>-\infty\wedge d<\infty\wedge d\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + b\left(\begin{cases}\frac{2(dx)^{\frac{5}{2}}}{5d} & \text{for } d\neq 0\\ 0 & \text{otherwise}\end{cases}\right)\operatorname{acos}(cx)$$

```
[In] integrate((d*x)**(3/2)*(a+b*acos(c*x)),x)
```

```
[Out] a*Piecewise((2*(d*x)**(5/2)/(5*d), Ne(d, 0)), (0, True)) + b*c*Piecewise((d
**(3/2)*x**(7/2)*gamma(7/4)*hyper((1/2, 7/4), (11/4,), c**2*x**2*exp_polar(
2*I*pi))/(5*gamma(11/4)), (d > -oo) & (d < oo) & Ne(d, 0)), (0, True)) + b*
Piecewise((2*(d*x)**(5/2)/(5*d), Ne(d, 0)), (0, True))*acos(c*x)
```

Maxima [F]

$$\int (dx)^{3/2}(a+b\arccos(cx))\,dx = \int (dx)^{\frac{3}{2}}\left(b\arccos(cx)+a\right)dx$$

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x)),x, algorithm="maxima")
```

```
[Out] 1/25*(10*b*c^3*d^(3/2)*x^(5/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) -
 (4*b*c^3*d*x^(5/2) + 50*b*c^4*d*integrate(1/5*sqrt(c*x + 1)*sqrt(-c*x + 1)
*x^(5/2)/(c^2*x^2 - 1), x) + 20*b*c*d*sqrt(x) - 5*(2*b*d*arctan(sqrt(c)*sqr
t(x)) - b*d*log((c*x - 1)/(c*x + 2*sqrt(c)*sqrt(x) + 1)))*sqrt(c))*sqrt(d))
/c^3
```

Giac [F]

$$\int (dx)^{3/2}(a + b \arccos(cx))\, dx = \int (dx)^{\frac{3}{2}} (b \arccos(cx) + a)\, dx$$

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x)),x, algorithm="giac")
[Out] integrate((d*x)^(3/2)*(b*arccos(c*x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}(a + b \arccos(cx))\, dx = \int (a + b \operatorname{acos}(c\, x))\ (d\, x)^{3/2}\, dx$$

```
[In] int((a + b*acos(c*x))*(d*x)^(3/2),x)
[Out] int((a + b*acos(c*x))*(d*x)^(3/2), x)
```

3.205 $\int \sqrt{dx}(a+b\arccos(cx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 88

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx=-\frac{4b\sqrt{dx}\sqrt{1-c^2x^2}}{9c}+\frac{2(dx)^{3/2}(a+b\arccos(cx))}{3d}+\frac{4b\sqrt{d}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{9c^{3/2}}$$

```
[Out] 2/3*(d*x)^(3/2)*(a+b*arccos(c*x))/d+4/9*b*EllipticF(c^(1/2)*(d*x)^(1/2)/d^(
1/2),I)*d^(1/2)/c^(3/2)-4/9*b*(d*x)^(1/2)*(-c^2*x^2+1)^(1/2)/c
```

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {4724, 327, 335, 227}

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx=\frac{2(dx)^{3/2}(a+b\arccos(cx))}{3d}+\frac{4b\sqrt{d}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{9c^{3/2}}-\frac{4b\sqrt{1-c^2x^2}\sqrt{dx}}{9c}$$

```
[In] Int[Sqrt[d*x]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (-4*b*Sqrt[d*x]*Sqrt[1 - c^2*x^2])/(9*c) + (2*(d*x)^(3/2)*(a + b*ArcCos[c*x
]))/(3*d) + (4*b*Sqrt[d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]], -1]
)/(9*c^(3/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(dx)^{3/2}(a+b\arccos(cx))}{3d}+\frac{(2bc)\int\frac{(dx)^{3/2}}{\sqrt{1-c^2x^2}}\,dx}{3d}\\
&=-\frac{4b\sqrt{dx}\sqrt{1-c^2x^2}}{9c}+\frac{2(dx)^{3/2}(a+b\arccos(cx))}{3d}+\frac{(2bd)\int\frac{1}{\sqrt{dx}\sqrt{1-c^2x^2}}\,dx}{9c}\\
&=-\frac{4b\sqrt{dx}\sqrt{1-c^2x^2}}{9c}+\frac{2(dx)^{3/2}(a+b\arccos(cx))}{3d}+\frac{(4b)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx,x,\sqrt{dx}\right)}{9c}\\
&=-\frac{4b\sqrt{dx}\sqrt{1-c^2x^2}}{9c}+\frac{2(dx)^{3/2}(a+b\arccos(cx))}{3d}+\frac{4b\sqrt{d}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{9c^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.28

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx = \frac{2}{9}\sqrt{dx}\left(3ax-\frac{2b\sqrt{1-c^2x^2}}{c}+3bx\arccos(cx)\right.$$
$$\left.-\frac{2ib\sqrt{-\frac{1}{c}}\sqrt{1-\frac{1}{c^2x^2}}\sqrt{x}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right),-1\right)}{\sqrt{1-c^2x^2}}\right)$$

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x]),x]
```

```
[Out] (2*Sqrt[d*x]*(3*a*x - (2*b*Sqrt[1 - c^2*x^2])/c + 3*b*x*ArcCos[c*x] - ((2*I
)*b*Sqrt[-c^(-1)]*Sqrt[1 - 1/(c^2*x^2)]*Sqrt[x]*EllipticF[I*ArcSinh[Sqrt[-c
^(-1)]/Sqrt[x]], -1])/Sqrt[1 - c^2*x^2]))/9
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.35

method	result	size
derivativedivides	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dx)^{\frac{3}{2}}\arccos(cx)}{3}+\frac{2c\left(-\frac{d^2\sqrt{dx}\sqrt{-c^2x^2+1}}{3c^2}+\frac{d^2\sqrt{-cx+1}\sqrt{cx+1}\,\text{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)}{3c^2\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$	119
default	$\frac{\frac{2(dx)^{\frac{3}{2}}a}{3}+2b\left(\frac{(dx)^{\frac{3}{2}}\arccos(cx)}{3}+\frac{2c\left(-\frac{d^2\sqrt{dx}\sqrt{-c^2x^2+1}}{3c^2}+\frac{d^2\sqrt{-cx+1}\sqrt{cx+1}\,\text{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)}{3c^2\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$	119
parts	$\frac{2a(dx)^{\frac{3}{2}}}{3d}+\frac{2b\left(\frac{(dx)^{\frac{3}{2}}\arccos(cx)}{3}+\frac{2c\left(-\frac{d^2\sqrt{dx}\sqrt{-c^2x^2+1}}{3c^2}+\frac{d^2\sqrt{-cx+1}\sqrt{cx+1}\,\text{EllipticF}\left(\sqrt{dx}\sqrt{\frac{c}{d}},i\right)}{3c^2\sqrt{\frac{c}{d}}\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$	121

```
[In] int((a+b*arccos(c*x))*(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(1/3*(d*x)^(3/2)*a+b*(1/3*(d*x)^(3/2)*arccos(c*x)+2/3*c/d*(-1/3/c^2*d^2
*(d*x)^(1/2)*(-c^2*x^2+1)^(1/2)+1/3/c^2*d^2/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c*x
+1)^(1/2)/(-c^2*x^2+1)^(1/2)*EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \sqrt{dx}(a+b\arccos(cx))\,dx = -\frac{2\left(2\sqrt{-c^2d}b\text{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)-\left(3\,bc^3x\arccos\left(cx\right)+3\,ac^3x-2\sqrt{-c^2x^2+1}bc^2\right)\sqrt{dx}\right)}{9\,c^3}$$

```
[In] integrate((a+b*arccos(c*x))*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/9*(2*sqrt(-c^2*d)*b*weierstrassPInverse(4/c^2, 0, x) - (3*b*c^3*x*arccos
(c*x) + 3*a*c^3*x - 2*sqrt(-c^2*x^2 + 1)*b*c^2)*sqrt(d*x))/c^3
```

Sympy [A] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.97

$$\begin{aligned}\int \sqrt{dx}(a+b\arccos(cx))\,dx &= a\left(\begin{cases}\frac{2(dx)^{\frac{3}{2}}}{3d} & \text{for } d\neq 0\\ 0 & \text{otherwise}\end{cases}\right)\\ &+ bc\left(\begin{cases}\frac{\sqrt{d}x^{\frac{5}{2}}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle| c^2x^2e^{2i\pi}\right)}{3\Gamma\left(\frac{9}{4}\right)} & \text{for } d>-\infty\wedge d<\infty\wedge d\neq 0\\ 0 & \text{otherwise}\end{cases}\right)\\ &+ b\left(\begin{cases}\frac{2(dx)^{\frac{3}{2}}}{3d} & \text{for } d\neq 0\\ 0 & \text{otherwise}\end{cases}\right)\operatorname{acos}\left(cx\right)\end{aligned}$$

```
[In] integrate((a+b*acos(c*x))*(d*x)**(1/2),x)
```

```
[Out] a*Piecewise((2*(d*x)**(3/2)/(3*d), Ne(d, 0)), (0, True)) + b*c*Piecewise((s
qrt(d)*x**(5/2)*gamma(5/4)*hyper((1/2, 5/4), (9/4,), c**2*x**2*exp_polar(2*
I*pi))/(3*gamma(9/4)), (d > -oo) & (d < oo) & Ne(d, 0)), (0, True)) + b*Pie
cewise((2*(d*x)**(3/2)/(3*d), Ne(d, 0)), (0, True))*acos(c*x)
```

Maxima [F]

$$\int \sqrt{dx}(a + b\arccos(cx))\,dx = \int \sqrt{dx}(b\arccos(cx) + a)\,dx$$

[In] integrate((a+b*arccos(c*x))*(d*x)^(1/2),x, algorithm="maxima")

[Out] 1/9*(6*b*c^2*sqrt(d)*x^(3/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (18*b*c^3*integrate(1/3*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^(3/2)/(c^2*x^2 - 1), x) + 4*b*c^2*x^(3/2) + 3*(2*b*arctan(sqrt(c)*sqrt(x)) + b*log((c*x - 1)/(c*x + 2*sqrt(c)*sqrt(x) + 1)))*sqrt(c))*sqrt(d))/c^2

Giac [F]

$$\int \sqrt{dx}(a + b\arccos(cx))\,dx = \int \sqrt{dx}(b\arccos(cx) + a)\,dx$$

[In] integrate((a+b*arccos(c*x))*(d*x)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(d*x)*(b*arccos(c*x) + a), x)

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}(a + b\arccos(cx))\,dx = \int (a + b\,\mathrm{acos}(c\,x))\,\sqrt{d\,x}\,dx$$

[In] int((a + b*acos(c*x))*(d*x)^(1/2),x)

[Out] int((a + b*acos(c*x))*(d*x)^(1/2), x)

3.206 $\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 89

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}(a+b\arccos(cx))}{d} + \frac{4bE\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{c}\sqrt{d}} - \frac{4b\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{\sqrt{c}\sqrt{d}}$$

```
[Out] 4*b*EllipticE(c^(1/2)*(d*x)^(1/2)/d^(1/2),I)/c^(1/2)/d^(1/2)-4*b*EllipticF(
c^(1/2)*(d*x)^(1/2)/d^(1/2),I)/c^(1/2)/d^(1/2)+2*(a+b*arccos(c*x))*(d*x)^(1
/2)/d
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {4724, 335, 313, 227, 1213, 435}

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}(a+b\arccos(cx))}{d} - \frac{4b\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{\sqrt{c}\sqrt{d}} + \frac{4bE\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{c}\sqrt{d}}$$

```
[In] Int[(a + b*ArcCos[c*x])/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*(a + b*ArcCos[c*x]))/d + (4*b*EllipticE[ArcSin[(Sqrt[c]*Sqrt[d
*x])/Sqrt[d]], -1])/(Sqrt[c]*Sqrt[d]) - (4*b*EllipticF[ArcSin[(Sqrt[c]*Sqrt
[d*x])/Sqrt[d]], -1])/(Sqrt[c]*Sqrt[d])
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{dx}(a+b\arccos(cx))}{d} + \frac{(2bc)\int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}}\,dx}{d}$$

$$= \frac{2\sqrt{dx}(a+b\arccos(cx))}{d} + \frac{(4bc)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx, x, \sqrt{dx}\right)}{d^2}$$

$$= \frac{2\sqrt{dx}(a+b\arccos(cx))}{d} - \frac{(4b)\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx, x, \sqrt{dx}\right)}{d} + \frac{(4b)\text{Subst}\left(\int \frac{1+\frac{cx^2}{d}}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx, x, \sqrt{dx}\right)}{d}$$

$$= \frac{2\sqrt{dx}(a+b\arccos(cx))}{d} - \frac{4b\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right), -1\right)}{\sqrt{c}\sqrt{d}} + \frac{(4b)\text{Subst}\left(\int \frac{\sqrt{1+\frac{cx^2}{d}}}{\sqrt{1-\frac{cx^2}{d}}}\,dx, x, \sqrt{dx}\right)}{d}$$

$$= \frac{2\sqrt{dx}(a+b\arccos(cx))}{d} + \frac{4bE\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{\sqrt{c}\sqrt{d}} - \frac{4b\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right), -1\right)}{\sqrt{c}\sqrt{d}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx = \frac{2x\left(3(a+b\arccos(cx)) + 2bcx\,\text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{3}{4}, \frac{7}{4}, c^2x^2\right)\right)}{3\sqrt{dx}}$$

```
[In] Integrate[(a + b*ArcCos[c*x])/Sqrt[d*x],x]
[Out] (2*x*(3*(a + b*ArcCos[c*x]) + 2*b*c*x*Hypergeometric2F1[1/2, 3/4, 7/4, c^2*
x^2]))/(3*Sqrt[d*x])
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{2\sqrt{dx}\,a+2b\left(\sqrt{dx}\,\arccos(cx)-\frac{2\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	98
default	$\frac{2\sqrt{dx}\,a+2b\left(\sqrt{dx}\,\arccos(cx)-\frac{2\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	98
parts	$\frac{2a\sqrt{dx}}{d}+\frac{2b\left(\sqrt{dx}\,\arccos(cx)-\frac{2\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	101

```
[In] int((a+b*arccos(c*x))/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((d*x)^(1/2)*a+b*((d*x)^(1/2)*arccos(c*x)-2/(c/d)^(1/2)*(-c*x+1)^(1/2)*
(c*x+1)^(1/2)/(-c^2*x^2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)-Elli
pticE((d*x)^(1/2)*(c/d)^(1/2),I))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.58

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx = \frac{2\left(2\sqrt{-c^2d}b\mathrm{weierstrassZeta}\left(\frac{4}{c^2},0,\mathrm{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)\right)+(bc\arccos{(cx)}+ac)\sqrt{dx}\right)}{cd}$$

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*(2*sqrt(-c^2*d)*b*weierstrassZeta(4/c^2, 0, weierstrassPInverse(4/c^2, 0,
 x)) + (b*c*arccos(c*x) + a*c)*sqrt(d*x))/(c*d)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*acos(c*x))/(d*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{\sqrt{dx}}\,dx = \int \frac{b\arccos{(cx)}+a}{\sqrt{dx}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] (2*b*c*sqrt(d)*sqrt(x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (2*b*c^
2*d*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(c^2*d*x^2 - d), x) + 4*
b*c*sqrt(x) - (2*b*arctan(sqrt(c)*sqrt(x)) - b*log((c*x - 1)/(c*x + 2*sqrt(
c)*sqrt(x) + 1)))*sqrt(c))*sqrt(d))/(c*d)
```

Giac [F]

$$\int \frac{a + b \arccos(cx)}{\sqrt{dx}} \, dx = \int \frac{b \arccos(cx) + a}{\sqrt{dx}} \, dx$$

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + b \arccos(cx)}{\sqrt{dx}} \, dx = \int \frac{a + b \operatorname{acos}(c x)}{\sqrt{d x}} \, dx$$

```
[In] int((a + b*acos(c*x))/(d*x)^(1/2),x)
[Out] int((a + b*acos(c*x))/(d*x)^(1/2), x)
```

3.207 $\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 55

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx = -\frac{2(a+b\arccos(cx))}{d\sqrt{dx}} - \frac{4b\sqrt{c}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{d^{3/2}}$$

```
[Out] -4*b*EllipticF(c^(1/2)*(d*x)^(1/2)/d^(1/2),I)*c^(1/2)/d^(3/2)-2*(a+b*arccos
(c*x))/d/(d*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {4724, 335, 227}

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx = -\frac{2(a+b\arccos(cx))}{d\sqrt{dx}} - \frac{4b\sqrt{c}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{d^{3/2}}$$

```
[In] Int[(a + b*ArcCos[c*x])/(d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCos[c*x]))/(d*Sqrt[d*x]) - (4*b*Sqrt[c]*EllipticF[ArcSin[(Sqr
t[c]*Sqrt[d*x])/Sqrt[d]], -1])/d^(3/2)
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(a+b\arccos(cx))}{d\sqrt{dx}}-\frac{(2bc)\int\frac{1}{\sqrt{dx}\sqrt{1-c^2x^2}}\,dx}{d}\\
&= -\frac{2(a+b\arccos(cx))}{d\sqrt{dx}}-\frac{(4bc)\text{Subst}\left(\int\frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx,x,\sqrt{dx}\right)}{d^2}\\
&= -\frac{2(a+b\arccos(cx))}{d\sqrt{dx}}-\frac{4b\sqrt{c}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{d^{3/2}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.69

$$\int\frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx=\frac{2x\left(-a-b\arccos(cx)+\frac{2ib\sqrt{-\frac{1}{c}}c^2\sqrt{1-\frac{1}{c^2x^2}}x^{3/2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{-\frac{1}{c}}}{\sqrt{x}}\right),-1\right)}{\sqrt{1-c^2x^2}}\right)}{(dx)^{3/2}}$$

```
[In] Integrate[(a + b*ArcCos[c*x])/(d*x)^(3/2),x]
[Out] (2*x*(-a - b*ArcCos[c*x] + ((2*I)*b*Sqrt[-c^(-1)]*c^2*Sqrt[1 - 1/(c^2*x^2)]
*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[-c^(-1)]/Sqrt[x]], -1])/Sqrt[1 - c^2*x^2]
))/(d*x)^(3/2)
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

method	result	size
derivativedivides	$\frac{-\frac{2a}{\sqrt{dx}}+2b\left(-\frac{\arccos(cx)}{\sqrt{dx}}-\frac{2c\sqrt{-cx+1}\,\sqrt{cx+1}\,\mathrm{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	85
default	$\frac{-\frac{2a}{\sqrt{dx}}+2b\left(-\frac{\arccos(cx)}{\sqrt{dx}}-\frac{2c\sqrt{-cx+1}\,\sqrt{cx+1}\,\mathrm{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	85
parts	$-\frac{2a}{\sqrt{dx}\,d}+\frac{2b\left(-\frac{\arccos(cx)}{\sqrt{dx}}-\frac{2c\sqrt{-cx+1}\,\sqrt{cx+1}\,\mathrm{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{d}$	87

```
[In] int((a+b*arccos(c*x))/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-a/(d*x)^(1/2)+b*(-1/(d*x)^(1/2)*arccos(c*x)-2*c/d/(c/d)^(1/2)*(-c*x+1
)^(1/2)*(c*x+1)^(1/2)/(-c^2*x^2+1)^(1/2)*EllipticF((d*x)^(1/2)*(c/d)^(1/2),
I)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx=\frac{2\left(2\sqrt{-c^2d}bx\mathrm{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)-\left(bc\arccos\left(cx\right)+ac\right)\sqrt{dx}\right)}{cd^2x}$$

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] 2*(2*sqrt(-c^2*d)*b*x*weierstrassPInverse(4/c^2, 0, x) - (b*c*arccos(c*x) +
 a*c)*sqrt(d*x))/(c*d^2*x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx=\text{Exception raised: TypeError}$$

```
[In] integrate((a+b*acos(c*x))/(d*x)**(3/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx = \int \frac{b\arccos(cx)+a}{(dx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -(2*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (2*b*c*d^2*integrate(sqr
t(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(c^2*d^2*x^3 - d^2*x), x) - (2*b*arctan(1
/(sqrt(c)*sqrt(x))) - b*log(-(c*x - 1)/(c*x + 2*sqrt(c)*sqrt(x) + 1)))*sqrt
(c))*sqrt(x))/(d^(3/2)*sqrt(x))
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx = \int \frac{b\arccos(cx)+a}{(dx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(dx)^{3/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{(d\,x)^{3/2}}\,dx$$

```
[In] int((a + b*acos(c*x))/(d*x)^(3/2),x)
```

```
[Out] int((a + b*acos(c*x))/(d*x)^(3/2), x)
```

3.208 $\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 125

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx = \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}} + \frac{4bc^{3/2}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{3d^{5/2}} - \frac{4bc^{3/2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{3d^{5/2}}$$

```
[Out] -2/3*(a+b*arccos(c*x))/d/(d*x)^(3/2)+4/3*b*c^(3/2)*EllipticE(c^(1/2)*(d*x)^
(1/2)/d^(1/2),I)/d^(5/2)-4/3*b*c^(3/2)*EllipticF(c^(1/2)*(d*x)^(1/2)/d^(1/2
),I)/d^(5/2)+4/3*b*c*(-c^2*x^2+1)^(1/2)/d^2/(d*x)^(1/2)
```

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {4724, 331, 335, 313, 227, 1213, 435}

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx = -\frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}} - \frac{4bc^{3/2}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right),-1\right)}{3d^{5/2}} + \frac{4bc^{3/2}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle|-1\right)}{3d^{5/2}} + \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}}$$

```
[In] Int[(a + b*ArcCos[c*x])/(d*x)^(5/2),x]
```

```
[Out] (4*b*c*Sqrt[1 - c^2*x^2])/(3*d^2*Sqrt[d*x]) - (2*(a + b*ArcCos[c*x]))/(3*d*
(d*x)^(3/2)) + (4*b*c^(3/2)*EllipticE[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d]],
```

```
-1])/(3*d^(5/2)) - (4*b*c^(3/2)*EllipticF[ArcSin[(Sqrt[c]*Sqrt[d*x])/Sqrt[d
]], -1])/(3*d^(5/2))
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 313

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Dist[-q^(-1), Int[1/Sqrt[a + b*x^4], x], x] + Dist[1/q, Int[(1 + q*x^2)/Sqr
t[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

Rule 331

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Dist[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1))), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 335

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^n
))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 1213

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Dist[d/Sq
rt[a], Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
d, e}, x] && NegQ[c/a] && EqQ[c*d^2 + a*e^2, 0] && GtQ[a, 0]
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
```

```
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}} - \frac{(2bc)\int \frac{1}{(dx)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3d}$$

$$= \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}} + \frac{(2bc^3)\int \frac{\sqrt{dx}}{\sqrt{1-c^2x^2}}\,dx}{3d^3}$$

$$= \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}} + \frac{(4bc^3)\,\text{Subst}\left(\int \frac{x^2}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx, x, \sqrt{dx}\right)}{3d^4}$$

$$= \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}} - \frac{(4bc^2)\,\text{Subst}\left(\int \frac{1}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx, x, \sqrt{dx}\right)}{3d^3} + \frac{(4bc^2)\,\text{Subst}\left(\int \frac{1+\frac{cx^2}{d}}{\sqrt{1-\frac{c^2x^4}{d^2}}}\,dx, x, \sqrt{dx}\right)}{3d^3}$$

$$= \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}} - \frac{4bc^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right), -1\right)}{3d^{5/2}} + \frac{(4bc^2)\,\text{Subst}\left(\int \frac{\sqrt{1+\frac{cx^2}{d}}}{\sqrt{1-\frac{cx^2}{d}}}\,dx, x, \sqrt{dx}\right)}{3d^3}$$

$$= \frac{4bc\sqrt{1-c^2x^2}}{3d^2\sqrt{dx}} - \frac{2(a+b\arccos(cx))}{3d(dx)^{3/2}} + \frac{4bc^{3/2}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right)\middle| -1\right)}{3d^{5/2}} - \frac{4bc^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{dx}}{\sqrt{d}}\right), -1\right)}{3d^{5/2}}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.54

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx = \frac{2x\left(-3\left(a-2bcx\sqrt{1-c^2x^2}+b\arccos(cx)\right)+2bc^3x^3\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c\right.}{9(dx)^{5/2}}$$

```
[In] Integrate[(a + b*ArcCos[c*x])/(d*x)^(5/2),x]
[Out] (2*x*(-3*(a - 2*b*c*x*Sqrt[1 - c^2*x^2] + b*ArcCos[c*x]) + 2*b*c^3*x^3*Hype
rgeometric2F1[1/2, 3/4, 7/4, c^2*x^2]))/(9*(d*x)^(5/2))
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.03

method	result	si
derivativedivides	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}+2b\left(-\frac{\arccos(cx)}{3(dx)^{\frac{3}{2}}}-\frac{2c\left(-\frac{\sqrt{-c^2x^2+1}}{\sqrt{dx}}+\frac{c\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$	12
default	$\frac{-\frac{2a}{3(dx)^{\frac{3}{2}}}+2b\left(-\frac{\arccos(cx)}{3(dx)^{\frac{3}{2}}}-\frac{2c\left(-\frac{\sqrt{-c^2x^2+1}}{\sqrt{dx}}+\frac{c\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$	12
parts	$-\frac{2a}{3(dx)^{\frac{3}{2}}d}+\frac{2b\left(-\frac{\arccos(cx)}{3(dx)^{\frac{3}{2}}}-\frac{2c\left(-\frac{\sqrt{-c^2x^2+1}}{\sqrt{dx}}+\frac{c\sqrt{-cx+1}\,\sqrt{cx+1}\,\left(\text{EllipticF}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)-\text{EllipticE}\left(\sqrt{dx}\,\sqrt{\frac{c}{d}},i\right)\right)}{d\sqrt{\frac{c}{d}}\,\sqrt{-c^2x^2+1}}\right)}{3d}\right)}{d}$	13

```
[In] int((a+b*arccos(c*x))/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(-1/3*a/(d*x)^(3/2)+b*(-1/3/(d*x)^(3/2)*arccos(c*x)-2/3*c/d*(-(-c^2*x^2
+1)^(1/2)/(d*x)^(1/2)+c/d/(c/d)^(1/2)*(-c*x+1)^(1/2)*(c*x+1)^(1/2)/(-c^2*x^
2+1)^(1/2)*(EllipticF((d*x)^(1/2)*(c/d)^(1/2),I)-EllipticE((d*x)^(1/2)*(c/d
)^(1/2),I)))))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx=\frac{2\left(2\sqrt{-c^2d}bcx^2\text{weierstrassZeta}\left(\frac{4}{c^2},0,\text{weierstrassPInverse}\left(\frac{4}{c^2},0,x\right)\right)+\left(2\sqrt{-c^2x^2+\cdots\right.\right.}{3\,d^3x^2}$$

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*(2*sqrt(-c^2*d)*b*c*x^2*weierstrassZeta(4/c^2, 0, weierstrassPInverse(4
/c^2, 0, x)) + (2*sqrt(-c^2*x^2 + 1)*b*c*x - b*arccos(c*x) - a)*sqrt(d*x))/
(d^3*x^2)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*acos(c*x))/(d*x)**(5/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx = \int \frac{b\arccos{(cx)}+a}{(dx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*b*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) - (6*b*c*d^3*x*integra
te(1/3*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(c^2*d^3*x^4 - d^3*x^2), x) + (
2*b*c*x*arctan(1/(sqrt(c)*sqrt(x))) + b*c*x*log(-(c*x - 1)/(c*x + 2*sqrt(c)
*sqrt(x) + 1)))*sqrt(c))*sqrt(x))/(d^(5/2)*x^(3/2))
```

Giac [F]

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx = \int \frac{b\arccos{(cx)}+a}{(dx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*arccos(c*x) + a)/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\arccos(cx)}{(dx)^{5/2}}\,dx = \int \frac{a+b\,\mathrm{acos}(c\,x)}{{(d\,x)}^{5/2}}\,dx$$

```
[In] int((a + b*acos(c*x))/(d*x)^(5/2),x)
```

```
[Out] int((a + b*acos(c*x))/(d*x)^(5/2), x)
```

3.209 $\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx=\frac{2(dx)^{7/2}(a+b\arccos(cx))^2}{7d}+\frac{8bc(dx)^{9/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},c^2x^2\right)}{63d^2}+\frac{16b^2c^2(dx)^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};c^2x^2\right)}{693d^3}$$

```
[Out] 2/7*(d*x)^(7/2)*(a+b*arccos(c*x))^2/d+8/63*b*c*(d*x)^(9/2)*(a+b*arccos(c*x)
)*hypergeom([1/2, 9/4],[13/4],c^2*x^2)/d^2+16/693*b^2*c^2*(d*x)^(11/2)*hype
rgeom([1, 11/4, 11/4],[13/4, 15/4],c^2*x^2)/d^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4724, 4806}

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx=\frac{16b^2c^2(dx)^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};c^2x^2\right)}{693d^3}+\frac{8bc(dx)^{9/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},c^2x^2\right)(a+b\arccos(cx))}{63d^2}+\frac{2(dx)^{7/2}(a+b\arccos(cx))^2}{7d}$$

```
[In] Int[(d*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (2*(d*x)^(7/2)*(a + b*ArcCos[c*x])^2)/(7*d) + (8*b*c*(d*x)^(9/2)*(a + b*Arc
Cos[c*x])*Hypergeometric2F1[1/2, 9/4, 13/4, c^2*x^2])/(63*d^2) + (16*b^2*c^
```

```
2*(d*x)^(11/2)*HypergeometricPFQ[{1, 11/4, 11/4}, {13/4, 15/4}, c^2*x^2])/(
693*d^3)
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4806

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(dx)^{7/2}(a+b\arccos(cx))^2}{7d} + \frac{(4bc)\int \frac{(dx)^{7/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx}{7d}\\ &= \frac{2(dx)^{7/2}(a+b\arccos(cx))^2}{7d}\\ &\quad+ \frac{8bc(dx)^{9/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{9}{4},\frac{13}{4},c^2x^2\right)}{63d^2}\\ &\quad+ \frac{16b^2c^2(dx)^{11/2}\,{}_3F_2\left(1,\frac{11}{4},\frac{11}{4};\frac{13}{4},\frac{15}{4};c^2x^2\right)}{693d^3}\end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 234 vs. $2(109) = 218$.

Time = 10.98 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.15

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx = \frac{(dx)^{5/2}\left(882a^2x^3 + \frac{84ab\left(-2\sqrt{1-c^2x^2}(5+3c^2x^2)+21c^3x^3\arccos(cx)+10\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^2x^2\right)\right)}{c^3}\right.}{}$$

```
[In] Integrate[(d*x)^(5/2)*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] ((d*x)^(5/2)*(882*a^2*x^3 + (84*a*b*(-2*Sqrt[1 - c^2*x^2]*(5 + 3*c^2*x^2) +
 21*c^3*x^3*ArcCos[c*x] + 10*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2]))/c^
3 + (b^2*(-16*c*x*(35 + 9*c^2*x^2) - 168*Sqrt[1 - c^2*x^2]*(5 + 3*c^2*x^2)*
ArcCos[c*x] + 882*c^3*x^3*ArcCos[c*x]^2 + 840*Sqrt[1 - c^2*x^2]*ArcCos[c*x]
*Hypergeometric2F1[3/4, 1, 5/4, c^2*x^2] + (105*Sqrt[2]*c*Pi*x*Hypergeometr
icPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2*x^2])/(Gamma[5/4]*Gamma[7/4])))/c^3))/
(3087*x^2)
```

Maple [F]

$$\int (dx)^{\frac{5}{2}} (a + b \arccos(cx))^2 \, dx$$

```
[In] int((d*x)^(5/2)*(a+b*arccos(c*x))^2,x)
```

```
[Out] int((d*x)^(5/2)*(a+b*arccos(c*x))^2,x)
```

Fricas [F]

$$\int (dx)^{5/2}(a + b \arccos(cx))^2 \, dx = \int (dx)^{\frac{5}{2}} (b \arccos(cx) + a)^2 \, dx$$

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")
```

```
[Out] integral((b^2*d^2*x^2*arccos(c*x)^2 + 2*a*b*d^2*x^2*arccos(c*x) + a^2*d^2*x
^2)*sqrt(d*x), x)
```

Sympy [F(-1)]

Timed out.

$$\int (dx)^{5/2}(a + b \arccos(cx))^2 \, dx = \text{Timed out}$$

```
[In] integrate((d*x)**(5/2)*(a+b*acos(c*x))**2,x)
```

```
[Out] Timed out
```

Maxima [F]

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx = \int (dx)^{\frac{5}{2}}\left(b\arccos\left(cx\right)+a\right)^2\,dx$$

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")
```

```
[Out] 2/7*b^2*d^(5/2)*x^(7/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 1/42
*a^2*c^2*d^(5/2)*(4*(3*c^2*x^(7/2) + 7*x^(3/2))/c^4 + 42*arctan(sqrt(c)*sqr
t(x))/c^(11/2) + 21*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(11/
2)) + 14*a*b*c^2*d^(5/2)*integrate(1/7*x^(9/2)*arctan(sqrt(c*x + 1)*sqrt(-c
*x + 1)/(c*x))/(c^2*x^2 - 1), x) - 4*b^2*c*d^(5/2)*integrate(1/7*sqrt(c*x +
 1)*sqrt(-c*x + 1)*x^(7/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*
x^2 - 1), x) - 1/6*a^2*d^(5/2)*(4*x^(3/2)/c^2 + 6*arctan(sqrt(c)*sqrt(x))/c
^(7/2) + 3*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(7/2)) - 14*a
*b*d^(5/2)*integrate(1/7*x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))
/(c^2*x^2 - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x)^(5/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{5/2}(a+b\arccos(cx))^2\,dx = \int \left(a+b\,\mathrm{acos}\left(c\,x\right)\right)^2\,{\left(d\,x\right)}^{5/2}\,dx$$

```
[In] int((a + b*acos(c*x))^2*(d*x)^(5/2),x)
```

```
[Out] int((a + b*acos(c*x))^2*(d*x)^(5/2), x)
```

3.210 $\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{2(dx)^{5/2}(a+b\arccos(cx))^2}{5d}+\frac{8bc(dx)^{7/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},c^2x^2\right)}{35d^2}+\frac{16b^2c^2(dx)^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};c^2x^2\right)}{315d^3}$$

```
[Out] 2/5*(d*x)^(5/2)*(a+b*arccos(c*x))^2/d+8/35*b*c*(d*x)^(7/2)*(a+b*arccos(c*x)
)*hypergeom([1/2, 7/4],[11/4],c^2*x^2)/d^2+16/315*b^2*c^2*(d*x)^(9/2)*hyper
geom([1, 9/4, 9/4],[11/4, 13/4],c^2*x^2)/d^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4724, 4806}

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx=\frac{16b^2c^2(dx)^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};c^2x^2\right)}{315d^3}+\frac{8bc(dx)^{7/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},c^2x^2\right)(a+b\arccos(cx))}{35d^2}+\frac{2(dx)^{5/2}(a+b\arccos(cx))^2}{5d}$$

```
[In] Int[(d*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (2*(d*x)^(5/2)*(a + b*ArcCos[c*x])^2)/(5*d) + (8*b*c*(d*x)^(7/2)*(a + b*Arc
Cos[c*x])*Hypergeometric2F1[1/2, 7/4, 11/4, c^2*x^2])/(35*d^2) + (16*b^2*c^
```

```
2*(d*x)^(9/2)*HypergeometricPFQ[{1, 9/4, 9/4}, {11/4, 13/4}, c^2*x^2])/(315
*d^3)
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4806

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\text{integral} &= \frac{2(dx)^{5/2}(a+b\arccos(cx))^2}{5d} + \frac{(4bc)\int \frac{(dx)^{5/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx}{5d}\\ &= \frac{2(dx)^{5/2}(a+b\arccos(cx))^2}{5d}\\ &\quad + \frac{8bc(dx)^{7/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{7}{4},\frac{11}{4},c^2x^2\right)}{35d^2}\\ &\quad + \frac{16b^2c^2(dx)^{9/2}\,{}_3F_2\left(1,\frac{9}{4},\frac{9}{4};\frac{11}{4},\frac{13}{4};c^2x^2\right)}{315d^3}\end{aligned}$$

Mathematica [A] (verified)

Time = 3.37 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.61

$$\int (dx)^{3/2}(a+b\arccos(cx))^2\,dx = \frac{(dx)^{3/2}\left(4480a^2x+\frac{128b\left(-28a\sqrt{1-c^2x^2}+70acx\arccos(cx)+35bcx\arccos(cx)^2+28a\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\right.\right.}{c}\right.}{11}$$

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] ((d*x)^(3/2)*(4480*a^2*x + (128*b*(-28*a*Sqrt[1 - c^2*x^2] + 70*a*c*x*ArcCo
s[c*x] + 35*b*c*x*ArcCos[c*x]^2 + 28*a*Hypergeometric2F1[1/2, 3/4, 7/4, c^2
*x^2] + 20*b*c^2*x^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Hypergeometric2F1[1, 9/4
, 11/4, c^2*x^2]))/c + (525*Sqrt[2]*b^2*c^2*Pi*x^3*HypergeometricPFQ[{1, 9/
4, 9/4}, {11/4, 13/4}, c^2*x^2])/(Gamma[11/4]*Gamma[13/4])))/11200
```

Maple [F]

$$\int (dx)^{\frac{3}{2}} (a + b \arccos (cx))^2 \, dx$$

[In] int((d*x)^(3/2)*(a+b*arccos(c*x))^2,x)

[Out] int((d*x)^(3/2)*(a+b*arccos(c*x))^2,x)

Fricas [F]

$$\int (dx)^{3/2}(a + b\arccos(cx))^2 \, dx = \int (dx)^{\frac{3}{2}} (b\arccos (cx) + a)^2 \, dx$$

[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="fricas")

[Out] integral((b^2*d*x*arccos(c*x)^2 + 2*a*b*d*x*arccos(c*x) + a^2*d*x)*sqrt(d*x
), x)

Sympy [F]

$$\int (dx)^{3/2}(a + b\arccos(cx))^2 \, dx = \int (dx)^{\frac{3}{2}} (a + b\operatorname{acos}(cx))^2 \, dx$$

[In] integrate((d*x)**(3/2)*(a+b*acos(c*x))**2,x)

[Out] Integral((d*x)**(3/2)*(a + b*acos(c*x))**2, x)

Maxima [F]

$$\int (dx)^{3/2}(a + b\arccos(cx))^2 \, dx = \int (dx)^{\frac{3}{2}} (b\arccos (cx) + a)^2 \, dx$$

[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="maxima")

[Out] 2/5*b^2*d^(3/2)*x^(5/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 1/10
*a^2*c^2*d^(3/2)*(4*(c^2*x^(5/2) + 5*sqrt(x))/c^4 - 10*arctan(sqrt(c)*sqrt(
x))/c^(9/2) + 5*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(9/2)) +
 10*a*b*c^2*d^(3/2)*integrate(1/5*x^(7/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x +
1)/(c*x))/(c^2*x^2 - 1), x) - 4*b^2*c*d^(3/2)*integrate(1/5*sqrt(c*x + 1)*s
qrt(-c*x + 1)*x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*x^2 -
 1), x) - 1/2*a^2*d^(3/2)*(4*sqrt(x)/c^2 - 2*arctan(sqrt(c)*sqrt(x))/c^(5/2
) + log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(5/2)) - 10*a*b*d^(3
/2)*integrate(1/5*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*x
^2 - 1), x)

Giac [F(-2)]

Exception generated.

$$\int (dx)^{3/2}(a + b\arccos(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2}(a + b\arccos(cx))^2\,dx = \int (a + b\,\mathrm{acos}(c\,x))^2\,(d\,x)^{3/2}\,dx$$

```
[In] int((a + b*acos(c*x))^2*(d*x)^(3/2),x)
```

```
[Out] int((a + b*acos(c*x))^2*(d*x)^(3/2), x)
```

3.211 $\int \sqrt{dx}(a+b\arccos(cx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \frac{2(dx)^{3/2}(a+b\arccos(cx))^2}{3d} + \frac{8bc(dx)^{5/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)}{15d^2} + \frac{16b^2c^2(dx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{105d^3}$$

```
[Out] 2/3*(d*x)^(3/2)*(a+b*arccos(c*x))^2/d+8/15*b*c*(d*x)^(5/2)*(a+b*arccos(c*x)
)*hypergeom([1/2, 5/4],[9/4],c^2*x^2)/d^2+16/105*b^2*c^2*(d*x)^(7/2)*hyperg
eom([1, 7/4, 7/4],[9/4, 11/4],c^2*x^2)/d^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4724, 4806}

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \frac{16b^2c^2(dx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{105d^3} + \frac{8bc(dx)^{5/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)(a+b\arccos(cx))}{15d^2} + \frac{2(dx)^{3/2}(a+b\arccos(cx))^2}{3d}$$

```
[In] Int[Sqrt[d*x]*(a + b*ArcCos[c*x])^2,x]
```

```
[Out] (2*(d*x)^(3/2)*(a + b*ArcCos[c*x])^2)/(3*d) + (8*b*c*(d*x)^(5/2)*(a + b*Arc
Cos[c*x])*Hypergeometric2F1[1/2, 5/4, 9/4, c^2*x^2])/(15*d^2) + (16*b^2*c^2
*(d*x)^(7/2)*HypergeometricPFQ[{1, 7/4, 7/4}, {9/4, 11/4}, c^2*x^2])/(105*d
^3)
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4806

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{2(dx)^{3/2}(a+b\arccos(cx))^2}{3d} + \frac{(4bc)\int \frac{(dx)^{3/2}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx}{3d} \\
&= \frac{2(dx)^{3/2}(a+b\arccos(cx))^2}{3d} \\
&\quad + \frac{8bc(dx)^{5/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{4},\frac{9}{4},c^2x^2\right)}{15d^2} \\
&\quad + \frac{16b^2c^2(dx)^{7/2}\,{}_3F_2\left(1,\frac{7}{4},\frac{7}{4};\frac{9}{4},\frac{11}{4};c^2x^2\right)}{105d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.85

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx$$

$$= \frac{1}{27}\sqrt{dx}\left(\frac{2\left(9a^2cx-8b^2cx-12ab\sqrt{1-c^2x^2}+18abcx\arccos(cx)-12b^2\sqrt{1-c^2x^2}\arccos(cx)+9b^2cx\,\text{arc}\right.}{}\right.$$

$$\left. + \frac{3\sqrt{2}b^2\pi x\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};c^2x^2\right)}{\text{Gamma}\left(\frac{5}{4}\right)\text{Gamma}\left(\frac{7}{4}\right)}\right)$$

[In] Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x])^2,x]

[Out] (Sqrt[d*x]*((2*(9*a^2*c*x - 8*b^2*c*x - 12*a*b*Sqrt[1 - c^2*x^2] + 18*a*b*c*x*ArcCos[c*x] - 12*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x] + 9*b^2*c*x*ArcCos[c*x]^2 + 12*a*b*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2] + 12*b^2*Sqrt[1 - c^2*x^2]*ArcCos[c*x]*Hypergeometric2F1[3/4, 1, 5/4, c^2*x^2]))/c + (3*Sqrt[2]*b^2*Pi*x*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2*x^2])/(Gamma[5/4]*Gamma[7/4])))/27

Maple [F]

$$\int (a+b\arccos(cx))^2\sqrt{dx}dx$$

[In] int((a+b*arccos(c*x))^2*(d*x)^(1/2),x)

[Out] int((a+b*arccos(c*x))^2*(d*x)^(1/2),x)

Fricas [F]

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \int \sqrt{dx}(b\arccos(cx)+a)^2\,dx$$

[In] integrate((a+b*arccos(c*x))^2*(d*x)^(1/2),x, algorithm="fricas")

[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x), x)

Sympy [F]

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \int \sqrt{dx}(a+b\operatorname{acos}(cx))^2\,dx$$

[In] integrate((a+b*acos(c*x))**2*(d*x)**(1/2),x)

[Out] Integral(sqrt(d*x)*(a + b*acos(c*x))**2, x)

Maxima [F]

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \int \sqrt{dx}(b\arccos(cx)+a)^2\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^2*sqrt(d)*x^(3/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + 1/6*
a^2*c^2*sqrt(d)*(4*x^(3/2)/c^2 + 6*arctan(sqrt(c)*sqrt(x))/c^(7/2) + 3*log(
(c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(7/2)) + 6*a*b*c^2*sqrt(d)*i
ntegrate(1/3*x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*x^2 -
1), x) - 4*b^2*c*sqrt(d)*integrate(1/3*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^(3/2)
*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*x^2 - 1), x) - 1/2*a^2*sqr
t(d)*(2*arctan(sqrt(c)*sqrt(x))/c^(3/2) + log((c*sqrt(x) - sqrt(c))/(c*sqrt
(x) + sqrt(c)))/c^(3/2)) - 6*a*b*sqrt(d)*integrate(1/3*sqrt(x)*arctan(sqrt(
c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*x^2 - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccos(c*x))^2*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}(a+b\arccos(cx))^2\,dx = \int (a+b\,\mathrm{acos}(c\,x))^2\,\sqrt{d\,x}\,dx$$

```
[In] int((a + b*acos(c*x))^2*(d*x)^(1/2),x)
```

```
[Out] int((a + b*acos(c*x))^2*(d*x)^(1/2), x)
```

3.212 $\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 107

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}(a+b\arccos(cx))^2}{d} + \frac{8bc(dx)^{3/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x^2\right)}{3d^2} + \frac{16b^2c^2(dx)^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};c^2x^2\right)}{15d^3}$$

```
[Out] 8/3*b*c*(d*x)^(3/2)*(a+b*arccos(c*x))*hypergeom([1/2, 3/4],[7/4],c^2*x^2)/d
^2+16/15*b^2*c^2*(d*x)^(5/2)*hypergeom([1, 5/4, 5/4],[7/4, 9/4],c^2*x^2)/d^
3+2*(a+b*arccos(c*x))^2*(d*x)^(1/2)/d
```

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {4724, 4806}

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx = \frac{16b^2c^2(dx)^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};c^2x^2\right)}{15d^3} + \frac{8bc(dx)^{3/2}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x^2\right)(a+b\arccos(cx))}{3d^2} + \frac{2\sqrt{dx}(a+b\arccos(cx))^2}{d}$$

```
[In] Int[(a + b*ArcCos[c*x])^2/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*(a + b*ArcCos[c*x])^2)/d + (8*b*c*(d*x)^(3/2)*(a + b*ArcCos[c*
x])*Hypergeometric2F1[1/2, 3/4, 7/4, c^2*x^2])/(3*d^2) + (16*b^2*c^2*(d*x)^
(5/2)*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, c^2*x^2])/(15*d^3)
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4806

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{dx}(a+b\arccos(cx))^2}{d} + \frac{(4bc)\int \frac{\sqrt{dx}(a+b\arccos(cx))}{\sqrt{1-c^2x^2}}\,dx}{d}$$

$$= \frac{2\sqrt{dx}(a+b\arccos(cx))^2}{d} + \frac{8bc(dx)^{3/2}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},c^2x^2\right)}{3d^2} + \frac{16b^2c^2(dx)^{5/2}\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};c^2x^2\right)}{15d^3}$$

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.33

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx = \frac{3\sqrt{2}b^2c^2\pi x^3\,{}_3F_2\left(1,\frac{5}{4},\frac{5}{4};\frac{7}{4},\frac{9}{4};c^2x^2\right) + 8x\,\text{Gamma}\left(\frac{7}{4}\right)\text{Gamma}\left(\frac{9}{4}\right)\left(3(a+b\arccos(cx))^2 + 4abcx\,\text{Hypergeo}\right.}{12\sqrt{dx}\,\text{Gamma}\left(\frac{7}{4}\right)\text{G}}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^2/Sqrt[d*x],x]
```

```
[Out] (3*Sqrt[2]*b^2*c^2*Pi*x^3*HypergeometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, c^2*
x^2] + 8*x*Gamma[7/4]*Gamma[9/4]*(3*(a + b*ArcCos[c*x])^2 + 4*a*b*c*x*Hyper
geometric2F1[1/2, 3/4, 7/4, c^2*x^2] + 2*b^2*ArcCos[c*x]*Hypergeometric2F1[
1, 5/4, 7/4, c^2*x^2]*Sin[2*ArcCos[c*x]]))/(12*Sqrt[d*x]*Gamma[7/4]*Gamma[9
/4])
```

Maple [F]

$$\int \frac{(a+b\arccos{(cx)})^2}{\sqrt{dx}}dx$$

```
[In] int((a+b*arccos(c*x))^2/(d*x)^(1/2),x)
```

```
[Out] int((a+b*arccos(c*x))^2/(d*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2}{\sqrt{dx}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x)/(d*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*acos(c*x))**2/(d*x)**(1/2),x)
```

```
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2}{\sqrt{dx}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*b^2*sqrt(x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 + (a^2*c^2*
sqrt(d)*(4*sqrt(x)/(c^2*d) - 2*arctan(sqrt(c)*sqrt(x))/(c^(5/2)*d) + log((c
*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/(c^(5/2)*d)) + 4*a*b*c^2*sqrt(d)
*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d*x^3 -
d*x), x) - 8*b^2*c*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^(3/2)*a
rctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d*x^3 - d*x), x) + a^2*sqrt(
d)*(2*arctan(sqrt(c)*sqrt(x))/(sqrt(c)*d) - log((c*sqrt(x) - sqrt(c))/(c*sq
rt(x) + sqrt(c)))/(sqrt(c)*d)) - 4*a*b*sqrt(d)*integrate(sqrt(x)*arctan(sqr
t(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d*x^3 - d*x), x))*sqrt(d))/sqrt(d)
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2}{\sqrt{dx}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^2/sqrt(d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{\sqrt{d\,x}}\,dx$$

```
[In] int((a + b*acos(c*x))^2/(d*x)^(1/2),x)
[Out] int((a + b*acos(c*x))^2/(d*x)^(1/2), x)
```

3.213 $\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 105

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx = -\frac{2(a+b\arccos(cx))^2}{d\sqrt{dx}} - \frac{8bc\sqrt{dx}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^2x^2\right)}{d^2} - \frac{16b^2c^2(dx)^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};c^2x^2\right)}{3d^3}$$

```
[Out] -16/3*b^2*c^2*(d*x)^(3/2)*hypergeom([3/4, 3/4, 1],[5/4, 7/4],c^2*x^2)/d^3-2
*(a+b*arccos(c*x))^2/d/(d*x)^(1/2)-8*b*c*(a+b*arccos(c*x))*hypergeom([1/4,
1/2],[5/4],c^2*x^2)*(d*x)^(1/2)/d^2
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4724, 4806}

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx = -\frac{16b^2c^2(dx)^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};c^2x^2\right)}{3d^3} - \frac{8bc\sqrt{dx}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^2x^2\right)(a+b\arccos(cx))}{d^2} - \frac{2(a+b\arccos(cx))^2}{d\sqrt{dx}}$$

```
[In] Int[(a + b*ArcCos[c*x])^2/(d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCos[c*x])^2)/(d*Sqrt[d*x]) - (8*b*c*Sqrt[d*x]*(a + b*ArcCos[c
*x])*Hypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2])/d^2 - (16*b^2*c^2*(d*x)^(3/
2)*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4}, c^2*x^2])/(3*d^3)
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4806

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\arccos(cx))^2}{d\sqrt{dx}} - \frac{(4bc)\int \frac{a+b\arccos(cx)}{\sqrt{dx}\sqrt{1-c^2x^2}}\,dx}{d}$$

$$= -\frac{2(a+b\arccos(cx))^2}{d\sqrt{dx}} - \frac{8bc\sqrt{dx}(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},c^2x^2\right)}{d^2} - \frac{16b^2c^2(dx)^{3/2}\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};c^2x^2\right)}{3d^3}$$

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx = \frac{x\left(-\frac{\sqrt{2}b^2c^2\pi x^2\,{}_3F_2\left(\frac{3}{4},\frac{3}{4},1;\frac{5}{4},\frac{7}{4};c^2x^2\right)}{\text{Gamma}\left(\frac{5}{4}\right)\text{Gamma}\left(\frac{7}{4}\right)} - 2\left((a+b\arccos(cx))^2 + 4abcx\,\text{Hypergeometric2}\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^2/(d*x)^(3/2),x]
```

```
[Out] (x*(-((Sqrt[2]*b^2*c^2*Pi*x^2*HypergeometricPFQ[{3/4, 3/4, 1}, {5/4, 7/4},
c^2*x^2])/(Gamma[5/4]*Gamma[7/4])) - 2*((a + b*ArcCos[c*x])^2 + 4*a*b*c*x*H
ypergeometric2F1[1/4, 1/2, 5/4, c^2*x^2] + 2*b^2*ArcCos[c*x]*Hypergeometric
2F1[3/4, 1, 5/4, c^2*x^2]*Sin[2*ArcCos[c*x]])))/(d*x)^(3/2)
```

Maple [F]

$$\int \frac{(a+b\arccos{(cx)})^2}{(dx)^{\frac{3}{2}}}dx$$

[In] int((a+b*arccos(c*x))^2/(d*x)^(3/2),x)

[Out] int((a+b*arccos(c*x))^2/(d*x)^(3/2),x)

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2}{(dx)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*arccos(c*x))^2/(d*x)^(3/2),x, algorithm="fricas")

[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x)/(d^2*x^2), x)

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((a+b*acos(c*x))**2/(d*x)**(3/2),x)

[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx = \int \frac{(b\arccos{(cx)}+a)^2}{(dx)^{\frac{3}{2}}}\,dx$$

[In] integrate((a+b*arccos(c*x))^2/(d*x)^(3/2),x, algorithm="maxima")

[Out] -1/2*(4*b^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^2 - (a^2*c^2*sqrt(d)*(2*arctan(sqrt(c)*sqrt(x))/(c^(3/2)*d^2) + log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/(c^(3/2)*d^2)) + 4*a*b*c^2*sqrt(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^2*x^4 - d^2*x^2), x) + 8*b^2*c*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^2*x^4 - d^2*x^2), x) - a^2*sqrt(d)*(2*sqrt(

```
c)*arctan(sqrt(c)*sqrt(x))/d^2 + sqrt(c)*log((c*sqrt(x) - sqrt(c))/(c*sqrt(
x) + sqrt(c)))/d^2 + 4/(d^2*sqrt(x))) - 4*a*b*sqrt(d)*integrate(sqrt(x)*arc
tan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^2*x^4 - d^2*x^2), x))*d^(3/2
)*sqrt(x))/(d^(3/2)*sqrt(x))
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx=\int \frac{(b\arccos(cx)+a)^2}{(dx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^2/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}}\,dx=\int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d\,x)^{3/2}}\,dx$$

```
[In] int((a + b*acos(c*x))^2/(d*x)^(3/2),x)
[Out] int((a + b*acos(c*x))^2/(d*x)^(3/2), x)
```

3.214 $\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 109

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = -\frac{2(a+b\arccos(cx))^2}{3d(dx)^{3/2}} + \frac{8bc(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},c^2x^2\right)}{3d^2\sqrt{dx}} + \frac{16b^2c^2\sqrt{dx}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};c^2x^2\right)}{3d^3}$$

```
[Out] -2/3*(a+b*arccos(c*x))^2/d/(d*x)^(3/2)+8/3*b*c*(a+b*arccos(c*x))*hypergeom(
[-1/4, 1/2],[3/4],c^2*x^2)/d^2/(d*x)^(1/2)+16/3*b^2*c^2*hypergeom([1/4, 1/4
, 1],[3/4, 5/4],c^2*x^2)*(d*x)^(1/2)/d^3
```

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {4724, 4806}

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \frac{16b^2c^2\sqrt{dx}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};c^2x^2\right)}{3d^3} + \frac{8bc\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},c^2x^2\right)(a+b\arccos(cx))}{3d^2\sqrt{dx}} - \frac{2(a+b\arccos(cx))^2}{3d(dx)^{3/2}}$$

```
[In] Int[(a + b*ArcCos[c*x])^2/(d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCos[c*x])^2)/(3*d*(d*x)^(3/2)) + (8*b*c*(a + b*ArcCos[c*x])*H
ypergeometric2F1[-1/4, 1/2, 3/4, c^2*x^2])/(3*d^2*Sqrt[d*x]) + (16*b^2*c^2*
Sqrt[d*x]*HypergeometricPFQ[{1/4, 1/4, 1}, {3/4, 5/4}, c^2*x^2])/(3*d^3)
```

Rule 4724

```
Int[((a_.) + ArcCos[(c_.)*(x_)]*(b_.))^(n_.)*((d_.)*(x_))^(m_.), x_Symbol]
:> Simp[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^n/(d*(m + 1))), x] + Dist[b*c*(n
/(d*(m + 1))), Int[(d*x)^(m + 1)*((a + b*ArcCos[c*x])^(n - 1)/Sqrt[1 - c^2*
x^2]), x], x] /; FreeQ[{a, b, c, d, m}, x] && IGtQ[n, 0] && NeQ[m, -1]
```

Rule 4806

```
Int[(((a_.) + ArcCos[(c_.)*(x_)]*(b_.))*((f_.)*(x_))^(m_))/Sqrt[(d_) + (e_.
)*(x_)^2], x_Symbol] :> Simp[((f*x)^(m + 1)/(f*(m + 1)))*(a + b*ArcCos[c*x]
)*Simp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*Hypergeometric2F1[1/2, (1 + m)/2,
 (3 + m)/2, c^2*x^2], x] + Simp[b*c*((f*x)^(m + 2)/(f^2*(m + 1)*(m + 2)))*S
imp[Sqrt[1 - c^2*x^2]/Sqrt[d + e*x^2]]*HypergeometricPFQ[{1, 1 + m/2, 1 + m
/2}, {3/2 + m/2, 2 + m/2}, c^2*x^2], x] /; FreeQ[{a, b, c, d, e, f, m}, x]
&& EqQ[c^2*d + e, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\frac{2(a+b\arccos(cx))^2}{3d(dx)^{3/2}} - \frac{(4bc)\int \frac{a+b\arccos(cx)}{(dx)^{3/2}\sqrt{1-c^2x^2}}\,dx}{3d} \\
&= -\frac{2(a+b\arccos(cx))^2}{3d(dx)^{3/2}} + \frac{8bc(a+b\arccos(cx))\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},c^2x^2\right)}{3d^2\sqrt{dx}} \\
&\quad + \frac{16b^2c^2\sqrt{dx}\,{}_3F_2\left(\frac{1}{4},\frac{1}{4},1;\frac{3}{4},\frac{5}{4};c^2x^2\right)}{3d^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.82

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \frac{x\left(-8\,\text{Gamma}\left(\frac{7}{4}\right)\text{Gamma}\left(\frac{9}{4}\right)\left(3\left(a^2-8b^2c^2x^2+2b\left(a-2bcx\sqrt{1-c^2x^2}\right)\arccos\right.\right.\right.}{}$$

```
[In] Integrate[(a + b*ArcCos[c*x])^2/(d*x)^(5/2),x]
```

```
[Out] (x*(-8*Gamma[7/4]*Gamma[9/4]*(3*(a^2 - 8*b^2*c^2*x^2 + 2*b*(a - 2*b*c*x*Sqr
t[1 - c^2*x^2])*ArcCos[c*x] + b^2*ArcCos[c*x]^2) - 12*a*b*c*x*Hypergeometri
c2F1[-1/4, 1/2, 3/4, c^2*x^2] - 4*b^2*c^3*x^3*Sqrt[1 - c^2*x^2]*ArcCos[c*x]
*Hypergeometric2F1[1, 5/4, 7/4, c^2*x^2]) + 3*Sqrt[2]*b^2*c^4*Pi*x^4*Hyperg
eometricPFQ[{1, 5/4, 5/4}, {7/4, 9/4}, c^2*x^2]))/(36*(d*x)^(5/2)*Gamma[7/4
]*Gamma[9/4])
```

Maple [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccos(c*x))^2/(d*x)^(5/2),x)
[Out] int((a+b*arccos(c*x))^2/(d*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(5/2),x, algorithm="fricas")
[Out] integral((b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2)*sqrt(d*x)/(d^3*x^3),
 x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*acos(c*x))**2/(d*x)**(5/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(5/2),x, algorithm="maxima")
[Out] -1/6*((3*a^2*c^2*sqrt(d)*(2*arctan(sqrt(c)*sqrt(x))/(sqrt(c)*d^3) - log((c*
sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/(sqrt(c)*d^3)) - 36*a*b*c^2*sqrt(
d)*integrate(1/3*x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^
3*x^5 - d^3*x^3), x) - 24*b^2*c*sqrt(d)*integrate(1/3*sqrt(c*x + 1)*sqrt(-c
*x + 1)*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^3*x^5 - d
^3*x^3), x) - a^2*sqrt(d)*(6*c^(3/2)*arctan(sqrt(c)*sqrt(x))/d^3 - 3*c^(3/2
```

```
)*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/d^3 - 4/(d^3*x^(3/2))) +
 36*a*b*sqrt(d)*integrate(1/3*sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(
c*x))/(c^2*d^3*x^5 - d^3*x^3), x))*d^(5/2)*x^(3/2) + 4*b^2*arctan2(sqrt(c*x
 + 1)*sqrt(-c*x + 1), c*x)^2)/(d^(5/2)*x^(3/2))
```

Giac [F]

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^2}{(dx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^2/(d*x)^(5/2),x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^2/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\arccos(cx))^2}{(dx)^{5/2}}\,dx = \int \frac{(a+b\,\mathrm{acos}(c\,x))^2}{(d\,x)^{5/2}}\,dx$$

```
[In] int((a + b*acos(c*x))^2/(d*x)^(5/2),x)
[Out] int((a + b*acos(c*x))^2/(d*x)^(5/2), x)
```

3.215 $\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx=\frac{2(dx)^{5/2}(a+b\arccos(cx))^3}{5d}+\frac{6bc\text{Int}\left(\frac{(dx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}},x\right)}{5d}$$

```
[Out] 2/5*(d*x)^(5/2)*(a+b*arccos(c*x))^3/d+6/5*b*c*Unintegrable((d*x)^(5/2)*(a+b
*arccos(c*x))^2/(-c^2*x^2+1)^(1/2),x)/d
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx=\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx$$

```
[In] Int[(d*x)^(3/2)*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (2*(d*x)^(5/2)*(a + b*ArcCos[c*x])^3)/(5*d) + (6*b*c*Defer[Int][((d*x)^(5/2
)*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2], x])/(5*d)
```

Rubi steps

$$\text{integral}=\frac{2(dx)^{5/2}(a+b\arccos(cx))^3}{5d}+\frac{(6bc)\int\frac{(dx)^{5/2}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\,dx}{5d}$$

Mathematica [N/A]

Not integrable

Time = 45.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx=\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx$$

```
[In] Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x])^3,x]
[Out] Integrate[(d*x)^(3/2)*(a + b*ArcCos[c*x])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.82 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int (dx)^{\frac{3}{2}} \left(a+b\arccos\left(cx\right)\right)^{3}dx$$

```
[In] int((d*x)^(3/2)*(a+b*arccos(c*x))^3,x)
[Out] int((d*x)^(3/2)*(a+b*arccos(c*x))^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.94

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx=\int (dx)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)^{3}dx$$

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^3,x, algorithm="fricas")
[Out] integral((b^3*d*x*arccos(c*x)^3 + 3*a*b^2*d*x*arccos(c*x)^2 + 3*a^2*b*d*x*a
rccos(c*x) + a^3*d*x)*sqrt(d*x), x)
```

Sympy [N/A]

Not integrable

Time = 80.91 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx=\int (dx)^{\frac{3}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)^{3}\,dx$$

```
[In] integrate((d*x)**(3/2)*(a+b*acos(c*x))**3,x)
[Out] Integral((d*x)**(3/2)*(a + b*acos(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 3.50 (sec) , antiderivative size = 441, normalized size of antiderivative = 24.50

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx = \int (dx)^{\frac{3}{2}}\,(b\arccos{(cx)}+a)^3\,dx$$

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^3,x, algorithm="maxima")
```

```
[Out] 2/5*b^3*d^(3/2)*x^(5/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + 1/10
*a^3*c^2*d^(3/2)*(4*(c^2*x^(5/2) + 5*sqrt(x))/c^4 - 10*arctan(sqrt(c)*sqrt(
x))/c^(9/2) + 5*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(9/2)) +
 15*a*b^2*c^2*d^(3/2)*integrate(1/5*x^(7/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x
+ 1)/(c*x))^2/(c^2*x^2 - 1), x) + 15*a^2*b*c^2*d^(3/2)*integrate(1/5*x^(7/2
)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*x^2 - 1), x) - 6*b^3*c*d^
(3/2)*integrate(1/5*sqrt(c*x + 1)*sqrt(-c*x + 1)*x^(5/2)*arctan(sqrt(c*x +
1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*x^2 - 1), x) - 1/2*a^3*d^(3/2)*(4*sqrt(x)/c
^2 - 2*arctan(sqrt(c)*sqrt(x))/c^(5/2) + log((c*sqrt(x) - sqrt(c))/(c*sqrt(
x) + sqrt(c)))/c^(5/2)) - 15*a*b^2*d^(3/2)*integrate(1/5*x^(3/2)*arctan(sqr
t(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*x^2 - 1), x) - 15*a^2*b*d^(3/2)*int
egrate(1/5*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*x^2 - 1)
, x)
```

Giac [F(-2)]

Exception generated.

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((d*x)^(3/2)*(a+b*arccos(c*x))^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx)^{3/2}(a+b\arccos(cx))^3\,dx=\int (a+b\,\mathrm{acos}(c\,x))^3\,(d\,x)^{3/2}\,dx$$

```
[In] int((a + b*acos(c*x))^3*(d*x)^(3/2),x)
[Out] int((a + b*acos(c*x))^3*(d*x)^(3/2), x)
```

3.216 $\int \sqrt{dx}(a+b\arccos(cx))^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx=\frac{2(dx)^{3/2}(a+b\arccos(cx))^3}{3d}+\frac{2bc\text{Int}\left(\frac{(dx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}},x\right)}{d}$$

```
[Out] 2/3*(d*x)^(3/2)*(a+b*arccos(c*x))^3/d+2*b*c*Unintegrable((d*x)^(3/2)*(a+b*a
rccos(c*x))^2/(-c^2*x^2+1)^(1/2),x)/d
```

Rubi [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx=\int \sqrt{dx}(a+b\arccos(cx))^3\,dx$$

```
[In] Int[Sqrt[d*x]*(a + b*ArcCos[c*x])^3,x]
```

```
[Out] (2*(d*x)^(3/2)*(a + b*ArcCos[c*x])^3)/(3*d) + (2*b*c*Defer[Int][((d*x)^(3/2
)*(a + b*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2], x])/d
```

Rubi steps

$$\text{integral}=\frac{2(dx)^{3/2}(a+b\arccos(cx))^3}{3d}+\frac{(2bc)\int \frac{(dx)^{3/2}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\,dx}{d}$$

Mathematica [N/A]

Not integrable

Time = 141.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx=\int \sqrt{dx}(a+b\arccos(cx))^3\,dx$$

```
[In] Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x])^3,x]
[Out] Integrate[Sqrt[d*x]*(a + b*ArcCos[c*x])^3, x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.38 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \left(a+b\arccos\left(cx\right)\right)^{3}\sqrt{dx}dx$$

```
[In] int((a+b*arccos(c*x))^3*(d*x)^(1/2),x)
[Out] int((a+b*arccos(c*x))^3*(d*x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx=\int \sqrt{dx}{\left(b\arccos\left(cx\right)+a\right)}^{3}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^3*(d*x)^(1/2),x, algorithm="fricas")
[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) +
 a^3)*sqrt(d*x), x)
```

Sympy [N/A]

Not integrable

Time = 8.49 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \sqrt{dx}(a+b\arccos(cx))^3\,dx=\int \sqrt{dx}\left(a+b\,\mathrm{acos}\left(cx\right)\right)^{3}\,dx$$

```
[In] integrate((a+b*acos(c*x))**3*(d*x)**(1/2),x)
[Out] Integral(sqrt(d*x)*(a + b*acos(c*x))**3, x)
```

Maxima [N/A]

Not integrable

Time = 3.42 (sec) , antiderivative size = 418, normalized size of antiderivative = 23.22

$$\int \sqrt{dx}(a + b\arccos(cx))^3 \, dx = \int \sqrt{dx}(b\arccos(cx) + a)^3 \, dx$$

```
[In] integrate((a+b*arccos(c*x))^3*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/3*b^3*sqrt(d)*x^(3/2)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + 1/6*
a^3*c^2*sqrt(d)*(4*x^(3/2)/c^2 + 6*arctan(sqrt(c)*sqrt(x))/c^(7/2) + 3*log(
(c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(7/2)) + 3*a*b^2*c^2*sqrt(d)
*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*x^2 -
1), x) + 3*a^2*b*c^2*sqrt(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c
*x + 1)/(c*x))/(c^2*x^2 - 1), x) - 2*b^3*c*sqrt(d)*integrate(sqrt(c*x + 1)*
sqrt(-c*x + 1)*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*x^
2 - 1), x) - 1/2*a^3*sqrt(d)*(2*arctan(sqrt(c)*sqrt(x))/c^(3/2) + log((c*sq
rt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/c^(3/2)) - 3*a*b^2*sqrt(d)*integrat
e(sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*x^2 - 1), x) -
3*a^2*b*sqrt(d)*integrate(sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x)
)/(c^2*x^2 - 1), x)
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{dx}(a + b\arccos(cx))^3 \, dx = \text{Exception raised: RuntimeError}$$

```
[In] integrate((a+b*arccos(c*x))^3*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> an error occurred running a Giac command:
INPUT:sage2OUTPUT:sym2poly/r2sym(const gen & e,const index_m & i,const vect
eur & l) Error: Bad Argument Value
```

Mupad [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \sqrt{dx}(a + b\arccos(cx))^3 \, dx = \int (a + b\,\mathrm{acos}(c\,x))^3\,\sqrt{d\,x} \, dx$$

```
[In] int((a + b*acos(c*x))^3*(d*x)^(1/2),x)
```

```
[Out] int((a + b*acos(c*x))^3*(d*x)^(1/2), x)
```

3.217 $\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \frac{2\sqrt{dx}(a+b\arccos(cx))^3}{d} + \frac{6bc\text{Int}\left(\frac{\sqrt{dx}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}},x\right)}{d}$$

```
[Out] 2*(a+b*arccos(c*x))^3*(d*x)^(1/2)/d+6*b*c*Unintegrable((a+b*arccos(c*x))^2*
(d*x)^(1/2)/(-c^2*x^2+1)^(1/2),x)/d
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx$$

```
[In] Int[(a + b*ArcCos[c*x])^3/Sqrt[d*x],x]
```

```
[Out] (2*Sqrt[d*x]*(a + b*ArcCos[c*x])^3)/d + (6*b*c*Defer[Int][(Sqrt[d*x]*(a + b
*ArcCos[c*x])^2)/Sqrt[1 - c^2*x^2], x])/d
```

Rubi steps

$$\text{integral} = \frac{2\sqrt{dx}(a+b\arccos(cx))^3}{d} + \frac{(6bc)\int \frac{\sqrt{dx}(a+b\arccos(cx))^2}{\sqrt{1-c^2x^2}}\,dx}{d}$$

Mathematica [N/A]

Not integrable

Time = 73.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a + b \arccos(cx))^3}{\sqrt{dx}}\, dx = \int \frac{(a + b \arccos(cx))^3}{\sqrt{dx}}\, dx$$

```
[In] Integrate[(a + b*ArcCos[c*x])^3/Sqrt[d*x],x]
[Out] Integrate[(a + b*ArcCos[c*x])^3/Sqrt[d*x], x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(a + b \arccos (cx))^3}{\sqrt{dx}} dx$$

```
[In] int((a+b*arccos(c*x))^3/(d*x)^(1/2),x)
[Out] int((a+b*arccos(c*x))^3/(d*x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{(a + b \arccos(cx))^3}{\sqrt{dx}}\, dx = \int \frac{(b \arccos (cx) + a)^3}{\sqrt{dx}}\, dx$$

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(1/2),x, algorithm="fricas")
[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) +
 a^3)*sqrt(d*x)/(d*x), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*acos(c*x))**3/(d*x)**(1/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [N/A]

Not integrable

Time = 3.39 (sec) , antiderivative size = 458, normalized size of antiderivative = 25.44

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{\sqrt{dx}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(1/2),x, algorithm="maxima")
[Out] 1/2*(4*b^3*sqrt(x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + (a^3*c^2*
sqrt(d)*(4*sqrt(x)/(c^2*d) - 2*arctan(sqrt(c)*sqrt(x))/(c^(5/2)*d) + log((c
*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/(c^(5/2)*d)) + 6*a*b^2*c^2*sqrt(
d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*d*x^
3 - d*x), x) + 6*a^2*b*c^2*sqrt(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*s
qrt(-c*x + 1)/(c*x))/(c^2*d*x^3 - d*x), x) - 12*b^3*c*sqrt(d)*integrate(sqr
t(c*x + 1)*sqrt(-c*x + 1)*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x)
)^2/(c^2*d*x^3 - d*x), x) + a^3*sqrt(d)*(2*arctan(sqrt(c)*sqrt(x))/(sqrt(c)
*d) - log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/(sqrt(c)*d)) - 6*a*b
^2*sqrt(d)*integrate(sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(
c^2*d*x^3 - d*x), x) - 6*a^2*b*sqrt(d)*integrate(sqrt(x)*arctan(sqrt(c*x +
1)*sqrt(-c*x + 1)/(c*x))/(c^2*d*x^3 - d*x), x))*sqrt(d))/sqrt(d)
```

Giac [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{\sqrt{dx}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^3/sqrt(d*x), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{\sqrt{dx}}\,dx=\int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^3}{\sqrt{d\,x}}\,dx$$

```
[In] int((a + b*acos(c*x))^3/(d*x)^(1/2),x)
[Out] int((a + b*acos(c*x))^3/(d*x)^(1/2), x)
```

3.218 $\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = -\frac{2(a+b\arccos(cx))^3}{d\sqrt{dx}} - \frac{6bc\text{Int}\left(\frac{(a+b\arccos(cx))^2}{\sqrt{dx}\sqrt{1-c^2x^2}},x\right)}{d}$$

```
[Out] -2*(a+b*arccos(c*x))^3/d/(d*x)^(1/2)-6*b*c*Unintegrable((a+b*arccos(c*x))^2
/(d*x)^(1/2)/(-c^2*x^2+1)^(1/2),x)/d
```

Rubi [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx$$

```
[In] Int[(a + b*ArcCos[c*x])^3/(d*x)^(3/2),x]
```

```
[Out] (-2*(a + b*ArcCos[c*x])^3)/(d*Sqrt[d*x]) - (6*b*c*Defer[Int][(a + b*ArcCos[
c*x])^2/(Sqrt[d*x]*Sqrt[1 - c^2*x^2]), x])/d
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\arccos(cx))^3}{d\sqrt{dx}} - \frac{(6bc)\int \frac{(a+b\arccos(cx))^2}{\sqrt{dx}\sqrt{1-c^2x^2}}\,dx}{d}$$

Mathematica [N/A]

Not integrable

Time = 51.88 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(3/2),x]
[Out] Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(3/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 2.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{\frac{3}{2}}}dx$$

```
[In] int((a+b*arccos(c*x))^3/(d*x)^(3/2),x)
[Out] int((a+b*arccos(c*x))^3/(d*x)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(3/2),x, algorithm="fricas")
[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) +
 a^3)*sqrt(d*x)/(d^2*x^2), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

```
[In] integrate((a+b*acos(c*x))**3/(d*x)**(3/2),x)
[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo
```

Maxima [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 489, normalized size of antiderivative = 27.17

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(3/2),x, algorithm="maxima")
[Out] -1/2*(4*b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 - (a^3*c^2*sqrt(d)
*(2*arctan(sqrt(c)*sqrt(x))/(c^(3/2)*d^2) + log((c*sqrt(x) - sqrt(c))/(c*sq
rt(x) + sqrt(c)))/(c^(3/2)*d^2)) + 6*a*b^2*c^2*sqrt(d)*integrate(x^(5/2)*ar
ctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*d^2*x^4 - d^2*x^2), x) + 6*
a^2*b*c^2*sqrt(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*
x))/(c^2*d^2*x^4 - d^2*x^2), x) + 12*b^3*c*sqrt(d)*integrate(sqrt(c*x + 1)*
sqrt(-c*x + 1)*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*d^
2*x^4 - d^2*x^2), x) - a^3*sqrt(d)*(2*sqrt(c)*arctan(sqrt(c)*sqrt(x))/d^2 +
 sqrt(c)*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/d^2 + 4/(d^2*sqrt
(x))) - 6*a*b^2*sqrt(d)*integrate(sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x +
1)/(c*x))^2/(c^2*d^2*x^4 - d^2*x^2), x) - 6*a^2*b*sqrt(d)*integrate(sqrt(x)
*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^2*x^4 - d^2*x^2), x))*d^
(3/2)*sqrt(x))/(d^(3/2)*sqrt(x))
```

Giac [N/A]

Not integrable

Time = 0.80 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{3}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(3/2),x, algorithm="giac")
[Out] integrate((b*arccos(c*x) + a)^3/(d*x)^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{3/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^3}{{\left(d\,x\right)}^{3/2}} \,d x$$

```
[In] int((a + b*acos(c*x))^3/(d*x)^(3/2),x)
[Out] int((a + b*acos(c*x))^3/(d*x)^(3/2), x)
```

3.219 $\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx = -\frac{2(a+b\arccos(cx))^3}{3d(dx)^{3/2}} - \frac{2bc\text{Int}\left(\frac{(a+b\arccos(cx))^2}{(dx)^{3/2}\sqrt{1-c^2x^2}},x\right)}{d}$$

```
[Out] -2/3*(a+b*arccos(c*x))^3/d/(d*x)^(3/2)-2*b*c*Unintegrable((a+b*arccos(c*x))
^2/(d*x)^(3/2)/(-c^2*x^2+1)^(1/2),x)/d
```

Rubi [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx = \int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx$$

```
[In] Int[(a + b*ArcCos[c*x])^3/(d*x)^(5/2),x]
```

```
[Out] (-2*(a + b*ArcCos[c*x])^3)/(3*d*(d*x)^(3/2)) - (2*b*c*Defer[Int][(a + b*Arc
Cos[c*x])^2/((d*x)^(3/2)*Sqrt[1 - c^2*x^2]), x])/d
```

Rubi steps

$$\text{integral} = -\frac{2(a+b\arccos(cx))^3}{3d(dx)^{3/2}} - \frac{(2bc)\int \frac{(a+b\arccos(cx))^2}{(dx)^{3/2}\sqrt{1-c^2x^2}}\,dx}{d}$$

Mathematica [N/A]

Not integrable

Time = 37.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx = \int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx$$

```
[In] Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(5/2),x]
[Out] Integrate[(a + b*ArcCos[c*x])^3/(d*x)^(5/2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 1.59 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{\frac{5}{2}}}dx$$

```
[In] int((a+b*arccos(c*x))^3/(d*x)^(5/2),x)
[Out] int((a+b*arccos(c*x))^3/(d*x)^(5/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.78

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{5}{2}}}\,dx$$

```
[In] integrate((a+b*arccos(c*x))^3/(d*x)^(5/2),x, algorithm="fricas")
[Out] integral((b^3*arccos(c*x)^3 + 3*a*b^2*arccos(c*x)^2 + 3*a^2*b*arccos(c*x) +
 a^3)*sqrt(d*x)/(d^3*x^3), x)
```

Sympy [F(-2)]

Exception generated.

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx = \text{Exception raised: TypeError}$$

[In] integrate((a+b*acos(c*x))**3/(d*x)**(5/2),x)

[Out] Exception raised: TypeError >> Invalid comparison of non-real zoo

Maxima [N/A]

Not integrable

Time = 3.47 (sec) , antiderivative size = 491, normalized size of antiderivative = 27.28

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{5}{2}}}\,dx$$

[In] integrate((a+b*arccos(c*x))^3/(d*x)^(5/2),x, algorithm="maxima")

[Out] -1/6*(4*b^3*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)^3 + (3*a^3*c^2*sqrt(d)*(2*arctan(sqrt(c)*sqrt(x))/(sqrt(c)*d^3) - log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/(sqrt(c)*d^3)) - 18*a*b^2*c^2*sqrt(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*d^3*x^5 - d^3*x^3), x) - 18*a^2*b*c^2*sqrt(d)*integrate(x^(5/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^3*x^5 - d^3*x^3), x) - 12*b^3*c*sqrt(d)*integrate(sqrt(c*x + 1)*sqrt(-c*x + 1)*x^(3/2)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*d^3*x^5 - d^3*x^3), x) - a^3*sqrt(d)*(6*c^(3/2)*arctan(sqrt(c)*sqrt(x))/d^3 - 3*c^(3/2)*log((c*sqrt(x) - sqrt(c))/(c*sqrt(x) + sqrt(c)))/d^3 - 4/(d^3*x^(3/2))) + 18*a*b^2*sqrt(d)*integrate(sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))^2/(c^2*d^3*x^5 - d^3*x^3), x) + 18*a^2*b*sqrt(d)*integrate(sqrt(x)*arctan(sqrt(c*x + 1)*sqrt(-c*x + 1)/(c*x))/(c^2*d^3*x^5 - d^3*x^3), x))*d^(5/2)*x^(3/2))/(d^(5/2)*x^(3/2))

Giac [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx = \int \frac{(b\arccos(cx)+a)^3}{(dx)^{\frac{5}{2}}}\,dx$$

[In] integrate((a+b*arccos(c*x))^3/(d*x)^(5/2),x, algorithm="giac")

[Out] integrate((b*arccos(c*x) + a)^3/(d*x)^(5/2), x)

Mupad [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\arccos(cx))^3}{(dx)^{5/2}}\,dx=\int \frac{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^3}{{\left(d\,x\right)}^{5/2}}\,dx$$

```
[In] int((a + b*acos(c*x))^3/(d*x)^(5/2),x)
[Out] int((a + b*acos(c*x))^3/(d*x)^(5/2), x)
```

3.220 $\int \frac{(dx)^{3/2}}{a+b \arccos(cx)} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^{3/2}}{a + b \arccos(cx)} \, dx = \text{Int}\left(\frac{(dx)^{3/2}}{a + b \arccos(cx)}, x\right)$$

```
[Out] Unintegrable((d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^{3/2}}{a + b \arccos(cx)} \, dx = \int \frac{(dx)^{3/2}}{a + b \arccos(cx)} \, dx$$

```
[In] Int[(d*x)^(3/2)/(a + b*ArcCos[c*x]),x]
[Out] Defer[Int][(d*x)^(3/2)/(a + b*ArcCos[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^{3/2}}{a + b \arccos(cx)} \, dx$$

Mathematica [N/A]

Not integrable

Time = 2.90 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx$$

```
[In] Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x]),x]
[Out] Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(dx)^{\frac{3}{2}}}{a+b\arccos\left(cx\right)}dx$$

```
[In] int((d*x)^(3/2)/(a+b*arccos(c*x)),x)
[Out] int((d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{b\arccos\left(cx\right)+a}\,dx$$

```
[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(d*x)*d*x/(b*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 4.13 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{a+b\operatorname{acos}\left(cx\right)}\,dx$$

```
[In] integrate((d*x)**(3/2)/(a+b*acos(c*x)),x)
[Out] Integral((d*x)**(3/2)/(a + b*acos(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{b\arccos{(cx)}+a}\,dx$$

```
[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
[Out] integrate((d*x)^(3/2)/(b*arccos(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{b\arccos{(cx)}+a}\,dx$$

```
[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
[Out] integrate((d*x)^(3/2)/(b*arccos(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{a+b\arccos(cx)}\,dx=\int \frac{(d\,x)^{3/2}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

```
[In] int((d*x)^(3/2)/(a + b*acos(c*x)),x)
[Out] int((d*x)^(3/2)/(a + b*acos(c*x)), x)
```

3.221 $\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx = \text{Int}\left(\frac{\sqrt{dx}}{a+b\arccos(cx)},x\right)$$

```
[Out] Unintegrable((d*x)^(1/2)/(a+b*arccos(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx = \int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx$$

```
[In] Int[Sqrt[d*x]/(a + b*ArcCos[c*x]),x]
```

```
[Out] Defer[Int][Sqrt[d*x]/(a + b*ArcCos[c*x]), x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx$$

Mathematica [N/A]

Not integrable

Time = 2.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{dx}}{a + b \arccos(cx)} \, dx = \int \frac{\sqrt{dx}}{a + b \arccos(cx)} \, dx$$

```
[In] Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x]),x]
[Out] Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x]), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.96 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{dx}}{a + b \arccos(cx)} dx$$

```
[In] int((d*x)^(1/2)/(a+b*arccos(c*x)),x)
[Out] int((d*x)^(1/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a + b \arccos(cx)} \, dx = \int \frac{\sqrt{dx}}{b \arccos(cx) + a} \, dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(d*x)/(b*arccos(c*x) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{dx}}{a + b \arccos(cx)} \, dx = \int \frac{\sqrt{dx}}{a + b \operatorname{acos}(cx)} \, dx$$

```
[In] integrate((d*x)**(1/2)/(a+b*acos(c*x)),x)
[Out] Integral(sqrt(d*x)/(a + b*acos(c*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{dx}}{b\arccos(cx)+a}\,dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
[Out] integrate(sqrt(d*x)/(b*arccos(c*x) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{dx}}{b\arccos(cx)+a}\,dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x)),x, algorithm="giac")
[Out] integrate(sqrt(d*x)/(b*arccos(c*x) + a), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{a+b\arccos(cx)}\,dx=\int \frac{\sqrt{d\,x}}{a+b\,\mathrm{acos}\left(c\,x\right)}\,dx$$

```
[In] int((d*x)^(1/2)/(a + b*acos(c*x)),x)
[Out] int((d*x)^(1/2)/(a + b*acos(c*x)), x)
```

3.222 $\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx=\text{Int}\left(\frac{1}{\sqrt{dx}(a+b\arccos(cx))},x\right)$$

[Out] `Unintegrable(1/(a+b*arccos(c*x))/(d*x)^(1/2),x)`

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx=\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx$$

[In] `Int[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])),x]`

[Out] `Defer[Int][1/(Sqrt[d*x]*(a + b*ArcCos[c*x])), x]`

Rubi steps

$$\text{integral}=\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.47 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx$$

```
[In] Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])),x]
[Out] Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.97 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{(a+b\arccos{(cx)})\sqrt{dx}}dx$$

```
[In] int(1/(a+b*arccos(c*x))/(d*x)^(1/2),x)
[Out] int(1/(a+b*arccos(c*x))/(d*x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{dx}(b\arccos{(cx)}+a)}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x)/(b*d*x*arccos(c*x) + a*d*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.45 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{dx}\,(a+b\operatorname{acos}{(cx)})}\,dx$$

```
[In] integrate(1/(a+b*acos(c*x))/(d*x)**(1/2),x)
[Out] Integral(1/(sqrt(d*x)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{dx}(b\arccos{(cx)}+a)}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(d*x)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx = \int \frac{1}{\sqrt{dx}(b\arccos{(cx)}+a)}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\sqrt{d\,x}}\,dx$$

```
[In] int(1/((a + b*acos(c*x))*(d*x)^(1/2)),x)
[Out] int(1/((a + b*acos(c*x))*(d*x)^(1/2)), x)
```

3.223 $\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx = \text{Int}\left(\frac{1}{(dx)^{3/2}(a+b\arccos(cx))},x\right)$$

```
[Out] Unintegrable(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx$$

```
[In] Int[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])),x]
[Out] Defer[Int][1/((d*x)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx$$

Mathematica [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx=\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx$$

```
[In] Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])),x]
[Out] Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.94 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+b\arccos\left(cx\right)\right)}dx$$

```
[In] int(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x)
[Out] int(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x)
```

Fricas [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx=\int \frac{1}{(dx)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)}\,dx$$

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="fricas")
[Out] integral(sqrt(d*x)/(b*d^2*x^2*arccos(c*x) + a*d^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.53 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx=\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+b\operatorname{acos}\left(cx\right)\right)}\,dx$$

```
[In] integrate(1/(d*x)**(3/2)/(a+b*acos(c*x)),x)
[Out] Integral(1/((d*x)**(3/2)*(a + b*acos(c*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(b\arccos{(cx)}+a)}\,dx$$

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="maxima")
[Out] integrate(1/((d*x)^(3/2)*(b*arccos(c*x) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(b\arccos{(cx)}+a)}\,dx$$

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x)),x, algorithm="giac")
[Out] integrate(1/((d*x)^(3/2)*(b*arccos(c*x) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))}\,dx = \int \frac{1}{(a+b\,\mathrm{acos}\,(c\,x))\,\,(d\,x)^{3/2}}\,dx$$

```
[In] int(1/((a + b*acos(c*x))*(d*x)^(3/2)),x)
[Out] int(1/((a + b*acos(c*x))*(d*x)^(3/2)), x)
```

3.224 $\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \text{Int}\left(\frac{(dx)^{3/2}}{(a+b\arccos(cx))^2},x\right)$$

```
[Out] Unintegrable((d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx$$

```
[In] Int[(d*x)^(3/2)/(a + b*ArcCos[c*x])^2,x]
[Out] Defer[Int][(d*x)^(3/2)/(a + b*ArcCos[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 16.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx$$

```
[In] Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x])^2,x]
[Out] Integrate[(d*x)^(3/2)/(a + b*ArcCos[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.53 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(dx)^{\frac{3}{2}}}{(a+b\arccos(cx))^2}dx$$

```
[In] int((d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
[Out] int((d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(dx)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

```
[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(d*x)*d*x/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 10.48 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(a+b\,\mathrm{acos}\,(cx))^2}\,dx$$

[In] integrate((d*x)**(3/2)/(a+b*acos(c*x))**2,x)

[Out] Integral((d*x)**(3/2)/(a + b*acos(c*x))**2, x)

Maxima [N/A]

Not integrable

Time = 1.85 (sec) , antiderivative size = 181, normalized size of antiderivative = 10.06

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")

[Out] (sqrt(c*x + 1)*sqrt(-c*x + 1)*d^(3/2)*x^(3/2) - (b^2*c*arctan2(sqrt(c*x + 1
)*sqrt(-c*x + 1), c*x) + a*b*c)*sqrt(d)*integrate(1/2*(5*c^2*d*x^2 - 3*d)*s
qrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(a*b*c^3*x^2 - a*b*c + (b^2*c^3*x^2 - b
^2*c)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x))/(b^2*c*arctan2(sqrt(
c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)

Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx = \int \frac{(dx)^{\frac{3}{2}}}{(b\arccos(cx)+a)^2}\,dx$$

[In] integrate((d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")

[Out] integrate((d*x)^(3/2)/(b*arccos(c*x) + a)^2, x)

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{(dx)^{3/2}}{(a+b\arccos(cx))^2}\,dx=\int \frac{(d\,x)^{3/2}}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2}\,dx$$

```
[In] int((d*x)^(3/2)/(a + b*acos(c*x))^2,x)
[Out] int((d*x)^(3/2)/(a + b*acos(c*x))^2, x)
```

3.225 $\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx = \operatorname{Int}\left(\frac{\sqrt{dx}}{(a+b\arccos(cx))^2},x\right)$$

```
[Out] Unintegrable((d*x)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx$$

```
[In] Int[Sqrt[d*x]/(a + b*ArcCos[c*x])^2,x]
```

```
[Out] Defer[Int][Sqrt[d*x]/(a + b*ArcCos[c*x])^2, x]
```

Rubi steps

$$\text{integral} = \int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 16.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx$$

```
[In] Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x])^2,x]
[Out] Integrate[Sqrt[d*x]/(a + b*ArcCos[c*x])^2, x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.97 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{\sqrt{dx}}{\left(a+b\arccos\left(cx\right)\right)^{2}}dx$$

```
[In] int((d*x)^(1/2)/(a+b*arccos(c*x))^2,x)
[Out] int((d*x)^(1/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx = \int \frac{\sqrt{dx}}{\left(b\arccos\left(cx\right)+a\right)^{2}}\,dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(d*x)/(b^2*arccos(c*x)^2 + 2*a*b*arccos(c*x) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{dx}}{(a + b \arccos(cx))^2} \, dx = \int \frac{\sqrt{dx}}{(a + b \operatorname{acos}(cx))^2} \, dx$$

```
[In] integrate((d*x)**(1/2)/(a+b*acos(c*x))**2,x)
[Out] Integral(sqrt(d*x)/(a + b*acos(c*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 1.88 (sec) , antiderivative size = 181, normalized size of antiderivative = 10.06

$$\int \frac{\sqrt{dx}}{(a + b \arccos(cx))^2} \, dx = \int \frac{\sqrt{dx}}{(b \arccos(cx) + a)^2} \, dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
[Out] -((b^2*c*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)*sqrt(d)*integr
ate(1/2*(3*c^2*x^2 - 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(a*b*c^3*x^3 -
 a*b*c*x + (b^2*c^3*x^3 - b^2*c*x)*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*
x)), x) - sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*sqrt(x))/(b^2*c*arctan2(sqrt
(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{(a + b \arccos(cx))^2} \, dx = \int \frac{\sqrt{dx}}{(b \arccos(cx) + a)^2} \, dx$$

```
[In] integrate((d*x)^(1/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
[Out] integrate(sqrt(d*x)/(b*arccos(c*x) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{dx}}{(a+b\arccos(cx))^2}\,dx=\int \frac{\sqrt{d\,x}}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2} \,d x$$

```
[In] int((d*x)^(1/2)/(a + b*acos(c*x))^2,x)
[Out] int((d*x)^(1/2)/(a + b*acos(c*x))^2, x)
```

3.226 $\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{\sqrt{dx}(a+b\arccos(cx))^2},x\right)$$

`[Out] Unintegrable(1/(a+b*arccos(c*x))^2/(d*x)^(1/2),x)`

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx$$

`[In] Int[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2),x]`

`[Out] Defer[Int][1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2), x]`

Rubi steps

$$\text{integral} = \int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 40.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx$$

```
[In] Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2),x]
[Out] Integrate[1/(Sqrt[d*x]*(a + b*ArcCos[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.98 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(a+b\arccos\left(cx\right)\right)^{2}\sqrt{dx}}dx$$

```
[In] int(1/(a+b*arccos(c*x))^2/(d*x)^(1/2),x)
[Out] int(1/(a+b*arccos(c*x))^2/(d*x)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.17

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{\sqrt{dx}\left(b\arccos\left(cx\right)+a\right)^{2}}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x)/(b^2*d*x*arccos(c*x)^2 + 2*a*b*d*x*arccos(c*x) + a^2*d*x
), x)
```

Sympy [N/A]

Not integrable

Time = 4.30 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}\,(a+b\operatorname{acos}(cx))^2}\,dx$$

```
[In] integrate(1/(a+b*acos(c*x))**2/(d*x)**(1/2),x)
[Out] Integral(1/(sqrt(d*x)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.68 (sec) , antiderivative size = 196, normalized size of antiderivative = 10.89

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}(b\arccos(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="maxima")
[Out] -((b^2*c*d*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*d*x)*sqrt(d
)*integrate(1/2*(c^2*x^2 + 1)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(a*b*c^3
*d*x^4 - a*b*c*d*x^2 + (b^2*c^3*d*x^4 - b^2*c*d*x^2)*arctan2(sqrt(c*x + 1)*
sqrt(-c*x + 1), c*x)), x) - sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(d)*sqrt(x))/(
b^2*c*d*x*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*d*x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{\sqrt{dx}(b\arccos(cx)+a)^2}\,dx$$

```
[In] integrate(1/(a+b*arccos(c*x))^2/(d*x)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(d*x)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{dx}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2\,\sqrt{d\,x}}\,dx$$

```
[In] int(1/((a + b*acos(c*x))^2*(d*x)^(1/2)),x)
[Out] int(1/((a + b*acos(c*x))^2*(d*x)^(1/2)), x)
```

3.227 $\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \operatorname{Int}\left(\frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx$$

```
[In] Int[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2),x]
[Out] Defer[Int][1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Rubi steps

$$\text{integral} = \int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx$$

Mathematica [N/A]

Not integrable

Time = 22.86 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx$$

```
[In] Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2),x]
[Out] Integrate[1/((d*x)^(3/2)*(a + b*ArcCos[c*x])^2), x]
```

Maple [N/A] (verified)

Not integrable

Time = 0.98 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{(dx)^{\frac{3}{2}}\left(a+b\arccos\left(cx\right)\right)^{2}}dx$$

```
[In] int(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
[Out] int(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.83

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\left(b\arccos\left(cx\right)+a\right)^{2}}\,dx$$

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="fricas")
[Out] integral(sqrt(d*x)/(b^2*d^2*x^2*arccos(c*x)^2 + 2*a*b*d^2*x^2*arccos(c*x) +
 a^2*d^2*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 11.73 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(a+b\operatorname{acos}{(cx)})^2}\,dx$$

```
[In] integrate(1/(d*x)**(3/2)/(a+b*acos(c*x))**2,x)
[Out] Integral(1/((d*x)**(3/2)*(a + b*acos(c*x))**2), x)
```

Maxima [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 218, normalized size of antiderivative = 12.11

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(b\arccos{(cx)}+a)^2}\,dx$$

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="maxima")
[Out] ((b^2*c*d^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a*b*c*d^2*x^2)
*sqrt(d)*integrate(1/2*(c^2*x^2 - 3)*sqrt(c*x + 1)*sqrt(-c*x + 1)*sqrt(x)/(
a*b*c^3*d^2*x^5 - a*b*c*d^2*x^3 + (b^2*c^3*d^2*x^5 - b^2*c*d^2*x^3)*arctan2
(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x)), x) + sqrt(c*x + 1)*sqrt(-c*x + 1)*sqr
t(d)*sqrt(x))/(b^2*c*d^2*x^2*arctan2(sqrt(c*x + 1)*sqrt(-c*x + 1), c*x) + a
*b*c*d^2*x^2)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx = \int \frac{1}{(dx)^{\frac{3}{2}}\,(b\arccos{(cx)}+a)^2}\,dx$$

```
[In] integrate(1/(d*x)^(3/2)/(a+b*arccos(c*x))^2,x, algorithm="giac")
[Out] integrate(1/((d*x)^(3/2)*(b*arccos(c*x) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx)^{3/2}(a+b\arccos(cx))^2}\,dx=\int \frac{1}{{\left(a+b\,\mathrm{acos}\left(c\,x\right)\right)}^2\,{\left(d\,x\right)}^{3/2}}\,dx$$

```
[In] int(1/((a + b*acos(c*x))^2*(d*x)^(3/2)),x)
[Out] int(1/((a + b*acos(c*x))^2*(d*x)^(3/2)), x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCou
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count is
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"<
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<>T
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
```

```
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
```

```
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
```

```
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 (",
```

```
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well")
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_coun
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:
```

```
# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
```

```
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-t
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expn.
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
     m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
     m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
     return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
     return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
     m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
     return max(4,m1)    #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
     m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
     return max(5,m1)    #max(5,m1)
  elif is_appell_function(expn.operator()):
     m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
     return max(6,m1)    #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
           #is checked before calling the grading function that is passed.
           #but kept it here.
     m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
     return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
     return 9

#main function
def grade_antiderivative(result,optimal):



  if debug:
     print ("Enter grade_antiderivative for sagemath")
     print("Enter grade_antiderivative, result=",result)
     print("Enter grade_antiderivative, optimal=",optimal)
     print("type(anti)=",type(result))
     print("type(optimal)=",type(optimal))

  leaf_count_result  = tree_size(result) #leaf_count(result)
  leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

  #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)



  expnType_result  = expnType(result)
  expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger th
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_c
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resul


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```